

# Improved Relay Selection for MIMO-SDM Cooperative Communications

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**Abstract** In this paper, we propose two relay selection algorithms based on the signal-to-noise ratio (SNR) and the eigenvalue which achieve improved bit error rate (BER) performance compared with the previous one based on the mean square error (MSE) at the same complexity order.

**Keywords** Cooperative communication · Relay selection · MIMO · SDM

## 1 Introduction

The modern wireless communication is developing very fast in order to meet the human demand for high-speed data access. The last decade has witnessed various successful developments in the air interface technology. The most important development is probably the multiple-input multiple-output (MIMO) transmission [1]. MIMO transmission systems can be implemented in the form of either the transmit diversity [2] or spatial division multiplexing (SDM) [3]. The aim of the transmit diversity is to achieve diversity gain in order to reduce the bit error rate (BER) and thus increasing the link reliability. This transmit diversity scheme is also known as the space–time block code [2]. The MIMO-SDM systems, on the other hand, aim at achieving multiplexing gain in order to increase the spectral efficiency. For a centralized MIMO system where multiple antennas are placed at the transmitter and the receiver, it was shown in [4] that there is a trade-off between the diversity and multiplexing gain. This means that the centralized

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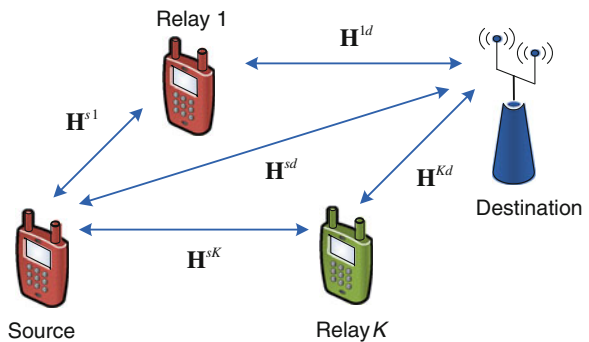
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MIMO systems do not achieve full diversity and multiplexing gain at the same time. In order to achieve both diversity and multiplexing gain, a so-called MIMO-SDM cooperative communication system was proposed in [5]. In this work the authors proposed three distributed relay selection schemes and a linear minimum mean square error (MMSE) combining scheme which achieve full diversity and full multiplexing gain at the same time. Among the three proposed selection algorithms based on maximum channel matrix norm, maximum channel harmonic mean, and minimum MSE, the MSE-based algorithm was shown to achieve the best BER performance [5]. In this paper, based on the idea of [6] for the case of MIMO-SDM, we developed two relay selection algorithms based on the signal to noise ratio (SNR) and eigenvalue. The two proposed algorithms have improved BER performance over the MSE-based algorithm while requiring the same complexity order.

## 2 System Model

We consider a MIMO-SDM cooperative communication network similar to [5] as illustrated in Fig. 1. The network consists of a source and a destination communicating with each other with the help of a relay node via a relaying path. Without loss of generality, we assume that all nodes (including source, destination and intermediate) are equipped with  $N = 2$  antennas for both transmission and reception. There are  $K$  capable intermediate nodes  $k = 1, 2, \dots, K$  between the source and the destination. Based on a distributed relay selection protocol [5, 7] the  $K$  intermediate nodes will interact with one another to select the best one to act as the relay (denoted by the index  $r$ ). The channels between nodes are assumed flat uncorrelated Rayleigh fading and unvarying during a transmission period. We denote  $\mathbf{H}^{sd}$ ,  $\mathbf{H}^{sk}$ ,  $\mathbf{H}^{kd}$  the channel matrices between the source and the destination, the source and the intermediate node  $k$  and the destination, respectively. The channel between a node  $a$  and a node  $b$  is denoted as the matrix  $\mathbf{H}^{ab} =$

**Fig. 1** System model of the MIMO cooperative communications



$[h_{11}^{ab}, h_{12}^{ab}, h_{21}^{ab}, h_{22}^{ab}]$  where  $h_{mn}^{ab}$  is the channel between the  $m$ th antenna of node  $b$  to the  $n$ th antenna of  $a$ .

The communication between the source and the destination involves two phases: relay selection and signal transmission. The relay selection is done using the distributed protocol as mentioned above while the signal transmission uses two time slots. In the first slot, the source transmits a signal vector  $\mathbf{s} = [s_1, s_2]^T$  consisting of two symbols  $s_1$  and  $s_2$  from the two antennas to both the destination and the relay. Here the superscript  $T$  denotes the matrix transpose. The received signal vector at the destination and the relay is given by  $\mathbf{y}_1 = \mathbf{H}^{sd}\mathbf{s} + \mathbf{z}_1, \mathbf{x}_r = \mathbf{H}^{sr}\mathbf{s} + \mathbf{z}_r$ , where  $\mathbf{z}_1$  and  $\mathbf{z}_r$  are the noise vector at the destination and relay  $r$ , respectively. In the second time slot, the relay performs amplifying-and-forwarding (AF) the received signal  $\mathbf{x}_r$  to the destination. The amplification matrix  $\mathbf{G}_r$  is a diagonal matrix with the amplification factor used for the  $i$ th branch given by [5]:  $g_i^r = \sqrt{E_s/N(E_s/N\|\mathbf{h}_i^{sr}\|^2 + 1)}$ , where  $E_s$  is the transmit symbol energy and  $\mathbf{h}_i$  is the  $i$ th row of the appropriate channel matrix. The received signal at the destination during the second time slot is given by [5]

$$\mathbf{y}_2 = \mathbf{H}^{rd}\mathbf{G}_r\mathbf{x}_r + \mathbf{z}_2 = \mathbf{H}^{rd}\mathbf{G}_r\mathbf{H}^{sr}\mathbf{s} + \mathbf{H}^{rd}\mathbf{G}_r\mathbf{z}_r + \mathbf{z}_2 \tag{1}$$

where  $\mathbf{z}_2$  is the noise vector at the destination in the second time slot. The destination will combine the received signal vectors  $\mathbf{y}_1$  and  $\mathbf{y}_2$  to obtain the received signal vector  $\mathbf{y}$ . Define  $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T]^T$ ,  $\mathbf{H} = [(\mathbf{H}^{sd})^T, (\mathbf{H}^{sr}\mathbf{H}^{rd})^T]^T$  and  $\mathbf{z} = [\mathbf{z}_1^T, (\mathbf{H}^{rd}\mathbf{G}_r\mathbf{z}_r + \mathbf{z}_2)^T]^T$ , the system equation is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{z}. \tag{2}$$

### 3 Proposed Relay Selection Algorithms

The proposed selection algorithms are performed in a distributed manner as described in [5, 7]. The intermediate nodes  $k = 1, 2, \dots, K$  are assumed to know the forward channel from itself to the destination  $\mathbf{H}^{kd}$  and the backward channel from it back to the source  $\mathbf{H}^{ks}$ . Due to reciprocity the channel  $\mathbf{H}^{sk}$  is assumed to be the same as  $\mathbf{H}^{ks}$ . Each node  $k$  will calculate the channel quality index (CQI) of the relaying path via itself. The node with the largest CQI, denoted by  $\kappa$  be selected as the relay.

**Table 1** Eigenvalue-based relay selection algorithm

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Input: $K, \mathbf{H}^{sk}, \mathbf{H}^{kd}$
For $k = 1$ to $K$
Calculate $\lambda_{1,2}^{sk}, \lambda_{1,2}^{kd}$ using (3)
Select $\lambda^k = \min\{\lambda_{1,2}^{sk}, \lambda_{1,2}^{kd}\}$
$\text{CQI}_k = \lambda^k$
$\kappa = \arg \max_k \{\text{CQI}_k\}$
End
Output: node $\kappa$ as relay $r$

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### 3.1 Eigenvalue-Based Relay Selection

The idea of selecting relay based on eigenvalues comes from the fact that in the MIMO systems eigenvalue of the channel matrix is considered the power gain of the channel [8]. As a result, the channel which has larger eigenvalues will have better power gain. The eigenvalues of the channel  $\mathbf{H}^{ab}$  is the solutions to the following characteristic equation  $\det(\mathbf{H}^{ab} - \lambda \mathbf{I}) = 0$ , where  $\mathbf{I}$  is an  $N \times N$  identity matrix and  $\det(\cdot)$  represents the determinant of the matrix formed by  $(\mathbf{H}^{ab} - \lambda \mathbf{I})$ . For an  $N \times N$  complex matrix  $\mathbf{H}^{ab}$  there are at most  $N$  distinct eigenvalues. For the  $2 \times 2$  channel matrix  $\mathbf{H}^{ab}$  considered here there are two eigenvalues given by

$$\lambda_{1,2}^{ab} = \frac{(h_{11}^{ab} + h_{22}^{ab}) \pm \sqrt{(h_{11}^{ab} + h_{22}^{ab})^2 - 4(h_{11}^{ab}h_{22}^{ab} - h_{12}^{ab}h_{21}^{ab})}}{2} \tag{3}$$

In order to obtain the associated CQI each intermediate node  $k$  will first select  $\lambda^k = \min\{\lambda_{1,2}^{sk}, \lambda_{1,2}^{kd}\}$  and then calculate  $\text{CQI}^k = \lambda^k$ . The max-min selection algorithm based on eigenvalues is summarized as pseudocodes in Table 1. It is worth noting that for the case of using a larger number of antennas, i.e.  $N > 2$ , the calculation of the eigenvalues as used in (3) is not straightforward and a more complicated calculation algorithm should be used.

### 3.2 SNR-Based Selection Algorithm

As SNR is inversely proportional to BER, selecting a relaying path with better SNR promises lower BER. In order to perform SNR-based relay selection, we assume that the destination uses the linear MMSE detector proposed in [5]. Based on this assumption intermediate nodes will calculate the received SNR at the destination via its relaying path. The CQI associated with each path will be assigned based on the calculated SNR. From [5] we can write the combining

**Table 2** SNR-based algorithm

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Input: $K, \mathbf{H}^{sk}, \mathbf{H}^{kd}, \mathbf{G}_k$
For $k = 1$ to $K$
Calculate $\text{SNR}_n^k$ using (6)
$\text{SNR}^k = \min\{\text{SNR}_1^k, \text{SNR}_2^k\}$
$\text{CQI}_k = \text{SNR}^k$
$\kappa = \arg \max_k \{\text{CQI}_k\}$
End
Output: node $\kappa$ as relay $r$

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weight matrix that the destination would use to combine the relaying signal with that from the direct path as follows

$$\mathbf{W}_2^k = \left[ \frac{E_s}{N} \mathbf{H}^{skd} (\mathbf{H}^{skd})^H + \sigma_{z_k}^2 \mathbf{H}^{kd} \mathbf{G}_k^2 (\mathbf{H}^{kd})^H + \sigma_{z_2}^2 \mathbf{I}_2 \right]^{-1} \frac{E_s}{N} \mathbf{H}^{skd}, \tag{4}$$

where  $\mathbf{H}^{skd} = \mathbf{H}^{sk} \mathbf{G}_k \mathbf{H}^{kd}$ ,  $\sigma_{z_k}^2$  and  $\sigma_{z_2}^2$  are the variance of the noise induced at node  $k$  and at the destination during the second time slot, respectively. For simplicity, we assume that  $\sigma_{z_d}^2 = \sigma_{z_1}^2 = \sigma_{z_2}^2$  and that  $\sigma_{z_k}^2 = \sigma_{z_d}^2$ . Since we use the assumption that the source sends two parallel streams, the estimated symbol  $\tilde{s}_n$  of  $s_n$  if node  $k$  acts as the relay would be

$$\tilde{s}_n = (\mathbf{w}_{2,n}^k)^H \mathbf{y}_2 = (\mathbf{w}_{2,n}^k)^H \mathbf{H}^{kd} \mathbf{G}_k \mathbf{H}^{sk} \mathbf{s} + (\mathbf{w}_{2,n}^k)^H (\mathbf{G}_k \mathbf{H}^{kd} \mathbf{z}_k + \mathbf{z}_2) \tag{5}$$

where  $\mathbf{w}_{2,n}^k$  is the  $n$ th column of  $\mathbf{W}_2^k$  and  $(\bullet)^H$  denotes the Hermitian operation. The received SNR at the destination is defined as follows

$$\text{SNR}_n^k = \frac{E_s \|(\mathbf{H}^{sk})^H \mathbf{G}_k^H (\mathbf{H}^{kd})^H \mathbf{w}_{2,n}^k\|^2}{N \left( \sigma_{z_k}^2 \| \mathbf{G}_k^H (\mathbf{H}^{kd})^H \mathbf{w}_{2,n}^k \|^2 + \sigma_{z_2}^2 \| \mathbf{w}_{2,n}^k \|^2 \right)} \tag{6}$$

From this equation the SNR-based relay algorithm as summarized in Table 2.

### 4 Complexity Analysis

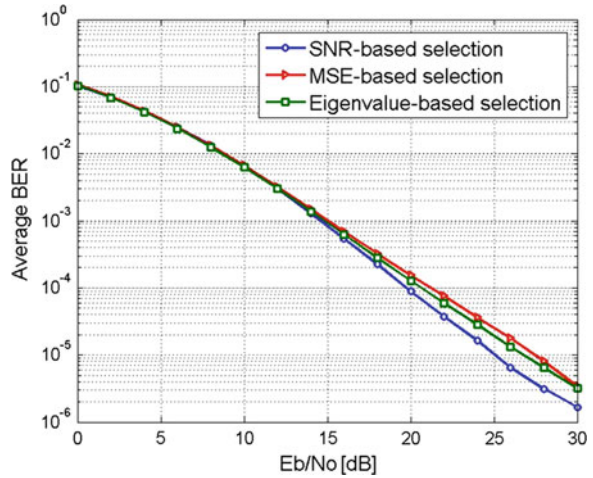
In order to compare the complexity of the proposed algorithms with that based on the MSE, we perform detailed calculation of the number of addition/subtraction, multiplication and division for all the case of complex–complex, complex–real, and real–real operations. These computational operations will be then converted into floating points (flops) for comparison. The complexity of the SNR-based algorithm involves mainly with calculating (5) and (7). The approximated complexity of the SNR-based algorithm is given by  $C_{\text{SNR}} = 36N^3 + 34N^2 + 28N + 5$  [flops]. The main complexity of the eigenvalue-based algorithm is used for computing the eigenvalues of the two square matrices  $\mathbf{H}^{sk}$ ,  $\mathbf{H}^{kd}$  both of the same

size  $N \times N$ . The complexity for calculating the eigenvalues using singular value decomposition is  $72N^3$  [flops]. The complexity of the MSE-based relay selection algorithm mainly involves with calculating equations (30), (34) and (35) in [5]. The number of computational operations required by the MSE-based algorithm is  $C_{\text{MSE}} = 20N^3 + 26N^2 + 4N + 3$  [flops]. Therefore, it is clear that all the algorithms have the same complexity order  $O(N^3)$ .

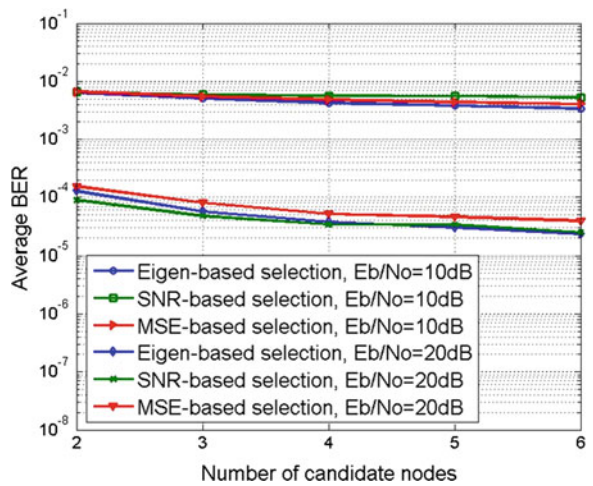
## 5 Simulation Results

In order to demonstrate the advantage of the proposed algorithms, we have performed Monte-Carlo simulations to obtain the average BER. In the first simulation, we use a simple model with three nodes, i.e., source, relay, and destination. In order to select the relay, we assume that there are two intermediate nodes within the coverage area of the source and destination. The proposed algorithms will be used to select the better node as the relay. The channels between the source to intermediate nodes and from the intermediate nodes to the destination are all assumed to undergo flat uncorrelated Rayleigh fading. All nodes are equipped with two antennas and transmit BPSK signal over the two parallel branches. The average symbol energy of each node is normalized to  $E_S$ . The destination employs the MMSE detector in [5] to estimate the transmit signal. In the second simulation, we use a similar model but the ratio  $E_b/N_0$  is fixed while the number of intermediate nodes is increased to analyze the effect of selecting a relay from a large number of nodes. In all simulations, BER of the MSE-based algorithm is also plotted for comparison. The average BER curves obtained using the proposed algorithms and the MSE based are shown in Fig. 2. It can be seen clearly from the figure that both the proposed algorithms have the same BER performance at the low  $E_b/N_0$  region but outperform the MSE-based for large  $E_b/N_0$ . Specifically, at  $\text{BER} = 10^{-5}$ , the proposed eigenvalue based algorithm has about 0.5 dB better  $E_b/N_0$  while the SNR based achieves up to 2.5 dB improvement. It is also clear that the gap between the SNR-based algorithm and the MSE-based is much larger than that of the eigenvalue-based. Figure 3 illustrates the BER performance of the three algorithms obtained at  $E_b/N_0 = 10, 20$  dB for the case of  $N = 2, 3, 4, 5, 6$ . It still can be seen that the two proposed algorithms achieve better BER performance than the MSE-based, particularly at high  $E_b/N_0$ . However, similar to [5] it is interesting to note that increasing the number of intermediate nodes does not achieve better improvement. This is the inherent property of the MIMO-SDM cooperative communication as explained in [5].

**Fig. 2** BER performance of different selection algorithm,  $N = 2$ ; 2 select 1



**Fig. 3** BER performance versus the number of candidate nodes



## 6 Conclusions

In this paper, we have proposed two relay selection algorithms based on SNR and eigenvalue for the MIMO-SDM cooperative communication networks. Both the proposed algorithms have better BER performance over the previous MSE-based algorithm. We have also carried out detailed complexity analysis to show that both the proposed algorithms and the MSE-based have the same complex order  $O(N^3)$ . The proposed SNR-based algorithm was shown to be the best candidate in terms of both BER performance and required complexity.

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