Physical Network Coding for Bidirectional Relay MIMO-SDM System

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***Abstract*—Physical-layer network coding (PNC) is a promising technique to increase transmission throughput over bidirectional relay wireless networks. In this paper, based on the work by Zhang and Liew, we propose a new scheme called multiple input multiple output spatial division multiplexing PNC (MIMO-SDM-PNC). By using MIMO-SDM, the proposed scheme achieves double multiplexing gain and also does not require that the end nodes know the information about the forward channel from the source nodes to the relay. Compared with the system by Zhang and Liew the proposed system achieves the same diversity order. The MMSE based detection was shown to have equivalent BER performance with that of the MIMO-PNC while the ZF based suffers about 2.4 dB power loss due to noise amplification problem.**

I. Introduction

The last decade has witnessed significant improvement in the transmission rate of data communication networks. A typical example that is worth mentioning is network coding [1]. Using linear processing at intermediate nodes network coding can reduce the number phases required for data exchange and thus increase the transmission throughput compared with the conventional forwarding. In the wireless ad hoc networks physical-layer network coding (PNC) can exploit the broadcast nature of the wireless environment to double the throughput in a bidirectional relay channel [2]–[7].

Before network coding, multiple-input multiple-output (MIMO) systems were known as the most effective way to increase the channel capacity in a point-to-point wireless communication system [8],[9]. A well-known realization of MIMO systems is the vertical Bell Labs space-time (V-BLAST) which uses the spatial division multiplexing at the transmitter and linear detection with successive interference cancellation at the receiver [9]. This MIMO-SDM system has been standardized as a radio interface for many wireless systems.

The combination of MIMO and PNC thus promises significant improvement in transmission rate and has become an attractive topic of research [3],[4],[6],[7]. In [3] Kim and Chun proposed a MIMO-PNC system which used the linear pre-equalizer at the source nodes to [1] increase multiplexing gain. In the proposed system the source nodes and the relay node are all equipped with multiple antennas. In order to perform pre-equalization the source nodes need to know the forward channels from them to the relay node. In the recent works [4],[6],[7] a MIMO-PNC system with a single antenna at the source nodes and two antennas at the relay node was proposed to increase the detection reliability. Using the equivalent MIMO channel model between the two source nodes and the relay node, several detection schemes were proposed and analyzed. The work by [4] proposed two linear detection based schemes using log-likelihood ratio and selective combining. In [6] Chung and *et al.* proposed linear zero forcing (ZF) and minimum mean square error (MMSE) detectors for QAM signalling. Recently, Zhang *et al.* extended the work in [4] by using a V-BLAST detector at the relay. It is noted that except [3], all other proposed systems only considered the case in which source nodes have only one antenna, and thus cannot achieve multiplexing gain.

Motivated by this open problem, in this paper we proposed a PNC system in which all source nodes are equipped with 2 antennas while the relay node has 4 antennas. The source nodes use MIMO-SDM to exchange their data via the relay. Since the MIMO-SDM allows transmission of 2 parallel streams the proposed system can achieve double multiplexing gain compared with the systems in [4],[6],[7]. In order to detect the network coded symbols the linear detection based LLR and selective combination proposed in [7] are extended to cope with self co-channel interference (CCI) among the two streams. In the broadcast phase to the destination nodes, multiple antenna transmission is used by the relay node and a simple MIMO fading equalizer is proposed at the destination to compensate for the channel effect. We have also analyzed the effect of selecting threshold on the BER performance in the selective combination. Simulation results show that our proposed MIMO-SDM-PNC system achieves the same diversity order of the MIMO-PNC in [4] but has double multiplexing gain. Compared with the system in [3] the source nodes in our system do not need information about the forward channel from them to the relay. In addition, as SDM rather than the pre-equalizing is used the end nodes, the structure of the end nodes is simpler. Simulation results show that the system with linear MMSE based detection has equivalent BER performance with that of the MIMO-PNC while that with the ZF based suffers about 2.4 dB power loss due to noise amplification problem.

The remainder of the paper is organized as follows. The idea of MIMO-PNC is briefly summarized in Sect. II. Our proposed MIMO-SDM-PNC is presented in Sect. III. Perfor-

[1]In the bidirectional relay network we refer the two end-nodes as either the source nodes or the destination nodes depending on the data flow direction.

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mance evaluation is carried out in Sect. IV and the paper is concluded in Sect. V.

II. Overview of MIMO-PNC Bidirectional Relaying System

A. System Model of MIMO-PNC

Consider a physical network coding scheme for a bidirectional relay system proposed in [4] as shown in Fig.1. The system consists of 2 source nodes N_1 and N_2 exchanging information with each other via the help of an intermediate relay R. It is assumed that there is no direct path between two source nodes and half-duplex mode is used for signalling. Source nodes have only one antenna while the relay is equipped with 2 antennas for both reception and transmission. The bidirectional relay system consists of two phases. At the first phase the two source nodes N_1 and N_2 transmit to the relay at the same time. The relay node detects and encodes the received signals from the two source nodes according to the PNC scheme. It then initiates the second phase by broadcasting the encoded symbols to both source nodes at the same time.

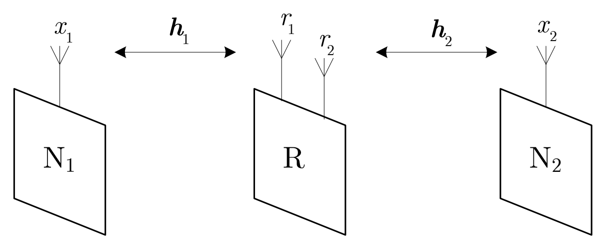

Fig. 1. System model of the bidirectional MIMO-PNC relay system.

Denote the transmit symbols from N_1 and N_2 as x_1 and x_2, respectively. The channels between the two nodes N_1, N_2 and the relay R are defined respectively as

$$\boldsymbol{h}_1 = [h_{11},\ h_{21}]^T \tag{1}$$

$$\boldsymbol{h}_2 = [h_{12},\ h_{22}]^T \tag{2}$$

where each element h_{ji} corresponds to the channel between the j-th receive antenna of relay R and source node N_i. The channels between the source nodes to the relay are assumed to undergo flat Rayleigh fading and each element h_{ji} is modeled by a complex Gaussian variable with zero mean and unit variance, i.e., $h_{ij} \sim \mathcal{N}_c(0, 1)$. The received signal at each receive branch of the relay R is affected by additive white Gaussian (AWGN) noise z_j. Noise element z_j is also modeled as a complex Gaussian random variable with zero mean but variance σ_z^2 for both dimensions.

The receive signal vector at the relay is given by

$$\boldsymbol{r} = \boldsymbol{h}_1 x_1 + \boldsymbol{h}_2 x_2 + \boldsymbol{z} \tag{3}$$

$$= \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \tag{4}$$

where $\boldsymbol{r} = [r_1, r_2]^T$ with $r_j, j = 1, 2$ being the received signal at the j-th antenna branch of the relay, $\boldsymbol{x} = [x_1,\ x_2]^T$, and the equivalent 2×2 MIMO channel $\boldsymbol{H}$ is given by

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = [\boldsymbol{h}_1,\ \boldsymbol{h}_2] \tag{5}$$

In the conventional PNC proposed in [2] the relay node will try to estimate the network coded symbol $x_1 \oplus x_2$, where notation $\oplus$ denotes the XOR operation. It then sends the estimate of this encoded symbol to both source nodes. Each source node will decode the received signal by performing XOR with its transmitted signal.

B. Signal Decoding for MIMO-PNC

For the bidirectional MIMO-PNC, Zhang and Liew proposed a linear detection based system model [4]. In the proposed model, the relay node estimates the combined symbols $x_1 + x_2$ and $x_1 - x_2$ but not x_1 and x_2 separately. It then converts these estimates to the nework coded symbol $x_1 \oplus x_2$ using PNC mapping. In order to facilitate the linear detection, the system model in (4) is transformed to

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \\ &= (\boldsymbol{H}\boldsymbol{D}^{-1})(\boldsymbol{D}\boldsymbol{x}) + \boldsymbol{z} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \boldsymbol{z} \end{aligned} \tag{6}$$

using the sum-difference matrix

$$\boldsymbol{D} = 2\boldsymbol{D}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{7}$$

Note that due to the property of the sum-difference matrix the transformed transmit vector can be expressed as follows

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}. \tag{8}$$

Since $\tilde{x}_1$ and $\tilde{x}_2$ are correlated with each other, it was shown in [4] that each of them can be easily mapped to $x_1 \oplus x_2$ using PNC mapping.

For linear detection, the estimate of $\tilde{\boldsymbol{x}}$ can be obtained using a combining weight matrix $\boldsymbol{G}$ corresponding to the transformed channel matrix $\tilde{\boldsymbol{H}}$ as follows

$$\boldsymbol{d} = \boldsymbol{G}\boldsymbol{r} = \boldsymbol{G}\tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \boldsymbol{G}\boldsymbol{z} \tag{9}$$

where $\boldsymbol{d} = [d_1, d_2]^T$. The combining weight matrix is given for ZF and MMSE respectively as follows

$$\boldsymbol{G}^{\mathrm{ZF}} = (\tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{H}})^{-1} \tilde{\boldsymbol{H}}^H \tag{10}$$

$$\boldsymbol{G}^{\mathrm{MMSE}} = \left(\tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{H}} + \sigma_z^2 \boldsymbol{I}_2\right)^{-1} \tilde{\boldsymbol{H}}^H. \tag{11}$$

Given the estimates d_1 and d_2 from (9), the relay node needs to find the estimates $\widehat{x_1 \oplus x_2}$ of the network coded symbol $x_1 \oplus x_2$. In order to do so, Zhang and Liew proposed two approaches based on Log-Likelihood Ratio (LLR) and selective combination [4].

In the LLR-based combination, LLR of $x_1 \oplus x_2$ given d_1 and d_2 is given by [4]

$$\begin{aligned} L\left(x_1 \oplus x_2 | d_1 d_2\right) &= \frac{P(d_1 d_2 \,| x_1 \oplus x_2 = 1)}{P(d_1 d_2 \,| x_1 \oplus x_2 = -1)} \\ &= \exp(2/\sigma_2^2 - 2/\sigma_1^2) \cosh(2d_1/\sigma_1^2) / \cosh(2d_2/\sigma_2^2) \end{aligned} \tag{12}$$

where $\sigma_i^2 = \sigma_z^2 \left(\boldsymbol{G}^H \boldsymbol{G}\right)_{i,i}$ is the noise variance in the i-th data stream after linear combining. The decision rule is [4]:

$$\widetilde{x_1 \oplus x_2} = \begin{cases} +1 \text{ if } L(x_1 \oplus x_2 \,|d_1 d_2\,) \geq 1 \\ -1 \text{ if } L(x_1 \oplus x_2 \,|d_1 d_2\,) < 1 \end{cases} \tag{13}$$

The LLR combination is optimum in terms of BER minimization. However, it is computationally complex and requires information of noise variance.

The selective combination is simpler as the decision can be made using either estimates d_1 or d_2 as follows

$$\widetilde{x_1 \oplus x_2} = \begin{cases} \operatorname{sign}\{|d_1| - \gamma\} & \text{if } \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{1,1} < \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{2,2} \\ \operatorname{sign}\{\gamma - |d_2|\} & \text{otherwise} \end{cases} \tag{14}$$

where γ the threshold and it is set to 1 in [4]. Compared with the LLR, the selective combination detection suffers power loss due to its simplicity.

III. Proposed System of MIMO-SDM-PNC

A. System Model of MIMO-SDM-PNC

We consider a more general case of MIMO-PNC in [4] where source nodes use multiple antennas and spatial division multiplexing as illustrated in Fig. 2. For the sake of simple presentation, we only use the case in which source nodes have 2 antennas and the relay has 4. Other assumptions are similar with those for MIMO-PNC in the previous section.

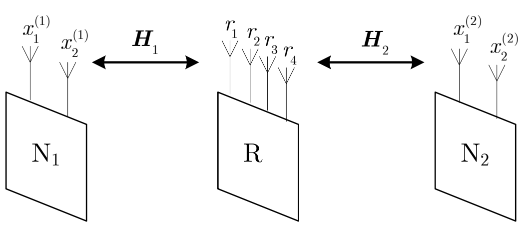

Fig. 2. System model of the bidirectional MIMO-SDM-PNC relay system.

In the MIMO-SDM system, each souce node transmits two parallel data streams

$$\boldsymbol{x}_1 = [x_1^{(1)},\, x_2^{(1)}]^T \tag{15}$$

$$\boldsymbol{x}_2 = [x_1^{(2)},\, x_2^{(2)}]^T. \tag{16}$$

The channels between the sources and the relay are defined as follows

$$\boldsymbol{H}_1 = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} \\ h_{21}^{(1)} & h_{22}^{(1)} \\ h_{31}^{(1)} & h_{32}^{(1)} \\ h_{41}^{(1)} & h_{42}^{(1)} \end{bmatrix}, \boldsymbol{H}_2 = \begin{bmatrix} h_{11}^{(2)} & h_{12}^{(2)} \\ h_{21}^{(2)} & h_{22}^{(2)} \\ h_{31}^{(2)} & h_{32}^{(2)} \\ h_{41}^{(2)} & h_{42}^{(2)} \end{bmatrix}. \tag{17}$$

Define the equivalent channel matrix and signal vector, respectively as

$$\boldsymbol{H} = \left[\boldsymbol{H}_1,\, \boldsymbol{H}_2\right] \tag{18}$$

$$\boldsymbol{x} = \left[\boldsymbol{x}_1^T,\, \boldsymbol{x}_2^T\right]^T. \tag{19}$$

The received signal vector at the relay node can be expressed similar to the MIMO-PNC case as

$$\boldsymbol{r} = \tfrac{1}{\sqrt{2}} \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \tag{20}$$

where in this case $\boldsymbol{r} = [r_1,\, r_2,\, ,r_3,\, r_4]^T$ and $\boldsymbol{z} = [z_1,\, z_2,\, ,z_3,\, z_4]^T$. The fraction $\frac{1}{\sqrt{2}}$ accounts for power normalization factor.

As for the MIMO-PNC case, the relay node needs to estimate the network coded symbols to be sent to the two source nodes. Since each source node transmits two symbols $x_1^{(i)},\ x_2^{(i)},\ i = 1, 2$, the network coded symbols are given $x_1^{(1)} \oplus x_1^{(2)}$ for node N_1 and $x_2^{(1)} \oplus x_2^{(2)}$ for node N_2. Methods to estimate these symbols are presented below.

B. Signal Detection at Relay Node for MIMO-SDM-PNC

Based on the idea of [4], we transform the system equation in (20) as follows:

$$\begin{aligned} \boldsymbol{r} &= \tfrac{1}{\sqrt{2}} \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \\ &= \tfrac{1}{\sqrt{2}} (\boldsymbol{H}\boldsymbol{U}^{-1})(\boldsymbol{U}\boldsymbol{x}) + \boldsymbol{z} = \tfrac{1}{\sqrt{2}} \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \boldsymbol{z} \end{aligned} \tag{21}$$

where the sum-difference is

$$\boldsymbol{U} = 2\boldsymbol{U}^{-1} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}. \tag{22}$$

It is worth noting that the equivalent transmit vector is given in the following form

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \tilde{x}_1^{(1)} \\ \tilde{x}_2^{(1)} \\ \tilde{x}_1^{(2)} \\ \tilde{x}_2^{(2)} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} + x_1^{(2)} \\ x_2^{(1)} + x_2^{(2)} \\ x_1^{(1)} - x_1^{(2)} \\ x_2^{(1)} - x_2^{(2)} \end{bmatrix}. \tag{23}$$

From system equation (21) we can apply the linear detection to estimate the equivalent transmit vector $\tilde{\boldsymbol{x}}$. The combining weight matrices for ZF and MMSE detector are still in the same form of (38) and (39). The estimated vector using linear detection is given by

$$\boldsymbol{y} = \begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_1^{(2)} \\ y_2^{(2)} \end{bmatrix} = \begin{bmatrix} \widetilde{x_1^{(1)} + x_1^{(2)}} \\ \widetilde{x_2^{(1)} + x_2^{(2)}} \\ \widetilde{x_1^{(1)} - x_1^{(2)}} \\ \widetilde{x_2^{(1)} - x_2^{(2)}} \end{bmatrix}. \tag{24}$$

It is worth noting that the estimates $y_i^{(1)}$ and $y_i^{(2)}$ are correlated, and we can apply the combination proposed for MIMO-PNC to estimate the network coded symbols $x_1^{(1)} \oplus x_1^{(2)}$, $x_2^{(1)} \oplus x_2^{(2)}$ for each source node.

For the LLR combination, the estimates of the network coded symbols are given by

$$L\left(x_1^{(1)} \oplus x_1^{(2)} \,\middle|\, y_1^{(1)} y_1^{(2)}\right) = \frac{P\left(y_1^{(1)} y_1^{(2)} \,\middle|\, x_1^{(1)} \oplus x_1^{(2)} = 1\right)}{P\left(y_1^{(1)} y_1^{(2)} \,\middle|\, x_1^{(1)} \oplus x_1^{(2)} = -1\right)} \tag{25}$$

$$L\left(x_2^{(1)} \oplus x_2^{(2)} \,\middle|\, y_2^{(1)} y_2^{(2)}\right) = \frac{P\left(y_2^{(1)} y_2^{(2)} \,\middle|\, x_2^{(1)} \oplus x_2^{(2)} = 1\right)}{P\left(y_2^{(1)} y_2^{(2)} \,\middle|\, x_2^{(1)} \oplus x_2^{(2)} = -1\right)} \tag{26}$$

From (25) and (26) we can calculate the LLRs as follows

$$\begin{aligned} \mathrm{LLR1} = \exp\left(\frac{-\left(y_1^{(2)}\right)^2}{2(\sigma_1^{(2)})^2}\right) \times \\ \left[\exp\left(\frac{-\left(y_1^{(1)}-2\right)^2}{2(\sigma_1^{(1)})^2}\right) + \exp\left(\frac{-\left(y_1^{(1)}+2\right)^2}{2(\sigma_1^{(1)})^2}\right)\right] \end{aligned} \tag{27}$$

$$\begin{aligned} \mathrm{LLR3} = \exp\left(\frac{-\left(y_1^{(1)}\right)^2}{2(\sigma_1^{(1)})^2}\right) \times \\ \left[\exp\left(\frac{-\left(y_1^{(2)}-2\right)^2}{2(\sigma_1^{(2)})^2}\right) + \exp\left(\frac{-\left(y_1^{(2)}+2\right)^2}{2(\sigma_1^{(2)})^2}\right)\right] \end{aligned} \tag{28}$$

$$\begin{aligned} \mathrm{LLR2} = \exp\left(\frac{-\left(y_2^{(2)}\right)^2}{2(\sigma_2^{(2)})^2}\right) \times \\ \left[\exp\left(\frac{-\left(y_2^{(1)}-2\right)^2}{2(\sigma_2^{(1)})^2}\right) + \exp\left(\frac{-\left(y_2^{(1)}+2\right)^2}{2(\sigma_2^{(1)})^2}\right)\right] \end{aligned} \tag{29}$$

$$\begin{aligned} \mathrm{LLR4} = \exp\left(\frac{-\left(y_2^{(1)}\right)^2}{2(\sigma_2^{(1)})^2}\right) \times \\ \left[\exp\left(\frac{-\left(y_2^{(2)}-2\right)^2}{2(\sigma_2^{(2)})^2}\right) + \exp\left(\frac{-\left(y_2^{(2)}+2\right)^2}{2(\sigma_2^{(2)})^2}\right)\right] \end{aligned} \tag{30}$$

where $\left(\sigma_k^{(i)}\right)^2 = \left(\boldsymbol{G}^H\boldsymbol{G}\right)_{k,k}\sigma_z^2$ is the noise variance associated with the k-th output stream of the k-th user from the linear detector.

Using equations (27) to (30) the decisions are made as follows:

$$\widetilde{x_1^{(1)} \oplus x_1^{(2)}} = \begin{cases} -1 & \text{if LLR1} \geq \text{LLR3} \\ +1 & \text{if LLR1} < \text{LLR3} \end{cases} \tag{31}$$

$$\widetilde{x_2^{(1)} \oplus x_2^{(2)}} = \begin{cases} -1 & \text{if LLR2} \geq \text{LLR4} \\ +1 & \text{if LLR2} < \text{LLR4} \end{cases} \tag{32}$$

Using the selective combination, the network coded symbols are given by

$$\widetilde{x_1^{(1)} \oplus x_1^{(2)}} = \begin{cases} \mathrm{sign}\left\{|y_1^{(1)}| - \gamma\right\} & \text{if } \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{1,1} < \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{3,3} \\ \mathrm{sign}\left\{\gamma - |y_1^{(2)}|\right\} & \text{otherwise} \end{cases}$$

$$\widetilde{x_2^{(1)} \oplus x_2^{(2)}} = \begin{cases} \mathrm{sign}\left\{|y_2^{(1)}| - \gamma\right\} & \text{if } \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{2,2} < \left(\boldsymbol{G}\boldsymbol{G}^H\right)_{4,4} \\ \mathrm{sign}\left\{\gamma - |y_2^{(2)}|\right\} & \text{otherwise} \end{cases}$$

C. Signal Detection at Destination Nodes

After the network coded symbols $\widetilde{x_1^{(1)} \oplus x_1^{(2)}}$ and $\widetilde{x_2^{(1)} \oplus x_2^{(2)}}$ have been successfully estimated, the relay will send them to the respective destination node i. There are several ways that the relay node can do so. If the relay node resource is available, it can encode the network coded symbols using space-time coding [11] to obtain further spatial diversity. If the relay knows the channel from itself to the source nodes, it can also use a precoding technique such as the maximal ratio transmission (MRT) [12] to optimize the transmit gain. When the resource is constrained, it can simply transmit the network coded symbols simultaneously over its antennas without coding. In this paper, for the sake of simplicity and fair comparison with [4], we will use this simple approach. However, as the relay node has four antennas while there are only two network coded symbols $\widetilde{x_1^{(1)} \oplus x_1^{(2)}}$ and $\widetilde{x_2^{(1)} \oplus x_2^{(2)}}$ to be sent, we can select two antennas for transmission. For simplicity, we assume that the first and the second antenna will be used. The first antenna transmits $\widetilde{x_1^{(1)} \oplus x_1^{(2)}}$, and the second $\widetilde{x_2^{(1)} \oplus x_2^{(2)}}$. Define the transmit vector from the relay as

$$\boldsymbol{x}_r = \left[\widetilde{x_1^{(1)} \oplus x_1^{(2)}},\ \widetilde{x_2^{(1)} \oplus x_2^{(2)}}\right]^T, \tag{33}$$

and the backward channels from the relay to the source nodes as

$$\hat{\boldsymbol{H}}_1 = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} \\ h_{21}^{(1)} & h_{22}^{(1)} \end{bmatrix} \tag{34}$$

$$\hat{\boldsymbol{H}}_2 = \begin{bmatrix} h_{11}^{(2)} & h_{12}^{(2)} \\ h_{21}^{(2)} & h_{22}^{(2)} \end{bmatrix} \tag{35}$$

we can write the signal vectors received at the source nodes as follows

$$\boldsymbol{u}_1 = \tfrac{1}{\sqrt{2}}\hat{\boldsymbol{H}}_1\boldsymbol{x}_r + \boldsymbol{n}_1. \tag{36}$$

$$\boldsymbol{u}_2 = \tfrac{1}{\sqrt{2}}\hat{\boldsymbol{H}}_2\boldsymbol{x}_r + \boldsymbol{n}_2. \tag{37}$$

where $\boldsymbol{n}_1 = \left[n_1^{(1)}, n_2^{(1)}\right]^T$, $\boldsymbol{n}_2 = \left[n_1^{(2)}, n_2^{(2)}\right]^T$ are the noise vectors at the two destination nodes, respectively.

Now, in order to detect the received symbols, we can also use the linear detectors such as ZF and MMSE at the destination nodes to estimate the network coded versions of $x_1^{(1)} \oplus x_1^{(2)}$, and $x_2^{(1)} \oplus x_2^{(2)}$. Assume that the destination nodes know the forward channel from themselves to the relay, the combining matrices of the detectors are given, respectively, by

$$\boldsymbol{G}_i^{\mathrm{ZF}} = \left(\hat{\boldsymbol{H}}_i{}^H\hat{\boldsymbol{H}}_i\right)^{-1}\hat{\boldsymbol{H}}_i{}^H \tag{38}$$

$$\boldsymbol{G}_i^{\mathrm{MMSE}} = \left(\hat{\boldsymbol{H}}_i^H\hat{\boldsymbol{H}}_i + \sigma_n^2\boldsymbol{I}_2\right)^{-1}\hat{\boldsymbol{H}}_i{}^H \tag{39}$$

where σ_n^2 is the noise variance at the destination nodes. The decision statistics of the network coded symbols after linear combining are given by

$$\hat{\boldsymbol{x}}_r^{(i)} = \boldsymbol{G}_i\boldsymbol{u}_i = \begin{bmatrix} \hat{x}_1^{(i)} \\ \hat{x}_2^{(i)} \end{bmatrix} = \begin{bmatrix} \widehat{x_1{}^{(1)} \oplus x_1{}^{(2)}} \\ \widehat{x_2{}^{(1)} \oplus x_2{}^{(2)}} \end{bmatrix}. \tag{40}$$

Now each destination node can use a quantization function to obtain the estimates of the network coded symbols as $\overline{x_1{}^{(1)} \oplus x_1{}^{(2)}} = \mathcal{Q}\big(\widehat{x_1{}^{(1)} \oplus x_1{}^{(2)}}\big)$ and $\overline{x_2{}^{(1)} \oplus x_2{}^{(2)}} = \mathcal{Q}\big(\widehat{x_2{}^{(1)} \oplus x_2{}^{(2)}}\big)$. The two nodes then simply perform XOR of the estimated coded symbols with their transmit symbols to obtain the received symbols.

D. Extension to the case with more antennas

From the above sections we can see that the proposed system has double multiplexing gain and thus double spectral efficiency compared with that in [4]. In order to further increase this gain, the two end nodes can use a large number of antennas. In general, if the number of antennas at the end nodes is N then we can increase the spectral efficiency by N times. In such a case, the relay node requires $2N$ antennas for reception and only N antennas for transmission. The sum-difference matrix can be then generalized as follows

$$\boldsymbol{U} = 2\boldsymbol{U}^{-1} = \begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{I}_N \\ \boldsymbol{I}_N & -\boldsymbol{I}_N \end{bmatrix}. \tag{41}$$

The price for this case is the increase in complexity and the number of antennas. As the complexity of the system mainly depends on that required by the linear detectors, increasing the number of antennas at the end nodes by N times will lead to an increase in complexity to $\mathcal{O}(N^3)$ at the end nodes and $\mathcal{O}([2N]^3)$ at the relay node.

IV. Performance Evaluation

In order to evaluate performance of the proposed system, we have carried out Monte-Carlo simulations. The network model is the same as in Fig. 2 for our proposed system and as in Fig. 1 for comparing with [4]. All channels are assumed undergo flat Rayleigh fading. In our simulations, BPSK was used for modulation and we assume that the E_s/N_0 at the all nodes are the same. At the relay node, both ZF and MMSE are used for symbol estimation while at the destination nodes only MMSE is used for the sake of quality improvement.

A. Threshold Selection for Selective Combination

As the threshold γ plays an important role in the system performance for selective combining, we first analyze its effect in order to choose the best value. Note that γ was simply set to 1 in [4]. In our analysis we set up simulations for the threshold range from 0.1 to 1.5. Three values of the E_s/N_0 for analysis were 0 dB, 10 dB, and 20 dB. Fig. 3 and 4 show the BER performance varying with γ for the ZF and MMSE detector. It is clearly seen that the best threshold for the ZF and MMSE is 1 and 0.7, respectively. The obtained value $\gamma = 1$ in the same with that used in [4] while $\gamma = 0.7$ is different. In the following analysis we will use these values for selective combination in our proposed system. The figures also show that the threshold γ has significant effect on the BER performance, particularly, at high E_s/N_0. Therefore, we need to be careful when selecting it in the system design. Failing to do so can degrade the system performance significantly.

B. BER Performance

Fig. 5 and 6 illustrate the BER performance of our proposed system MIMO-SDM-PNC using ZF and MMSE detection, respectively. In order to compare with the system in [4], BER performance of the the MIMO-PNC is also shown

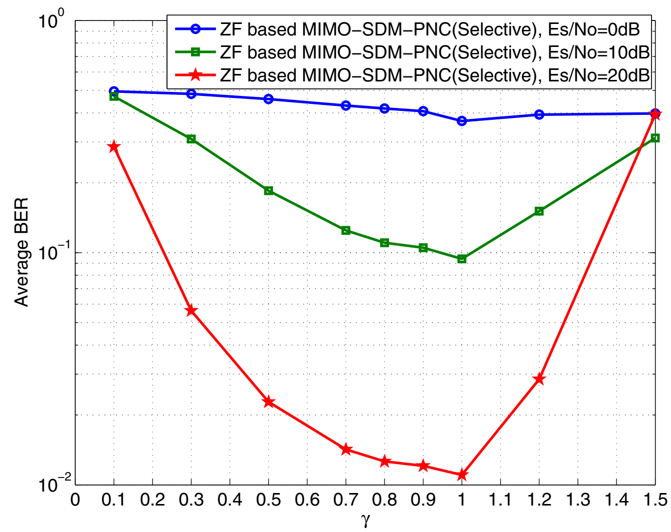

Fig. 3. Threslhold γ selection for ZF-based MIMO-SDM-PNC.

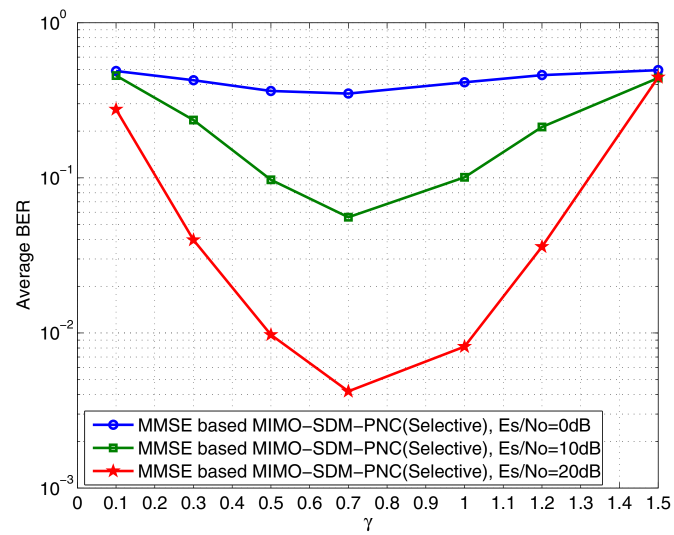

Fig. 4. Threslhold γ selection for MMSE-based MIMO-SDM-PNC.

in the figures for reference. It is clearly realized that the proposed MIMO-SDM-PNC achieve the same diversity order of the MIMO-PNC, particularly at high E_s/N_0. We can also see that the proposed MIMO-SDM-PNC with the MMSE-based detection provides almost the same BER performance with MIMO-PNC. However, with the ZF-based detection the proposed system suffers about 2.4 dB loss in power. This loss can be attributed to the noise amplification effect of the ZF detection. Compared with the MIMO-PNC this effect is more influenced in the MIMO-SDM-PNC system due to the problem of self co-channel interference (CCI) among two SDM transmit streams. Therefore, the MMSE detection will be better suitable for the MIMO-SDM-PNC system. However, it is worth noting that even having that power loss, the ZF-based detection provides double multiplexing gain compared to the MIMO-PNC system.

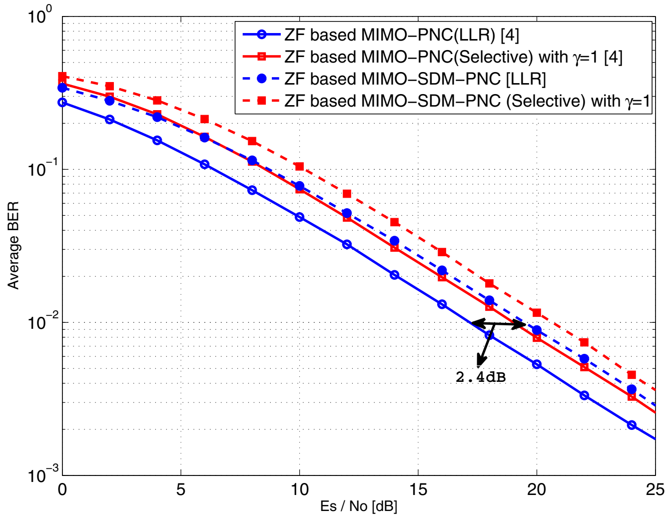

Fig. 5. BER performance of the MIMO-SDM-PNC using ZF-based detection.

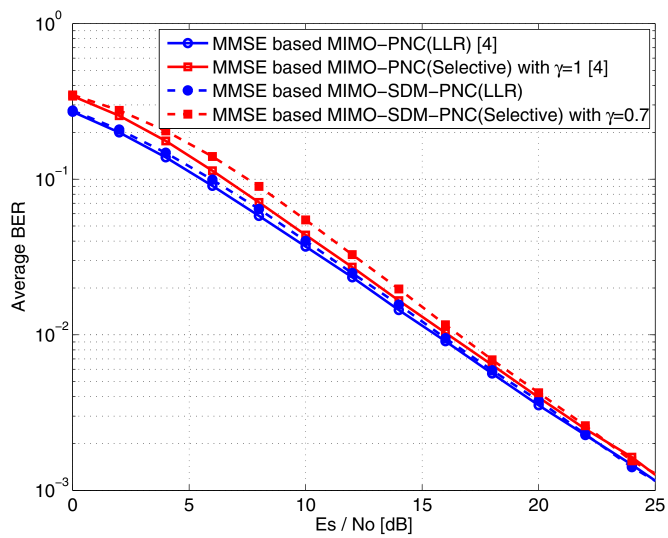

Fig. 6. BER performance of the MIMO-SDM-PNC using MMSE-based detection.

V. Conclusions

In this paper, we have proposed a PNC scheme for bidirectional relaying MIMO-SDM system based on the MIMO-PNC in [4]. In the proposed scheme, the relay node requires double number of antennas for reception but only equal number for transmission compared with the end nodes. The sum-difference matrix was developed to support multiple antenna systems. We have also analyzed the effect of selecting the decision threshold on the BER performance and found the best values for the ZF and MMSE detection based system. Compared with [4] the proposed system with MMSE detection has the approximately equivalent BER performance while that with ZF suffers power loss due to the effect of noise amplification in the self CCI channel. Thanks to MIMO-SDM the proposed system can achieve linear increase in the transmission rate when multiple antennas are used at the source nodes. However, it requires more antennas and complexity and thus is suitable for the system with available antennas and hardware resources.

VI. Acknowledgement

This work was supported by the Ministry of Science and Technology of Vietnam under Project 39/2012/HD/NDT grant.

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