Calculation of Stored Electromagnetic Powers and Q Factors of Very Small Normal-Mode Helical Antennas

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***Abstract*— Theoretical explanations of radiated, dissipated and stored energies of small antennas were shown in a textbook. However, practical values of these energies were not discussed previously. In this paper, these energies are obtained numerically through electromagnetic simulations. As for a study object, a normal-mode helical antenna is utilized, because stored electric and magnetic energies are simultaneously observed. First of all, it is shown that these two energies have the same value at the self-resonant conditions. Next, electric and magnetic stored powers are calculated from these energies and are compared with the input power. Moreover, antenna *Q* factors are also obtained. Adequateness of calculated results is ensured thorough comparing with other calculated results.**

I. INTRODUCTION

For designing high efficiencies at small antennas, to understand effects of stored electric and magnetic energies is important. Stored electric and magnetic energies and dissipated energies of antennas were detailed discussed in the Harrington's book [1]. However, actual values of individual energies were not shown before. Now, electromagnetic simulations become reliable and are expected to be useful for clarifying above mentioned energies.

In this paper, normal-mode helical antennas (NMHA) are selected for study objects, because stored electric and magnetic energies can be observed simultaneously [2]. First of all, antenna structures used in this study are explained with respect to self-resonant structures. And calculation process of stored electric and magnetic energies is shown. Next, stored energies are converted to stored powers and compared with the input power. Furthermore, antenna *Q* factors are obtained. And, *Q* factors obtained from three methods are compared and adequateness is ensured.

II. SELF-RESONANT STRUCTURES

Antenna structure of a normal-mode helical antenna is shown in Fig. 1. H_A, D_A and d express antenna height, diameter and wire diameter, respectively. a expresses the radius of a sphere enclosing the antenna. The feature of this antenna is to have two radiation sources such as very small electric current and magnetic current sources simultaneously [3]. And the electric source produces electric fields (E) and the magnetic source produces magnetic fields (H), respectively. The input impedance (Z_{in}) is given by the next expression.

$$Z_{in} = R_{rad} + R_{metal} + j\,(-X_C + X_L) \qquad (1)$$

Here, R_{rad} and R_{metal} indicate radiation and metallic loss resistances, respectively. And, R_{in} is used for $R_{rad} + R_{metal}$.

X_C and X_L indicate input capacitance and inductance, respectively. At $X_C = X_L$, the reactance part becomes zero. This situation is called the self-resonance. At this condition, stored electric and magnetic powers are supposed to be cancelled out. And all the input power is effectively utilized for the radiation.

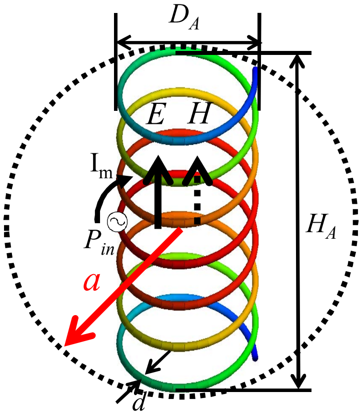

Fig. 1 Normal-mode helical antenna (NMHA)

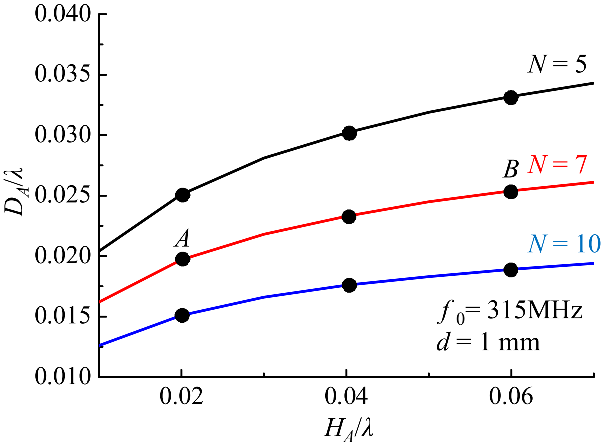

Fig. 2 Self-resonant structures

The self-resonant structures are determined through the next structural equation [2].

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$$600\pi \frac{19.7N(\frac{D_A}{\lambda})^2}{9\frac{D_A}{\lambda}+20\frac{H_A}{\lambda}} = \frac{279\frac{H_A}{\lambda}}{N\pi(0.92\frac{H_A}{\lambda}+\frac{D_A}{\lambda})^2} \qquad (2)$$

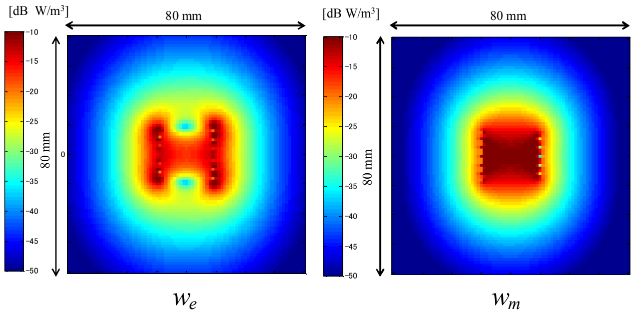
The self-resonant structures are shown in Fig. 2. D_A and H_A exist on a curve having a parameter N (number of turns). From the inclination of curves, D_A changes are rather small for the H_A changes. At the black circle points, simulation data are obtained.

III. STORED ELECTRIC AND MAGNETIC ENERGIES

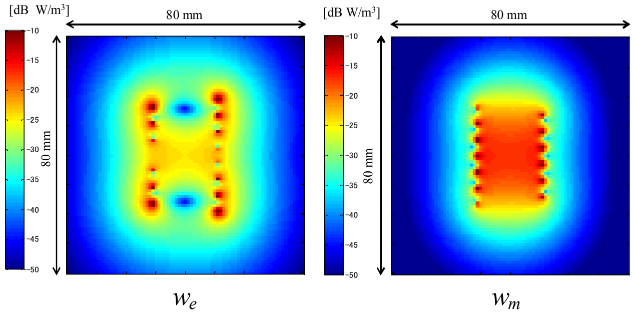
Calculation process of stored electro and magnetic energies and powers are explained in this section. And some discussions on comparisons of input and store powers are made. Moreover, stored power dependences on antenna lengths are considered.

A. Energy density distributions

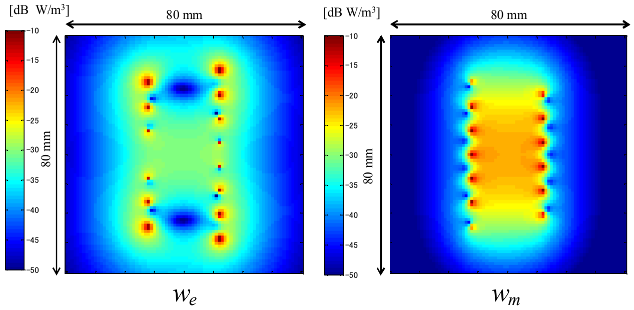
Electric (w_e) and magnetic (w_m) energy densities are obtained based on the calculated results of electric (E) and magnetic (H) fields around the NMHA with the following equations.



(a) $N = 7$, $H_A/\lambda = 0.02$



(b) $N = 7$, $H_A/\lambda = 0.04$



(c) $N = 7$, $H_A/\lambda = 0.06$

Fig. 3 Energy densities

$$w_e = \frac{\varepsilon}{2}E^2 \qquad (3)$$

$$w_m = \frac{\mu}{2}H^2 \qquad (4)$$

Calculated results of w_e and w_m are shown in Fig. 3. Here, antenna structures of $N = 7$ in Fig. 2 are used. And the input powers to the antennas are set 1 W. In the calculations, the input impedance miss matches are ignored. Thus, all antennas are receiving 1 W input power. At structures of small H_A values, w_e and w_m become very strong. The largest values of w_m at $H_A/\lambda = 0.02$, 0.04 and 0.06 are −10 dBW/m^3, −17 dBW/m^3 and −24 dBW/m^3, respectively.

The current amplitudes at the feed point (I_m) of structures in Fig.2 are shown in Fig. 4. I_m values increase rapidly in accordance with decreases of H_A. The input resistance (R_{in}) and radiation resistances (R_f) are shown in Fig. 5. R_{in} values decrease in accordance with decreases of H_A.

From the results of I_m and R_{in}, the antenna input power can be calculated with the next equation.

$$R_{in} \times (I_m)^2/2 = P_{in}\ [\mathrm{W}] \qquad (5)$$

All the results of Fig. 4 and Fig. 5 satisfy the results of $P_{in} = 1$ W.

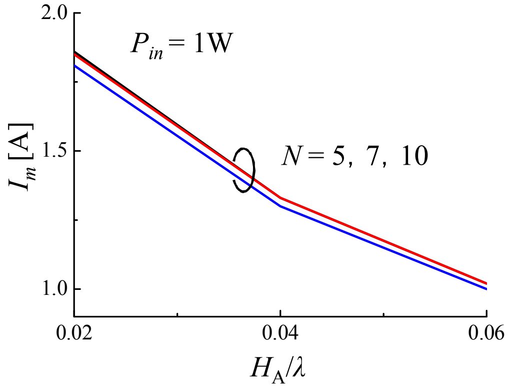


Fig. 4 Feed current

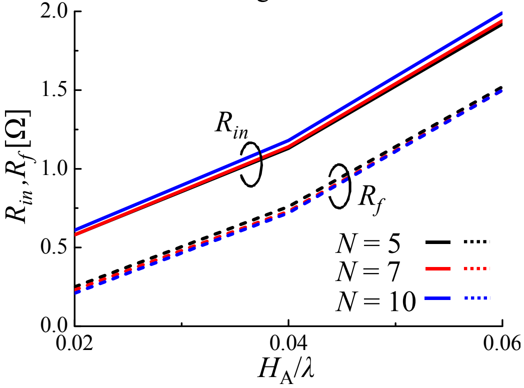


Fig. 5 Input impedance and radiation resistance

B. *Stored electric and magnetic powers*

By employing the results of Fig. 3, stored electric and magnetic energies are calculated through the following equations.

$$W_e = \int_V w_e dv = \frac{\varepsilon}{2}\int_V E^2 dv \tag{6}$$

$$W_m = \int_V w_m dv = \frac{\mu}{2}\int_V H^2 dv \tag{7}$$

As for the integration areas of *V*, the cubic of 80 mm lengths are used. The electric powers are estimated by the next expression [1].

$$P_{in} = P_f + P_d + j2\omega(W_m - W_e) \tag{8}$$

Here, P_{in} indicates the input power. P_f and P_d indicate radiated and dissipated powers, respectively.

As for stored powers, electric and magnetic powers are calculated by the following expressions.

$$P_e = 2\omega W_e \tag{9}$$

$$P_m = 2\omega W_m \tag{10}$$

Calculated results of P_e and P_m are shown in Fig. 6 [4]. In the case of integral calculation shown in Eqs.(6),(7), thin cylindrical area containing the antenna wire is omitted because irregular values appeared at E fields. Therefore, P_e values become somewhat smaller than P_m. By taking into account this calculation approximation, Good agreements of P_e and P_m are observed. From these agreements, the stored P_e and P_m cancel each other in Eq. (8). It is ensured that stored electric and magnetic powers are cancelled out at self-resonant structures. Moreover, values of P_e and P_m become between 500 W and 2000 W. These values are very large compare with the input power of 1 W. It is understand that the balanced situation is very critical. In very small deviations of structures, the balanced conditions are broken. These critical conditions imagine very narrow bandwidths.

Other features of P_e and P_m are dependences on *N* and H_A. These dependencies are discussed in the next section.

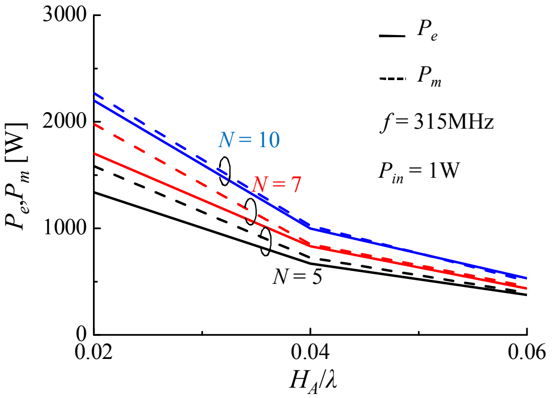


Fig. 6 Stored electric and magnetic powers

C. *P_e and P_m dependences on N and H_A*

In considering P_e and P_m values, expressions of Eq. (6) and Eq. (7) give the basic concepts.

As for P_m, magnetic field (*H*) changes are important. Magnetic fields shown in Fig. 7 are calculated by the next expression.

$$H = \int_l \frac{I(l)}{4\pi r^2} dl \tag{11}$$

Here, *I*(*l*) indicates current on the antenna wire. Therefore, *H* becomes proportional to the input current (I_m). Namely, P_m becomes proportional to I_m^2. Considering the tendencies of Fig. 4, P_m increases are recognized. Dependencies on *N* are anticipated from the relation of H to N as shown in Fig. 7. *H* is increased in accordance with *N* increases.

As for P_e, electric field (*E*) changes are important. As shown in Fig. 8, *E* is determined by charges (*Q*) at both ends. Then, the next proportion relation is given.

$$E \propto Q \tag{12}$$

At the same time, *Q* is related to the input current (I_m) as shown in the next expression.

$$\mathrm{Im} = \frac{dQ}{dt} = j\omega Q \tag{13}$$

As a result, *E* becomes proportional to I_m. Therefore, P_e becomes proportional to I_m^2. Considering the tendencies of Fig. 4, P_e increases are recognized.

Dependencies on *N* are recognized by comparing results of Fig. 8 (a) and (b). At larger *N* number, the cross sectional area containing *E* fields becomes small. Even if stored charge *Q* is unchanged, *E* fields inside *N* = 10 become larger than the *N* = 5 case. This tendencies explain the *N* dependence of P_e.

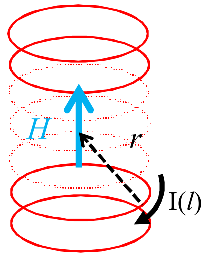
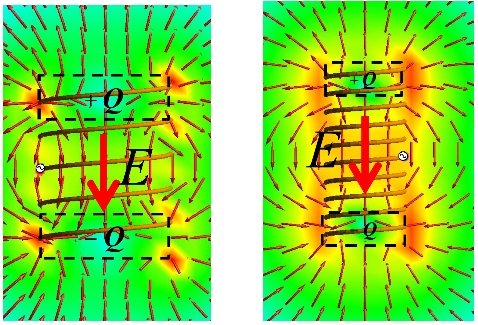


Fig. 7 Magnetic field expressions



(a) N = 5 (b) N = 10

Fig. 8 Electric field expressions at H_A/λ = 0.04

IV. ANTENNA Q FACTORS

In the previous section, stored powers (P_{st}) of electric and magnetic fields are numerically clarified. Then, the antenna Q factor can be calculated by the next expression.

$$Q = \frac{P_{st}}{P_f} \, or \, \frac{P_{st}}{P_f + P_d} \qquad (14)$$

Here, P_{st} represents P_e or P_m. And, P_f indicates the radiated power in Eq. (8). P_f and P_d are calculated by employing the results of Fig. 4 and Fig. 5. P_f is calculated by $R_f \times I_m^2/2$. P_d is calculated by $(R_{in} - R_f) \times I_m^2/2$. P_{st} / P_f is sometimes refered as an antenna Q. However, an actual antenna Q is given by $P_{st} / (P_f + P_d)$, because the antenna loss is included.

As for other method of calculating Q factors, following two methods are useful. In the reference [5], the lowest Q value of the next expression is given by J.S. McLearn.

$$Q_{lowest} = \frac{1}{k^3a^3} + \frac{1}{ka} \qquad (15)$$

In the reference [6], the relation of Q factor and bandwidth is given by the next expression.

$$Q_A = \frac{S-1}{\sqrt{S}} \cdot \frac{f_0}{f_B} \qquad (16)$$

Here, S indicates the VSWR value. f_0 and f_B indicate the center frequency and bandwidth, respectively.

Comparisons of Q factors based on Eq. (14) to Eq. (16) are shown in Fig. 9. The a at the horizontal axis indicates the radius of a sphere shown in Fig.1. So, the twice value of a corresponds to H_A. The broken line indicates the result of Eq. (15). Black circles indicate the results of Eq. (16). The black triangle indicates a measured result. In calculation of black circles and triangle, the bandwidths shown in Fig. 10 are used.

It is interesting that black circles and triangle agree rather well with $P_{st} /(P_f + P_d)$ results. Moreover, at small antenna structures near $a/\lambda = 0.015$, black circle, $P_{st} /(P_f + P_d)$ and Eq. (15) results agree well.

As a result, adequateness of P_{st} results shown in Fig. 6 is ensured.

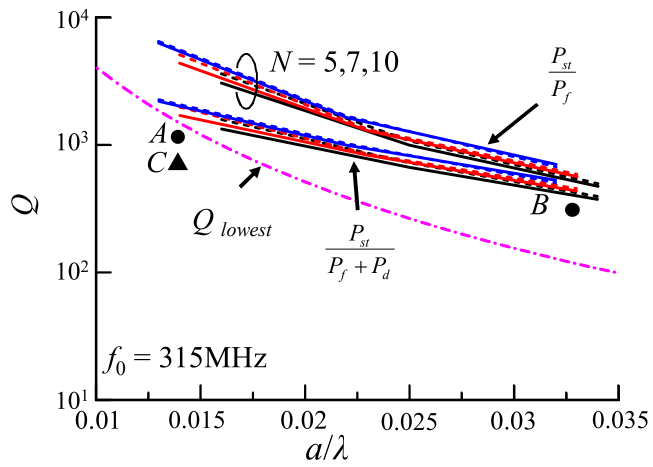


Fig. 9 Comparisons of Q factors

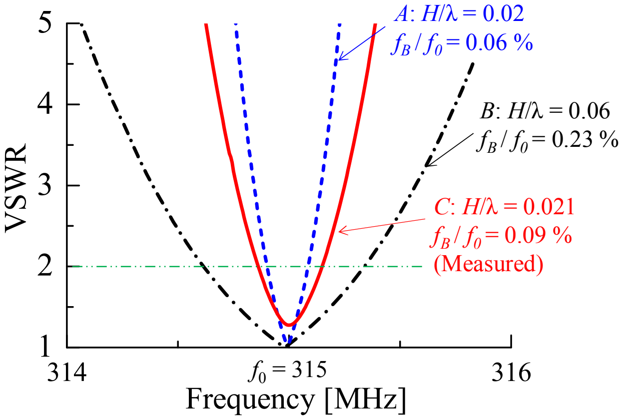


Fig. 10 Bandwidth characteristics

V. CONCLUSION

As a study object, normal-mode helical antennas are employed. From calculated results of electric and magnetic fields around antennas, stored electric and magnetic powers are obtained. The equivalence of two components at the self-resonant structures are shown numerically. The values of stored powers are shown to become 500 W to 2000 W at small antenna sizes. From stored power, antenna Q factors are estimated. As a result of good agreements of Q factors of three methods, adequateness of stored power results are ensured.

REFERENCES

[1] R.F. Harrington, *Time-Harmonic Electomagnetic Fields*, McGraw-Hill Book Company, pp.19-23, 1961.

[2] N.Q. Dinh, N. Michishita, Y. Yamada and K. Nakatani, "Deterministic Equation for Self Resonant Structures of Very Small Normal-Mode Helical Antenna," *IEICE Trans. Commun.* vol.E94-B, no.5, pp.1276-1279, May 2011.

[3] J.D. Kraus, Antennas, Second Edition, pp.333-338, McGraw-Hill Book Company, 1988.

[4] K. Mouri, N.Michishita and Y.Yamada, "Comparisons of stored electric and magnetic energies in self-resonant nomal-mode helical antennas," *The 29th Int. Rev. of Progress in ACES*, March 2013.

[5] J.S. McLearn, "A Re-examination of the Fundamental Limits on the Radiation Q of Electrically Small Antenna," *IEEE Trans. Antennas Propag.*, vol.44, no.5, pp.672-676, May 1996.

[6] S.R. Best, Antenna Engineering Handbook, fourth ed.; Small Antennas, J.L. Volakis ed., McGraw Hill, 2007.