Design of Two Way Relay Network Using Space-Time Block Coded Network Coding and Linear Detection

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***Abstract*—This paper presents a design of two-way two-hop relay network using physical layer network coding (PNC) in which multiple antennas are used at all nodes. For transmission over the multiple-input multiple-output channels, the Alamouti's space-time block code (STBC) is used for transmission while linear detection is used for signal estimation at all nodes. In order to facilitate linear detection, we develop an equivalent multiuser STBC model for the proposed network and design the sum-and-difference matrix which is suitable for signal estimation. Simulation results show that the proposed network achieves diversity order 2 with polynomial complexity.**

I. INTRODUCTION

Wireless ad-hoc networks are becoming more popular recently. It is easy to find their deployment in various applications such as distributed computing, search-and-rescue supports, inter-vehicle communications, as well as military deployment. A most important feature of wireless ad-hoc networks is capability to exchange information among network nodes with the help of intermediate nodes without requiring existing infrastructure. Communication in such relay networks is done in a two-way, also referred to as bidirectional, fashion. In order to improve network throughput as well as transmission reliability, large research efforts were paid on two-way relay networks [1]–[11]. A typical example of such a two-way relay network is the simple two-hop three-node network, where the two end nodes exchange their data with each other via a relay. For this network, physical-layer network coding (PNC) is known as an effective scheme to improve network throughput by exploit the broadcast nature of wireless channels [1]–[3],[5],[7]–[11].

Meanwhile, multiple-input multiple-output (MIMO) transmission techniques are also known as an effective method to achieve improved channel capacity and signal reception quality [12]–[14]. Thus a suitable combination of MIMO and PNC will promise further improvement in network throughput and system performance. Various MIMO-PNC schemes for two-way relaying channels can be found in the literature. These schemes can be roughly classified into 2 groups, namely distributed MIMO-PNC and full MIMO-PNC. In the distributed MIMO-PNC systems, the end nodes have two antennas while the relay has only single [4] or both end nodes have only single antenna while the relay has two antennas [5],[9],[10]. In order to differentiate the distributed with the original MIMO systems we will call the former systems MISO-PNC[1] and the latter SIMO-PNC[2]. In the full MIMO-PNC system multiple, commonly two, antennas are employed at all nodes [6]–[8],[11]. In this paper, we are interested in the full MIMO-PNC system and will call it simply by MIMO-PNC. Specifically, we aim at designing a simple two-way relay network using MIMO-PNC in order to achieve improved system performance with low complexity at all nodes. In order to fulfil this objective the Alamouti space-time block code (STBC) is used for MIMO encoding at both the end nodes as well as the relay. In order to reduce complexity the linear-based estimation of network coded symbols proposed in [5] will be used at the relay while only simple linear processing is proposed at the two end nodes for signal recovery. The difference of our proposed system to the previous MIMO-PNC ones is explained as follows. In the proposed MIMO-PNC system in [6] all nodes employ the Alamouti STBC for MIMO transmission. During the multiple access (MA) phase the relay (R) receives the transmitted signals from both the end nodes at the same time. The relay R in [6] used maximum likelihood (ML) detection to estimate the transmitted symbols from each antenna of each end node, i.e., $\left\langle s_1^{(1)}, s_2^{(1)} \right\rangle$ from node 1 (N_1) and $\left\langle s_1^{(2)}, s_2^{(2)} \right\rangle$ from node 2 (N_2). Network coding is then proposed to encode each of these pairs and broadcast them to N_1 and N_2 using the Alamouti STBC. ML detection is again used to recover the transmitted symbols from the partner at both N_1 and N_2. In our system, whereas, linear detection based on zero-forcing (ZF) or minimum mean squared error (MMSE) criterion is used to estimate the sum and difference of the corresponding pair of transmitted symbols from the two antennas of N_1 and N_2 at R, i.e., $\left\langle s_1^{(1)} + s_1^{(2)}, s_1^{(1)} - s_1^{(2)} \right\rangle$ and $\left\langle s_2^{(1)} + s_2^{(2)}, s_2^{(1)} - s_2^{(2)} \right\rangle$. Log-likelihood ratio based estimation or selective combining as proposed by [5] is then used to convert these sum and difference statistics into network coded symbols which have the following forms: $\left\langle s_1^{(1)} \oplus s_1^{(2)}, s_2^{(1)} \oplus s_2^{(2)} \right\rangle$. Due to this XOR network coding, only simple fading compensation and then XOR is used to recover the transmitted symbols. The system in [7] combines channel coding and PNC for the Alamouti STBC two-way relay system. In order to avoid the ML detection, pre-coding is designed for the two end nodes to facilitate multiuser

[1]Multiple Input Single Output
[2]Single Input Multiple Output

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detection during the MA phase. Our system, however, does not require complicated precoding for the two end nodes N_1 and N_2. In contrast, we design a simple sum-and-difference matrix to support signal detection at R. The MIMO-PNC system in [8] also considered the combination of STBC with PNC but for multi-hop relay network. Recently, we have also proposed a MIMO-PNC system in [11]. The proposed system, however, used spatial division multiplexing (SDM) for transmission in order to achieve multiplexing gain and thus different from the current proposal.

The contributions of the paper can be summarized as follows. A general model of multiuser STBC system which can adapt it to linear based estimation is developed for both MA and BC phase. In order to separate the multiuser signal at the relay, the sum-difference matrix is designed to separate each pair of signal components from respective antennas $i = 1, 2$ of the two end nodes, i.e., $s_i^{(1)} + s_i^{(2)}$ and $s_i^{(1)} - s_i^{(2)}$. We propose an effective network coding scheme which facilitates signal estimation using only simple fading compensation at the end nodes. Computer simulations are carried out and complexity analysis is given to show the efficiency of the proposed network.

The remainder of the paper is organized as follows. Sect. II presents a general model of MIMO-PNC for two-way relay channels. The proposed design of two-way relay MIMO communication network using STBC-PNC is described in Sect. III. Performance evaluation is presented in Sect.IV and, finally, conclusions are drawn in Sect. V.

II. System Model of Physical Layer Network Coding for Two-Way Relay MIMO Channels

We consider a network exchanging data over a two-way relay MIMO channel as illustrated in Fig 1. The network consists of two end nodes, denoted by N_1 and N_2, communicating with each other via the help of a relay (relaying node) R in semi-duplex mode. In a general model, all nodes are assumed to be equipped with multiple antennas. However, for simplicity, we assume further that the number of antennas of each node is the same and equal to 2, $N = 2$. The transmit vectors from each end node k denoted by $\boldsymbol{x}_k = \left[x_1^{(k)}, x_2^{(k)}\right]^T$, where $k = 1, 2$, contain complex symbols obtained from a M-ary modulation such as phase shift keying (PSK) or quadrature amplitude modulation (QAM) and a space-time encoding scheme such as the Alamouti's STBC [12] or spatial division multiplexing (SDM) [14] is used for MIMO communication. The MIMO channels between each end node and the relay, $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, are flat and undergo independent and identically distributed (i.i.d.) Rayleigh fading. Each element $h_{ji}^{(k)}$ of $\boldsymbol{H}_k$, denoting the channel between the i-th antenna of node k and j-th antenna of the relay R, is modelled using a complex Gaussian variable with zero mean and unit variance, i.e., $h_{ji}^{(k)} \sim \mathcal{N}_c(0, 1)$. The noise vector induced at each node receiver is assumed i.i.d. complex Gaussian distributed with mean zero and variance $(\sigma_z^{(k)})^2$. To simplify notation we assume that the noise variances at all nodes are the same, i.e., $\left(\sigma_z^{(1)}\right)^2 = \left(\sigma_z^{(2)}\right)^2 = \left(\sigma_z^{\mathrm{R}}\right)^2 = \sigma_z^2$.

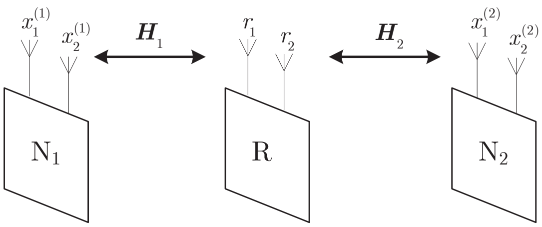


Fig. 1. Network model of a two-way relay MIMO-PNC channels.

The exchange of data between the two end nodes N_1 and N_2 is performed over two phases, i.e., MA and BC, using PNC. It is worth noting that different from a relaying multiple access (RMA) channel, the MA phase in PNC is non-orthogonal. As a result, the relay R will rely on an efficient signal detection scheme in order to recover the transmitted symbols from the two end nodes before encoding them using PNC. The received signal vector at the relay R during the MA phase is given by

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{H}_1\boldsymbol{x}_1 + \boldsymbol{H}_2\boldsymbol{x}_2 + \boldsymbol{z}_{\mathrm{R}} \\ &= \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z}_{\mathrm{R}} \end{aligned} \tag{1}$$

where $\boldsymbol{H} = [\boldsymbol{H}_1 \; \boldsymbol{H}_2] \in \mathbb{C}^{2\times 4}$ and $\boldsymbol{x} = [\boldsymbol{x}_1^T \; \boldsymbol{x}_2^T]^T \in \mathbb{C}^{4\times 1}$ and $\boldsymbol{z}_{\mathrm{R}} = [z_1^{\mathrm{R}} \; z_2^{\mathrm{R}}]^T \in \mathbb{C}^{2\times 1}$.

Upon receiving $\boldsymbol{r}$, the relay R would use a detector to estimate each transmitted symbol $x_1^{(1)}$, $x_2^{(1)}$ and $x_1^{(2)}$, $x_2^{(2)}$. However, in the MIMO-PNC channel instead of estimating each symbol independently the detector can efficiently detect the composite version of transmitted symbols: $x_1^{(1)} + x_1^{(2)}$, $x_1^{(1)} - x_1^{(2)}$ and $x_2^{(1)} + x_2^{(2)}$, $x_2^{(1)} - x_2^{(2)}$. These composite estimated symbols are mapped to the PNC symbols as follows [11]:

$$\left.\begin{matrix} x_1^{(1)} + x_1^{(2)} \\ x_1^{(1)} - x_1^{(2)} \end{matrix}\right\} \rightarrow x_1^{(1)} \oplus x_1^{(2)}, \tag{2}$$

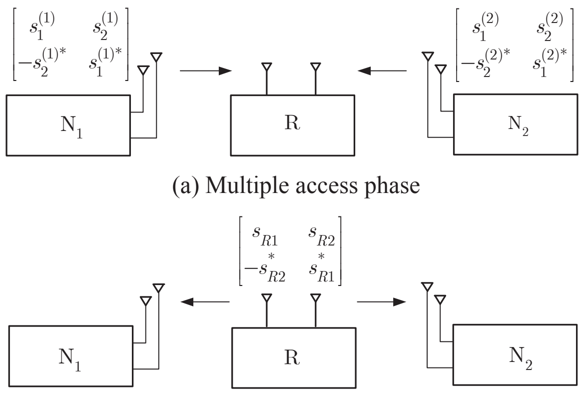
$$\left.\begin{matrix} x_2^{(1)} + x_2^{(2)} \\ x_2^{(1)} - x_2^{(2)} \end{matrix}\right\} \rightarrow x_2^{(1)} \oplus x_2^{(2)}. \tag{3}$$

During the next BC phase, these PNC symbols will be broadcast over the relay two antennas. The first antenna will transmit $s_1^{\mathrm{R}} \triangleq x_1^{(1)} \oplus x_1^{(2)}$ and the second $s_2^{\mathrm{R}} \triangleq x_2^{(1)} \oplus x_2^{(2)}$. The transmit vector of the relay is defined as $\boldsymbol{s}_{\mathrm{R}} \triangleq [s_1^{\mathrm{R}} \; s_2^{\mathrm{R}}]^T$. Assume that the channels between each end node and the relay are reciprocal, the received signal vectors at the two end nodes can be expressed respectively as

$$\boldsymbol{y}_1 = \boldsymbol{H}_1^T\boldsymbol{s}_{\mathrm{R}} + \boldsymbol{z}_1 \tag{4}$$

$$\boldsymbol{y}_2 = \boldsymbol{H}_2^T\boldsymbol{s}_{\mathrm{R}} + \boldsymbol{z}_2. \tag{5}$$

Each end node N_1 and N_2 will then estimate the transmitted PNC vector $\bar{\boldsymbol{s}}_{\mathrm{R}}$. Under the assumption that the estimated vector is correct, the end nodes will simply perform XOR operation of the estimated vector with its own transmitted vector $\boldsymbol{x}_k$ to obtain the transmitted vector from its partner. Specifically, we have $\bar{\boldsymbol{x}}_2 = \bar{\boldsymbol{s}}_{\mathrm{R}} \oplus \boldsymbol{x}_1$ at N_1 and $\bar{\boldsymbol{x}}_1 = \bar{\boldsymbol{s}}_{\mathrm{R}} \oplus \boldsymbol{x}_2$ at N_2



(a) Multiple access phase

(b) Broadcast phase

Fig. 2. System model of a two-way relay MIMO-STBC-PNC channel.

III. Proposed Two-Way Relay MIMO Communication Network Using STBC-PNC

A. Multiple Access Phase

The system of the MIMO-STBC-PNC under consideration is shown in Fig. 2. During the MA phase as shown in Fig. 2(a) transmission is divided into time slots. The two end nodes N_1 and N_2 use the Alamouti's STBC to encode the transmit symbols over two consecutive time slots denoted by t_1 and t_2. The objective of using STBC instead of SDM as in [11] is to achieve diversity gain rather than multiplexing gain. In order to enable the Alamouti's STBC transmission the channel in this case is further assumed to be quasi-static over at least two consecutive time slots. During the first time slot t_1 and second t_2 the two end nodes N_1 and N_2 transmit

$$\boldsymbol{X}_k = \begin{bmatrix} s_1^{(k)} & s_2^{(k)} \\ -s_2^{(k)*} & s_1^{(k)*} \end{bmatrix} \tag{6}$$

where the asterisk * denotes the complex conjugation.

The received signals at the relay R during the first time slot is given by

$$r_{1,t_1} = h_{11}^{(1)} s_1^{(1)} + h_{12}^{(1)} s_2^{(1)} + h_{11}^{(2)} s_1^{(2)} + h_{12}^{(2)} s_2^{(2)} + z_{1,t_1} \tag{7}$$

$$r_{2,t_1} = h_{21}^{(1)} s_1^{(1)} + h_{22}^{(1)} s_2^{(1)} + h_{21}^{(2)} s_1^{(2)} + h_{22}^{(2)} s_2^{(2)} + z_{2,t_1}, \tag{8}$$

and at the second time slot

$$r_{1,t_2} = h_{12}^{(1)} s_1^{(1)*} - h_{11}^{(1)} s_2^{(1)*} + h_{12}^{(2)} s_1^{(2)*} - h_{11}^{(2)} s_2^{(2)*} + z_{1,t_2} \tag{9}$$

$$r_{2,t_2} = h_{22}^{(1)} s_1^{(1)*} - h_{21}^{(1)} s_2^{(1)*} + h_{22}^{(2)} s_1^{(2)*} - h_{21}^{(2)} s_2^{(2)*} + z_{2,t_2} \tag{10}$$

where $r_{j,t_\ell}, i = 1,2, \ell = 1,2$, denotes the received signal at the j-th antenna of the relay during the ℓ-th time slot; $h_{ji}^{(k)}$ the channel between the i-th antenna of end node k and the j-th antenna of the relay; $s_1^{(k)}$ and $s_2^{(k)}$ the two transmit symbols from user k; z_{j,t_ℓ} the AWGN noise component induced at the j-th antenna of the relay during the ℓ-th time slot.

1) Signal Detection at Relay: The process of detecting the transmit symbols from the two end nodes is similar to the multiuser detection of STBC system. Therefore, we can apply the linear multiuser detection of STBC system in [15] to the current system. Taking complex conjugation over the received signal during the second time slot and using the method presented in [15], we can convert equations (7)–(10) into a convenient matrix form of the system equation as follows

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z} \tag{11}$$

where

$$\boldsymbol{s} \triangleq \left[\boldsymbol{s}_1^T \ \boldsymbol{s}_2^T\right]^T = \left[s_1^{(1)},\ s_2^{(1)},\ s_1^{(2)},\ s_2^{(2)}\right]^T \tag{12}$$

$$\boldsymbol{H} \triangleq \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & h_{11}^{(2)} & h_{12}^{(2)} \\ h_{12}^{(1)*} & -h_{11}^{(1)*} & h_{12}^{(2)*} & -h_{11}^{(2)*} \\ h_{21}^{(1)} & h_{22}^{(1)} & h_{21}^{(2)} & h_{22}^{(2)} \\ h_{22}^{(1)*} & -h_{21}^{(1)*} & h_{22}^{(2)*} & -h_{21}^{(2)*} \end{bmatrix} \tag{13}$$

$$\boldsymbol{z} \triangleq \left[z_{1,1},\ z_{1,2}^*,\ z_{2,1},\ z_{2,2}^*\right]^T \tag{14}$$

$$\boldsymbol{r} \triangleq \left[r_{1,1},\ r_{1,2},\ r_{2,1},\ r_{2,2}\right]^T \tag{15}$$

where $\boldsymbol{s}_1 \triangleq \left[s_1^{(1)},\ s_2^{(1)}\right]^T$, $\boldsymbol{s}_2 \triangleq \left[s_1^{(2)},\ s_2^{(2)}\right]^T$ the superscript ^{T} denotes the vector/matrix transposition.

Now as explained in Section II the objective is to estimate the following composite component vector

$$\hat{\boldsymbol{s}} = \begin{bmatrix} s_1^{(1)} + s_1^{(2)} \\ s_2^{(1)} + s_2^{(2)} \\ s_1^{(1)} - s_1^{(2)} \\ s_2^{(1)} - s_2^{(2)} \end{bmatrix} \triangleq \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \end{bmatrix}, \tag{16}$$

and the next step will be mapping this vector to the PNC symbols. Linear estimation can be a good choice for the sake of simple optimization and complexity minimization. In order to facilitate linear estimation we can express (11) as follows

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z} \\ &= (\boldsymbol{H}\boldsymbol{D}^{-1})(\boldsymbol{D}\boldsymbol{s}) + \boldsymbol{z} = \hat{\boldsymbol{H}}\hat{\boldsymbol{s}} + \boldsymbol{z} \end{aligned} \tag{17}$$

where $\boldsymbol{D}$ is the sum-difference matrix, $\hat{\boldsymbol{H}} \triangleq \boldsymbol{H}\boldsymbol{D}^{-1}$ and $\hat{\boldsymbol{s}} \triangleq \boldsymbol{D}\boldsymbol{s}$. Note that the system equation in (17) can be considered as an equivalent form of the MIMO-SDM system in [11] since the transmit matrices $\boldsymbol{X}_k$ in (6) are now converted to a single vector $\boldsymbol{s}$. In order to have the network coded symbols in the form of (2) and (3) the sum-difference matrix $\boldsymbol{D}$ is given by:

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{I}_2 \\ \boldsymbol{I}_2 & -\boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}. \tag{18}$$

Note that this sum-difference matrix has the same form of that for MIMO-SDM-PNC in [11] due to the equivalent model. For the case the number of antennas is larger than 2, i.e. $N > 2$, the sum-difference matrix can be easily extended with $\boldsymbol{I}_N$.

Using the sum-difference matrix $\boldsymbol{D}$ the equivalent transmit vector $\hat{\boldsymbol{s}}$ can be expressed as

$$\hat{\boldsymbol{s}} \triangleq \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \end{bmatrix} = \begin{bmatrix} s_1^{(1)} + s_1^{(2)} \\ s_2^{(1)} + s_2^{(2)} \\ s_1^{(1)} - s_1^{(2)} \\ s_2^{(1)} - s_2^{(2)} \end{bmatrix}. \tag{19}$$

Since the symbols $\hat{s}_1$, $\hat{s}_3$, and symbols $\hat{s}_2$, $\hat{s}_4$ are correlated with each other in pairs and can be easily mapped to $s_1^{(1)} \oplus s_1^{(2)}$, $s_2^{(1)} \oplus s_2^{(2)}$ using the PNC mapping in (2) and (3). From here we can utilize the method for MIMO-SDM-PNC in [11] to estimate the network coded symbols of $\hat{\boldsymbol{s}}$ using both linear based log-likelihood ratio (LLR) and selective combining estimation. First, the linear estimation using ZF and MMSE method has the weight combining matrices given respectively as

$$\boldsymbol{G}^{\mathrm{ZF}} = \left(\hat{\boldsymbol{H}}^H \hat{\boldsymbol{H}}\right)^{-1} \hat{\boldsymbol{H}}^H \tag{20}$$

$$\boldsymbol{G}^{\mathrm{MMSE}} = \left(\hat{\boldsymbol{H}}^H \hat{\boldsymbol{H}} + \sigma_z^2 \boldsymbol{I}_2\right)^{-1} \hat{\boldsymbol{H}}^H. \tag{21}$$

The output of the linear detector is given by

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{r} = \boldsymbol{G}\hat{\boldsymbol{H}}\hat{\boldsymbol{s}} + \boldsymbol{G}\boldsymbol{z} \tag{22}$$

where we have removed the superscripts ZF and MMSE to simplify our presentation. Next let us define

$$\boldsymbol{y} \triangleq \left[y_1^{(1)}\ y_2^{(1)}\ y_1^{(2)}\ y_2^{(2)}\right]^T. \tag{23}$$

Now the LLRs associated with each output of $\boldsymbol{y}$ for BPSK modulation are given by[3]

$$\mathrm{LLR}_1 = \exp\left(\frac{-\left(y_1^{(2)}\right)^2}{2(\sigma_1^{(2)})^2}\right) \times \left[\exp\left(\frac{-\left(y_1^{(1)}-2\right)^2}{2(\sigma_1^{(1)})^2}\right) + \exp\left(\frac{-\left(y_1^{(1)}+2\right)^2}{2(\sigma_1^{(1)})^2}\right)\right] \tag{24}$$

$$\mathrm{LLR}_3 = \exp\left(\frac{-\left(y_1^{(1)}\right)^2}{2(\sigma_1^{(1)})^2}\right) \times \left[\exp\left(\frac{-\left(y_1^{(2)}-2\right)^2}{2(\sigma_1^{(2)})^2}\right) + \exp\left(\frac{-\left(y_1^{(2)}+2\right)^2}{2(\sigma_1^{(2)})^2}\right)\right] \tag{25}$$

$$\mathrm{LLR}_2 = \exp\left(\frac{-\left(y_2^{(2)}\right)^2}{2(\sigma_2^{(2)})^2}\right) \times \left[\exp\left(\frac{-\left(y_2^{(1)}-2\right)^2}{2(\sigma_2^{(1)})^2}\right) + \exp\left(\frac{-\left(y_2^{(1)}+2\right)^2}{2(\sigma_2^{(1)})^2}\right)\right] \tag{26}$$

$$\mathrm{LLR}_4 = \exp\left(\frac{-\left(y_2^{(1)}\right)^2}{2(\sigma_2^{(1)})^2}\right) \times \left[\exp\left(\frac{-\left(y_2^{(2)}-2\right)^2}{2(\sigma_2^{(2)})^2}\right) + \exp\left(\frac{-\left(y_2^{(2)}+2\right)^2}{2(\sigma_2^{(2)})^2}\right)\right] \tag{27}$$

where $\left(\sigma_i^{(k)}\right)^2 = \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{i,i}\sigma_z^2$ denotes variance of the residual noise at the i-th output of the k-th user from the linear detector. The decisions are then made for the network coded symbols as follows:

$$\overline{s_1^{(1)} \oplus s_1^{(2)}} = \begin{cases} -1 & \text{if } \mathrm{LLR}_1 \geq \mathrm{LLR}_3 \\ +1 & \text{if } \mathrm{LLR}_1 < \mathrm{LLR}_3 \end{cases} \tag{28}$$

$$\overline{s_2^{(1)} \oplus s_2^{(2)}} = \begin{cases} -1 & \text{if } \mathrm{LLR}_2 \geq \mathrm{LLR}_4 \\ +1 & \text{if } \mathrm{LLR}_2 < \mathrm{LLR}_4 \end{cases} \tag{29}$$

In the case the selective combining is used in order to reduce further complexity, yet at the cost of reduced performance, the network coded symbols are decided as

$$\overline{s_1^{(1)} \oplus s_1^{(2)}} = \begin{cases} \mathrm{sgn}\left\{|y_1^{(1)}| - \gamma\right\} & \text{if } \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{1,1} < \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{3,3} \\ \mathrm{sgn}\left\{\gamma - |y_1^{(2)}|\right\} & \text{otherwise} \end{cases}$$

$$\overline{s_2^{(1)} \oplus s_2^{(2)}} = \begin{cases} \mathrm{sgn}\left\{|y_2^{(1)}| - \gamma\right\} & \text{if } \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{2,2} < \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{4,4} \\ \mathrm{sgn}\left\{\gamma - |y_2^{(2)}|\right\} & \text{otherwise} \end{cases}$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function

B. Broadcast Phase

Upon obtaining the network coded symbols $\overline{s_1 \oplus s_3}$, $\overline{s_2 \oplus s_4}$, the relay R will broadcast these symbols to the two end nodes N_1 and N_2. This broadcast transmission can be done using either spatial multiplexing as in [11] or STBC as the two end-nodes do. In this paper, we use the STBC as illustrated in Fig. 2(b) in order to achieve diversity gain and reduce the computational complexity of the detectors at the end nodes. Denote the estimated network coded symbols as

$$s_{\mathrm{R}1} \triangleq \overline{s_1 \oplus s_3} \tag{30}$$

$$s_{\mathrm{R}2} \triangleq \overline{s_2 \oplus s_4}. \tag{31}$$

Using the Alamouti's STBC these symbols will be encoded and sent through the two antennas using the encoding matrix $\boldsymbol{X}$ in (6) as

$$\boldsymbol{X}_{\mathrm{R}} = \begin{bmatrix} s_{\mathrm{R}1} & s_{\mathrm{R}2} \\ -s_{\mathrm{R}2}^* & s_{\mathrm{R}1}^* \end{bmatrix}. \tag{32}$$

Each end node will perform linear processing on the received vector and detect the transmitted vector $\boldsymbol{s}_{\mathrm{R}} = [s_{\mathrm{R}1},\ s_{\mathrm{R}2}]^T$ from the relay and then try to recover the transmitted vector from the other end node by performing XOR of the estimated vector with its transmitted vector. Using the equivalent model in (11), we can express the received signal vector at the two end nodes after processing over two time slots as follows

$$\underbrace{\begin{bmatrix} u_{1,t_1}^{(k)} \\ u_{1,t_2}^{(k)*} \\ u_{2,t_1}^{(k)} \\ u_{2,t_2}^{*(k)} \end{bmatrix}}_{\boldsymbol{u}_k} = \underbrace{\begin{bmatrix} h_{11}^{(k)} & h_{12}^{(k)} \\ h_{12}^{(k)*} & -h_{11}^{(k)*} \\ h_{21}^{(k)} & h_{22}^{(k)} \\ h_{22}^{(k)*} & -h_{21}^{(k)*} \end{bmatrix}}_{\boldsymbol{H}_k} \underbrace{\begin{bmatrix} s_{\mathrm{R}1} \\ s_{\mathrm{R}2} \end{bmatrix}}_{\boldsymbol{s}_{\mathrm{R}}} + \underbrace{\begin{bmatrix} n_{1,t_1}^{(k)} \\ n_{1,t_2}^{(k)*} \\ n_{2,t_1}^{(k)} \\ n_{2,t_2}^{(k)*} \end{bmatrix}}_{\boldsymbol{n}_k} \tag{33}$$

where $u_{i,t_\ell}^{(k)}$ and $n_{i,t_\ell}^{(k)}$ denote respectively the received signal and induced noise at the i-th antenna of node k at the t_ℓ time slot; $h_{ij}^{(k)}$ denotes the channel from the j-th antenna of the

[3] Due to limited space, detailed derivation of LLRs is obmitted in this paper.

relay to the i-th antenna of node k. In practical systems, this channel $h_{ij}^{(k)}$ might be different from the inverse channel $h_{ji}^{(k)}$ from node k to the relay. This difference will affect the channel estimation process in each direction. However, for simplicity we assume that all channels are slowly varying such that the channels in both directions stay the same during an end-to-end transmission duration. This assumption is appropriate for ad hoc networks where network nodes operate under nomadic mode. Under this assumption, we can conveniently use the same notation $h_{ji}^{(k)}$ to express the channel in both directions. Using appropriate notations we can simply express (33) as

$$\boldsymbol{u}_k = \boldsymbol{H}_k \boldsymbol{s}_{\mathrm{R}} + \boldsymbol{n}_k. \tag{34}$$

It is noted that since $\boldsymbol{H}_k^H \boldsymbol{H}_k = \sum_i \sum_j |h_{ji}^{(k)}|^2 \boldsymbol{I}_2$ each end node can easily estimate $\boldsymbol{s}_{\mathrm{R}}$ from the received vector $\boldsymbol{u}_k$ by equalizing the fading effect using an equalization matrix $\boldsymbol{F}_k = \boldsymbol{H}_k^H$ as

$$\hat{\boldsymbol{s}}_{\mathrm{R}}^{(k)} = \boldsymbol{H}_k^H \boldsymbol{u}_k = \begin{bmatrix} \hat{s}_{\mathrm{R}1}^{(k)} \\ \hat{s}_{\mathrm{R}2}^{(k)} \end{bmatrix} = \begin{bmatrix} \widehat{s_1 \oplus s_3} \\ \widehat{s_2 \oplus s_4} \end{bmatrix}. \tag{35}$$

A quantization function will be then used to make decision on the transmitted network coded symbols as $\bar{\boldsymbol{s}}_{\mathrm{R}}^{(k)} = \mathcal{Q}(\hat{\boldsymbol{s}}_{\mathrm{R}}^{(k)})$ where each component is independently quantized as

$$\overline{s_1 \oplus s_3} = \mathcal{Q}\big(\widehat{s_1 \oplus s_3}\big) \tag{36}$$

$$\overline{s_2 \oplus s_4} = \mathcal{Q}\big(\widehat{s_2 \oplus s_4}\big) \tag{37}$$

The last operation is to perform XOR of the quantized vector $\bar{\boldsymbol{s}}_{\mathrm{R}}^{(k)}$ with its own transmitted vector $\boldsymbol{s}_k$ to recover the transmitted vector from the other side.

IV. Performance Evaluation

In this section we present our simulation results to evaluate the BER performance of the proposed system. The network model is as illustrated Fig. 2. In our simulations, slowly varying uncorrelated flat Rayleigh fading is assumed. The transmitted bits are modulated using BPSK with the equal average E_s/N_0 at all nodes.

A. Threshold Selection for Selective Combining

In the first investigation, we analyse the effect of choosing the threshold γ for the case of selective combining on the BER performance. The threshold was varied from 0 to 2 for 3 typical values of E_s/N_0, namely, 0 dB, 10 dB and 20 dB. Figs. 3 and 4 show the simulated averaged BER versus E_s/N_0 for the case using ZF and MMSE estimation, respectively. It can be seen clearly from the figures that the best value is $\gamma = 1.2$ for ZF and $\gamma = 1$ for MMSE, particularly at the high E_s/N_0 region. We will use these values in our following simulation for the case of selective combining.

B. BER Performance Analysis

Figs. 5 and 6 show the end-to-end BER performance of the proposed MIMO-STBC-PNC system using ZF and MMSE estimation, respectively. In order to compare with the proposed system in [5], BER performance of SIMO-PNC is also shown

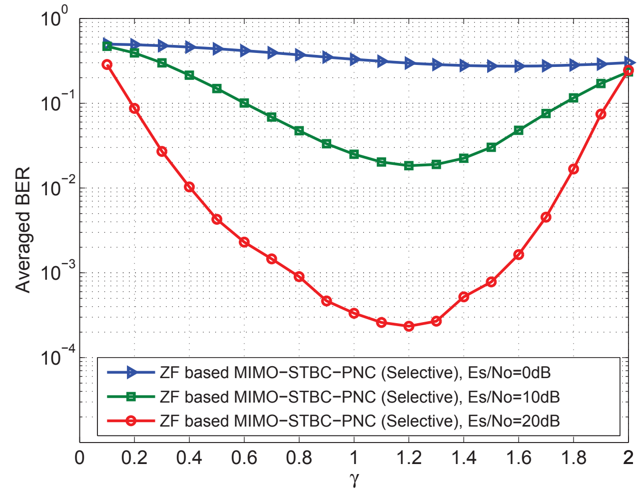


Fig. 3. Threshold variation in MIMO-STBC-PNC system using ZF estimation.

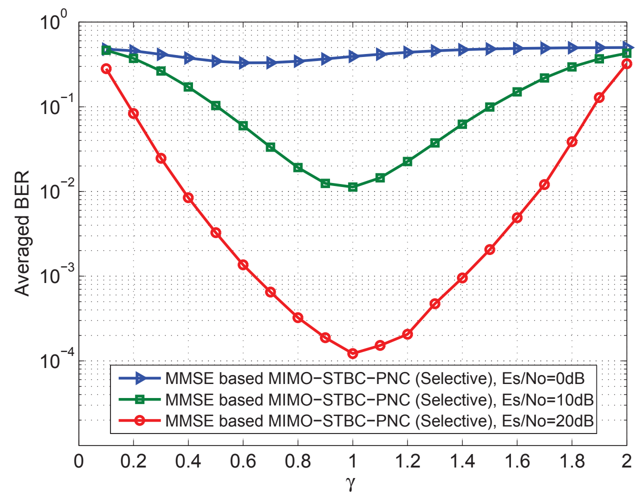


Fig. 4. Threshold variation in MIMO-STBC-PNC system using MMSE estimation.

in the figures for reference. It is shown clearly that the proposed MIMO-STBC-PNC system significantly outperforms the SIMO-PNC system due to having higher diversity order. We can see that our proposed system can double diversity order compared to the SIMO-PNC system in [5]. Apart from achieving the same diversity order of 2, Fig. 6 also indicates the E_s/N_0 gain of about 1 dB over the 2×1 Alamouti STBC if MMSE is used for estimation. Comparing BER curves in Fig.5 and Fig. 6 shows the improvement in performance detection using MMSE over ZF. The difference is about 2.5 dB for the case using LLR and 2 dB for selective combining.

C. Complexity Notes

In order to compare the complexity of our proposed MIMO-STBC-PNC network with the previous MISO-PNC in [5], we note that the increase in computational complexity at the two end nodes is not significant since only linear processing is used for both transmission and reception. The complexity at the

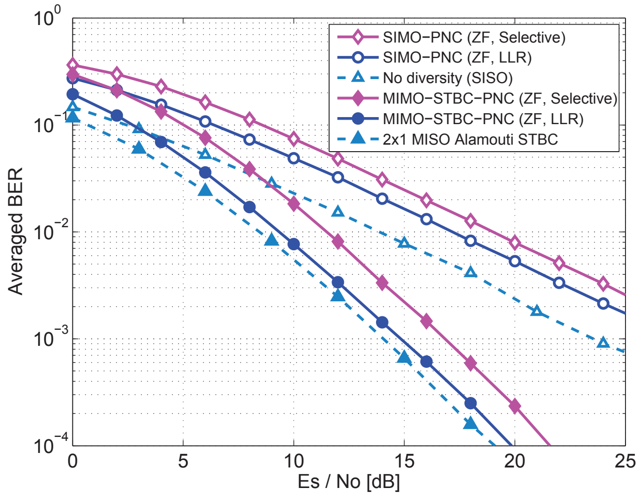


Fig. 5. BER performance of the proposed MIMO-STBC-PNC system using ZF detection at relay.

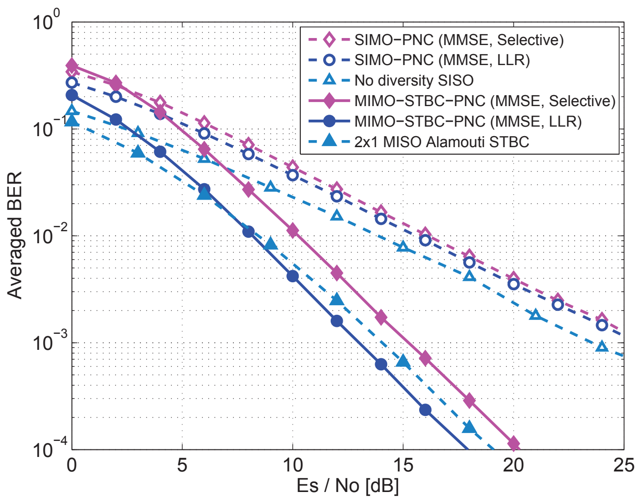


Fig. 6. BER performance of the proposed MIMO-STBC-PNC system using MMSE detection at relay.

relay is, however, increased since the linear detector needs to inverse a larger (double size) matrix to compute the combining weight $\boldsymbol{G}$. The complexity order in our proposed system will be $C_{\text{Proposed}} \sim \mathcal{O}([2N]^3)$. The complexity in the system of [5] is $C_{\text{Ref.[5]}} \sim \mathcal{O}(2[N]^3)$ as the processing is done for two consecutive time slots. As $N = 2$ for the proposed system the increase in complexity is acceptable.

When comparing with the counterpart MIMO-STBC-PNC in [6], we note that both networks use a same system model. The network in [6], however, uses ML detection at all nodes N_1, N_2, and R. As a result, its complexity at all nodes can be expressed as $C_{\text{Ref.[6]}} \sim \mathcal{O}(M^{2N})$. This complexity is an exponential function of the modulation order M and the number of antenna N. Clearly it will become significantly large when larger order modulation $M \geq 4$ is used as well as if the network is extended to the case with more antennas $N > 2$.

V. Conclusions

In this paper we have proposed a design of a two-way relay network using the Alamouti's space-time block coded physical layer network coding. Thanks to the proper design of the sum-difference matrix to separate the multiuser signals the proposed network can use only linear processing at all nodes to estimate signal to save the computational complexity. Using computer simulations, we have demonstrated that the network achieves diversity order 2 with polynomial complexity.

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