Interpolative Reasoning Approach to Sparse General Type-2 Fuzzy Rules Based on the Reduced Grid Representation

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***Abstract*—Interpolative reasoning is one of the most interested problems with various approaches for type-1 fuzzy sets, interval type-2 fuzzy sets, recently. However, the related methods have not mentioned general type-2 fuzzy sets yet because of their computational complexity. The paper deals with an approach to representation theorem of general type-2 fuzzy sets using the reduced grid. A computational schema for interpolative reasoning of sparse general type-2 fuzzy rules is also introduced. This schema is not depended on the shape of membership functions. Beside, the parallelizing schema for GPU platform is proposed to speed-up the algorithms. The proposed methods are implemented on both of GPU and CPU platforms with various membership functions.**

***Index Terms*—Interpolative reasoning; sparse fuzzy rule; general type-2 fuzzy sets; GPU parallel computing;**

I. Introduction

Interpolative reasoning is to find the fuzzy interpolated result for sparse fuzzy rules to reduce the complexity of fuzzy models. Many approaches have proposed to gain the quality interpolated fuzzy sets. However, almost approaches are for type-1 fuzzy sets. Recently, some interpolative reasoning methods have mentioned to interval type-2 fuzzy sets. For type-1 fuzzy sets, Koczy and Hirota [14] firstly proposed a reasoning approach using linear interpolation based on the proportion of distances between fuzzy sets. P.Baranyi et al [8] proposed an interpolation methodology based on the interpolation of relations in which the fuzzy and semantic-relations are used. Z. Huang et al [9] proposed an approach to fuzzy interpolation using means of scale and move transformations in which interpolating method is designed for complex polygon, Gaussian or other bell-shaped fuzzy membership functions. For technique of interpolation for sparse fuzzy rules based on polygon representation of membership functions, S.M.Chen et al [10], [11], [12] proposed several methods. In [10], authors had generalized the membership functions as polygon fuzzy sets for interpolative reasoning based the ratio of fuzziness using the area of fuzzy sets. A technique of interpolation based on α-cuts and ration transformation [11] had been also proposed for various membership functions. The weighted fuzzy interpolative reasoning [12] based on weighted increment transformation and weighted ratio transformation techniques was also proposed using polygonal membership functions of fuzzy sets.

Type-2 fuzzy models are normally very complex to inference, especially general type-2 fuzzy logic systems. Many studies have done to reduce the computational complexity for general type-2 fuzzy sets [1], [2], [3], [5], [7]. Several techniques of interpolative reasoning have proposed for interval type-2 fuzzy sets based on the ratio of fuzziness [19] or combination of ratio of fuzziness and genetic algorithms to learn optimal interval type-2 Gaussian fuzzy sets [20]. However, no technique related to interpolation reasoning have mentioned for general type-2 fuzzy sets because of the computational complexity.

The paper deals with an approach to interpolative reasoning for sparse general type-2 fuzzy rules based on the reduced grid representation and the weight of rules. For type-2 fuzzy sets, computation approaches depend on their representation theorem. Hence, this paper has approached the interpolative reasoning based on reduced grid. The reduced grid representation for general type-2 fuzzy sets is for optimizing the memory instead of grid representation. A process of interpolative reasoning for sparse general type-2 fuzzy rules has proposed with some experiments. A diagram of parallel computation on GPU platform is also introduced with various experiments in comparison with CPU platform. The summarised data shows the advantage of the proposed methods.

The paper is organized as follows: II presents an overview on type-2 fuzzy sets; III introduces reduced grid representation for general type-2 fuzzy sets; IV presents the proposed process of interpolative reasoning for sparse general fuzzy rules; V introduces implementation and discussion; VI is conclusion and future works.

II. Preliminaries

A. Type-2 fuzzy sets

A type-2 fuzzy set in X is denoted $\tilde{A}$, and its membership grade of $x \in X$ is $\mu_{\tilde{A}}(x,u), u \in J_x \subseteq [0,1]$, which is a type-1 fuzzy set in [0, 1]. The elements of domain of $\mu_{\tilde{A}}(x,u)$ are called primary memberships of x in $\tilde{A}$ and memberships

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of primary memberships in $\mu_{\tilde{A}}(x, u)$ are called secondary memberships of x in $\tilde{A}$.

Definition 2.1: A $type-2\ fuzzy\ set$, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$, i.e.,

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u))|\forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (1)$$

or

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x, u))/(x, u), J_x \subseteq [0, 1] \quad (2)$$

in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$.

At each value of x, say $x = x'$, the 2-D plane whose axes are u and $\mu_{\tilde{A}}(x', u)$ is called a $vertical\ slice$ of $\mu_{\tilde{A}}(x, u)$. A *secondary membership function* is a vertical slice of $\mu_{\tilde{A}}(x, u)$. It is $\mu_{\tilde{A}}(x = x', u)$ for $x \in X$ and $\forall u \in J_{x'} \subseteq [0, 1]$, i.e.

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} f_{x'}(u)/u, J_{x'} \subseteq [0, 1] \quad (3)$$

in which $0 \leq f_{x'}(u) \leq 1$.

Type-2 fuzzy sets are called an interval type-2 fuzzy sets if the secondary membership function has $f_{x'}(u) = 1\ \forall u \in J_x$ i.e. a type-2 fuzzy set is defined as follows:

Definition 2.2: An *interval type-2 fuzzy set* $\tilde{A}$ is characterized by an interval type-2 membership function $\mu_{\tilde{A}}(x, u) = 1$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$, i.e.,

$$\tilde{A} = \{((x, u), 1)|\forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (4)$$

Uncertainty of $\tilde{A}$, denoted FOU, is union of primary functions i.e. $FOU(\tilde{A}) = \bigcup_{x\in X} J_x$. Upper/lower bounds of membership function (UMF/LMF), denoted $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, of $\tilde{A}$.

B. Operations

Let $\tilde{A}, \tilde{B}$ be type-2 fuzzy sets whose secondary membership grades are $f_x(u), g_x(w)$, respectively. Theoretic operations of type-2 fuzzy sets such as union, intersection and complement are described [1] as follows:

$$\mu_{\tilde{A}\cup\tilde{B}}(x) = \mu_{\tilde{A}}(x) \sqcup \mu_{\tilde{B}}(x) = \int_u\int_v (f_x(u) \star g_x(w))/(u \vee w) \quad (5)$$

$$\mu_{\tilde{A}\cap\tilde{B}}(x) = \mu_{\tilde{A}}(x) \sqcap \mu_{\tilde{B}}(x) = \int_u\int_v (f_x(u) \star g_x(w))/(u \star w) \quad (6)$$

$$\mu_{\tilde{A}}(x) = \mu_{\neg\tilde{A}}(x) = \int_u (f_x(u))/(1 - u) \quad (7)$$

where $\vee$, $\star$ are t-cornorm, t-norm, respectively.

Join operation will be used for fusion of interpolated type-2 fuzzy sets. The following is description in the case of the discretized domain. Suppose more than one calculation of u and w gives the same point $u \vee w$, for example, $u_1 \vee w_1 = \theta^*$ and $u_2 \vee w_2 = \theta^*$. Then within the computation of (5), we would have

$$f_x(u_1) \star g_x(w_1)/\theta^* + f_x(u_2) \star g_x(w_2)/\theta^* \quad (8)$$

where + denotes union. Combining these two terms for the common θ^* is a type-1 computation in which t-conorm can be used, e.g. the maximum.

If $\theta \in F \sqcup G$, the possible $\{u, w\}$ pairs that can give θ as the result of the maximum operation are $\{u, \theta\}$ where $u \in (-\infty, \theta]$ and $\{\theta, w\}$ where $w \in (-\infty, \theta]$. The process of finding the membership of θ in $\tilde{A} \sqcup \tilde{B}$ can be computed as follows:

$$f_{F\sqcup G}(\theta) = \phi_1(\theta) \vee \phi_2(\theta) \quad (9)$$

where

$$\phi_1(\theta) = \sup_{u\in(-\infty,\theta]} \{f_x(u)\wedge g_x(\theta)\} = g_x(\theta)\wedge \sup_{u\in(-\infty,\theta]} \{f_x(u)\} \quad (10)$$

and

$$\phi_2(\theta) = f_x(\theta) \wedge \sup_{w\in(-\infty,\theta]} \{g_x(w)\} \quad (11)$$

III. Interpolative Reasoning for General Type-2 Fuzzy Rules

A. The reduced grid representation of general T2FS

In [4], [6], a grid representation of general type-2 fuzzy sets has been proposed for speed-up computation based on GPU platform. This section re-describes with some improvement of grid representation, called *reduced grid*, for computation of interpolative reasoning of sparse general type-2 fuzzy rules. This grid only describes cells that belong to the FOU of type-2 fuzzy sets. Hence, some concepts are described as follows:

Let X be domain of type-2 fuzzy set $\tilde{A}$ and $U = [u_{min}, u_{max}] \subseteq [0, 1]$ be secondary domain of $\tilde{A}$. Suppose that $X = [x_{min}, x_{max}]$. The space $X \times U$ can be divided into a grid being union of $M \times N$ cells, in which $M = [(x_{max}-x_{min})/dx]$ and $N = [(u_{max}-u_{min})/du]$. According to this way, a sub type-2 fuzzy set $\tilde{A}_{ij}$ in domain of the cell (i, j) is described as follows:

$$\tilde{A}_{ij} = \{((x, u), \mu_{\tilde{A}}(x, u)|x \in X_i = [\underline{x}_i, \overline{x}_i], u \in U_j = [\underline{u}_j, \overline{u}_j]\} \quad (12)$$

in which $\underline{x}_i = x_{min} + i * dx$, $\overline{x}_i = \underline{x}_i + dx$, $\underline{u}_j = u_{min} + j * du$, $\overline{u}_j = \underline{u}_j + du$, $i = \overline{0, N-1}$ and $j = \overline{0, M-1}$.

To define grid type-2 fuzzy set, called $\tilde{A}^g$, an approximate representation of sub type-2 fuzzy set $\tilde{A}_{ij}$ by a cell type-2 fuzzy set $\tilde{A}^c_{ij}$ is introduced.

Provide that (x, u) is a point in the cell (i, j) then the membership grade at (x, u) of $\tilde{A}$ is computed as:

$$f_{ij}(x, u) = h_{\tilde{A}^c_{ij}}(x, u) = \sum_{k=1}^{4} f^{(k)}/4 \quad (13)$$

in which $f^{(k)}$ is membership grade at the k^{th} vertex and $h_{\tilde{A}^c_{ij}}$ is also called the height of the cell $\tilde{A}^c_{ij}$.

Definition 3.1: A cell type-2 fuzzy set, denoted $\tilde{A}^c_{ij}$, is approximate representation of sub type-2 fuzzy set $\tilde{A}_{ij}$ and is defined as follows:

$$\tilde{A}^c_{ij} = \{((x, u), \mu_{\tilde{A}^c_{ij}}(x, u))|x \in [\underline{x}_i, \overline{x}_i], u \in [\underline{u}_i, \overline{u}_i]\} \quad (14)$$

in which $\mu_{\tilde{A}^c_{ij}}(x,u)) = f_{ij}(x,u)$, $\sum_{k=1}^{4} f^{(k)} > 0$.

Definition 3.2: A reduced grid type-2 fuzzy set, denoted $\tilde{A}^g$, is union of above defined cell type-2 fuzzy set, i.e

$$\tilde{A}^g = \bigcup_{i=0}^{N-1} \bigcup_{j=0}^{M-1} \tilde{A}^c_{ij} \tag{15}$$

$$FOU_{\tilde{A}} \simeq FOU_{\tilde{A}^g} = \bigcup_{i=0}^{N-1} \bigcup_{j=0}^{M-1} FOU_{\tilde{A}^c_{ij}} \tag{16}$$

Definition 3.3: The degree of approximation (DoA) is the difference between original type-2 fuzzy set $\tilde{A}$ and reduced grid type-2 fuzzy set $\tilde{A}^g$ and is defined as follows:

$$DoA = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \triangle_{ij}/n_c \tag{17}$$

in which $\triangle_{ij} = |\mu_{\tilde{A}}(x^c_i, u^c_j) - f_{ij}(x^c_i, u^c_j)|$, $x^c_i = \frac{1}{2}(\underline{x}_i + \overline{x}_i)$, $u^c_i = \frac{1}{2}(\underline{u}_i + \overline{u}_i)$ and n_c is the number of cells.

Example 3.1: Let $\tilde{A}$ is a general type-2 fuzzy set. The feature membership functions of $\tilde{A}$ are described as follows:

FOU is Gaussian function with upper MF and lower MF as follows:

Upper MF of FOU:

$$f_u(x) = \begin{cases} e^{-\frac{1}{2}(\frac{x-m_1}{\sigma})^2} & if\ x < m_1 \\ 1 & if\ m_1 \le x \le m_2 \\ e^{-\frac{1}{2}(\frac{x-m_2}{\sigma})^2} & if\ x > m_2 \end{cases} \tag{18}$$

Lower MF of FOU:

$$f_l(x) = \begin{cases} e^{-\frac{1}{2}(\frac{x-m_2}{\sigma})^2} & if\ x < \frac{m_1+m_2}{2} \\ e^{-\frac{1}{2}(\frac{x-m_1}{\sigma})^2} & if\ otherwise \end{cases} \tag{19}$$

where $m_1 = 3.0$, $m_2 = 4.0$ and $\sigma = 0.5$.

The next feature of $\tilde{A}$ is set of points where $\mu_{\tilde{A}}(x,u) = 1.0$, involves points belong to the MF described as follows:

$$f_m(x) = e^{-\frac{1}{2}(\frac{x-(m_1+m_2)/2}{\sigma})^2} \tag{20}$$

The secondary membership function at x of $\tilde{A}$ are Gaussian functions that are described as follows:

$$g(u) = \begin{cases} e^{-\frac{1}{2}(\frac{u}{\sigma_1})^2} & if\ u \ge u_0 \\ e^{-\frac{1}{2}(\frac{u}{\sigma_2})^2} & if\ u < u_0 \end{cases} \tag{21}$$

in which $u_0 = f_m(x)$, $\sigma_1 = 3.035 * |\overline{u} - u_0|$ and $\sigma_2 = 3.035 * |\underline{u} - u_0|$.

Fig. 1 depicts a reduced grid T2FS with various parameters of M, N. The implementation has gained the results as Table I.

TABLE I
GRID TYPE-2 FUZZY SETS WITH VARIOUS PARAMETERS

M × N	DoA	M × N	DoA
64 × 16	0.05493	1024 × 256	0.00083
256 × 64	0.00745	4096 × 1024	0.00009

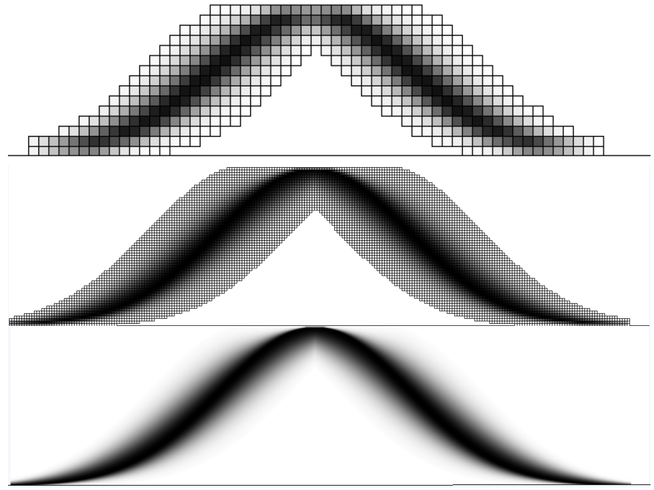

Fig. 1. Reduced grid type-2 fuzzy set; Top: $M = 64, N = 16$; Middle: $M = 256, N = 64$; Bottom: $M = 1024, N = 256$.

B. Proposed method for interpolative reasoning of sparse general type-2 fuzzy rules

Let us consider the following fuzzy rules interpolation schema in which $\tilde{A}_{i,j}$, $\tilde{B}_i$ $(i = 1, 2, \ldots, k,\ j = 1, 2, \ldots, m)$ are reduced grid - based general T2FSs.

Rule 1: **If** X_1 is $\tilde{A_{1,1}}$ and X_2 is $\tilde{A_{1,2}}$ and ... and X_m is $\tilde{A_{1,m}}$ **Then** Y is $\tilde{B}_1$

Rule 2: **If** X_1 is $\tilde{A_{2,1}}$ and X_2 is $\tilde{A_{2,2}}$ and ... and X_m is $\tilde{A_{2,m}}$ **Then** Y is $\tilde{B}_2$

$\vdots$

Rule n: **If** X_1 is $\tilde{A_{n,1}}$ and X_2 is $\tilde{A_{n,2}}$ and ... and X_m is $\tilde{A_{n,m}}$ **Then** Y is $\tilde{B}_n$.

Observation: X_1 is $\tilde{A}^*_1$ and X_2 is $\tilde{A}^*_2$ and ... and X_m is $\tilde{A^*_m}$

Conclusion: Y is B^*.

For interpolative reasoning of above fuzzy rule schema, we define some concepts related to a general T2FS.

Definition 3.4: Let $\tilde{A}$ be the reduced grid general T2FS with grid size $M \times N$. A representative point of $\tilde{A}$ is its centroid and representative value, denoted $Rep(\tilde{A})$, is computed as follows:

$$Re(\tilde{A}) = \frac{\sum_{0 \le i < M, 0 \le j < N} \left(c_{\tilde{A}^c_{ij}} \times v_{\tilde{A}^c_{ij}}\right)}{\sum_{0 \le i < M, 0 \le j < N} \left(v_{\tilde{A}^c_{ij}}\right)} \tag{22}$$

where $c_{\tilde{A}^c_{ij}}, v_{\tilde{A}^c_{ij}}$ is the centroid and the volume of the cell T2FS $\tilde{A}^c_{ij}$ that are computed as $c_{\tilde{A}^c_{ij}} = (\underline{x_i} + \overline{x_i})/2$ and $v_{\tilde{A}^c_{ij}} = dx \times du \times h_{\tilde{A}^c_{ij}}$.

For each rule, its weight is defined as the rate between the one and observation in comparison with other ones. The rule's weights are computed as the way described in [20], i.e., assume that $\tilde{A}_{l_j,j}$, $\tilde{A}_{r_j,j}$ are the left nearest antecedent fuzzy set and the right nearest antecedent fuzzy set with respect to the observation fuzzy set $\tilde{A}_j^*$, respectively, where $1 \le l_j \le n$, $1 \le r_j \le n$ and $1 \le j \le m$, then let

$$w_k = min_{1\le j\le m}\left(1 - \frac{|Re(\tilde{A}_j^*) - Re(\tilde{A}_{k,j})|}{Re(\tilde{A}_{l_j,j}) - Re(\tilde{A}_{r_j,j})}\right) \quad (23)$$

$$W_k = \frac{w_k}{\sum_{i=1}^{n} w_k} \quad (24)$$

The proposed approach for interpolative reasoning of reduced grid general type-2 fuzzy rules is described as follows:

Step 1: Compute the representative values (using equation (22)) of all reduced grid general T2FSs of the observation $\tilde{A}_j^*$, $0 \le j \le m$.

Step 2: For the rule k $(0 \le k \le n)$, compute the representative values (using equation (22)) of all reduced grid general T2FSs appearing in the antecedent part of the rule $\tilde{A}_{k,j}$, $0 \le j \le m$.

Step 3: Compute the weights of all rules in the fuzzy rule schema W_k, $0 \le k \le n$, using the equations (23) and (24).

Step 4: Compute the representative values of consequence type-2 fuzzy sets $\tilde{B}_k$, $0 \le k \le n$. The representative value of $\tilde{B}^*$ is computed as follows:

$$Re(\tilde{B}^*) = \sum_{k=1}^{n} W_k \times Re(\tilde{B}_k) \quad (25)$$

Step 5: The algorithm for computing membership grade of $\tilde{B}^*$ is described as the following algorithms:

1. Compute $k_0 = \left[\frac{Re(\tilde{B}^*) - x_{min}}{dx}\right]$, where x_{min} is smallest value of the domain X, dx is size of grid along x-axis and [ ] is integer part operation.

2. As the same way, the indices in the grid of the representative values of the consequence T2Fs $\tilde{B}_k$ are computed as follows:

$$k_i = \left[\frac{Re(\tilde{B}_i) - x_{min}}{dx}\right] \quad (26)$$

where $0 \le i \le n$.

3. Let $s_{\tilde{B}^*,j} = \{u_{k_1}, u_{k_1+1}, ..., u_{k_2} | 0 \le k_1 \le k_2 < N\}$ be the set of vertices in the grid of $\tilde{B}_j$ along vertical slide x_j then the support $\|s_{\tilde{B}^*,j}\| \le N$. Compute the membership grade at vertices of $\tilde{B}^*$ is described as follows:

3.1. *The meet operation:* $s_{\tilde{C},i} = Meet(s_{\tilde{A},i},\ s_{\tilde{B},i})$

1) Call $i_1 = min\{k | h_{\tilde{A}}(x_i, u_k) > 0$ or $h_{\tilde{A}}(x_i, u_{k+1}) > 0\}$
$i_2 = max_k\{k | h_{\tilde{A}}(x_i, u_k) > 0$ or $h_{\tilde{A}}(x_i, u_{k+1}) > 0\}$.
As similar way, j_1, j_2 are responding to $\tilde{B}$.

2)
for $k = 0$ to $N-1$ **do**
  a) $ip = k > i_1 ? k : i_1$.
  b) $f_1 = max_{j=ip}^{N-1} h_{\tilde{A}}(x_i, u_j)$
  c) $f_2 = max_{j=ip}^{N-1} h_{\tilde{B}}(x_i, u_j)$
  d) $t_1 = min(h_{\tilde{B}}(x_i, u_j), f_1)$.
  e) $t_2 = min(h_{\tilde{A}}(x_i, u_j), f_2)$.
  f) $h_{\tilde{C}}(x_i, u_k) = max(t_1, t_2)$;
end for

3.2. *The algorithm for computing membership grades*
for $i = 0$ to $M-1$ **do**
  1) $x_0 = k_1 - (k_0 - i)$;
  2) $s_{\tilde{C},i} = s_{\tilde{B}_1,x_0}$.
  for $j = 2$ to n **do**
    a) $x_0 = k_j - (k_0 - i)$;
    b) $s_{\tilde{D},i} = Meet(s_{\tilde{C},i}, s_{\tilde{B}_j,x_0})$;
    c) $s_{\tilde{C},i} \longleftarrow s_{\tilde{D},i}$;
  end for
end for

The proposed method of interpolative reasoning is general computational process for sparse type-2 fuzzy rules based on reduced grid representation. This method is not depended on the shape of membership functions, i.e can be applied for Triangular/Trapezoid functions, Gaussian functions, Bell-shaped functions or other shaped functions.

C. GPU implementation

GPU is the computing platform for speeding up problems with huge computational complexity. The reduced grid representation results in the high accuracy by using smaller size of grid, i.e. it takes the huge computational time. GPU-based acceleration is the suitable approach for this problems.

Firstly, we organize memory for parallelizing the process of interpolation reasoning related the data structures and move or copy easy between host memory and device memory. Fig. 2 shows the way to organize a memory for reduced grid general T2FS. In which, each vertical slide of the grid is stored in a memory segment with $k_2^i - k_1^i + 2$ blocks. Two head blocks are used to store indices of the first index and last index of $s_{\tilde{A},i}$, i.e. k_1^i and k_2^i. The remaining blocks are used to store the membership grades of vertices in $s_{\tilde{A},i}$. In the case of $\|s_{\tilde{A},i}\| = \oslash$, memory is only used one blocks for "-1" flag to skip the slide. These memory segments are joined to an one-dimensional array. Hence, total of the used memory is less than $\sum_{i=1}^{M}(k_2^i - k_1^i + 2)$ blocks.

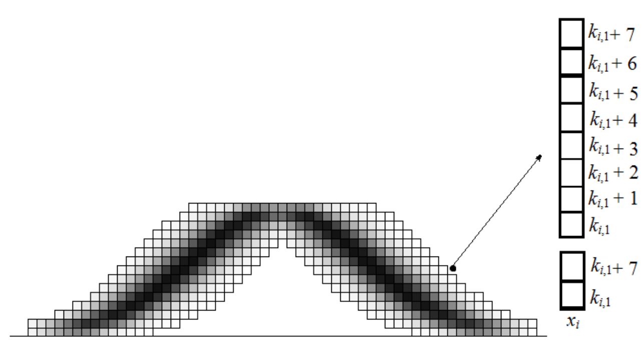

Fig. 2. Memory for reduced grid general T2FS with $k_{i,1} = 0$.

Secondly, we have organized procedures which are implemented on GPU platform, involving *ComputeRep*, *ComputeConclusion*. *ComputeRep* is to compute the representative value of the reduced grid general T2FS. If there are more than one T2FS, a computational matrix on GPU device is made to compute parallel all of these T2FSs. In that case, the memory of grids is 2-dimentional matrix described as Fig. 3. *ComputeConclusion* is a procedure to compute parallel the membership grade at grid's vertices of $\tilde{B}^*$ as described in *Step 5.3*. A 2-dimensional computational matrix is set up to compute these membership grades concurrently.

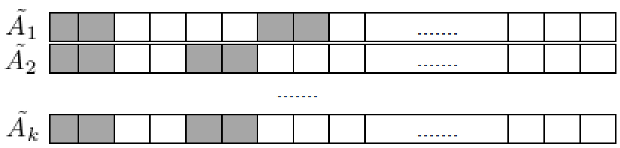

Fig. 3. Memory used in the case of multi T2FSs.

Computational schema on both of GPU and CPU platforms is described as Fig. 4 in which two color-filled blocks (no. 1 and 5) are implemented on GPU. Other blocks are implemented on CPU involving block no. 2, 3 and 4.

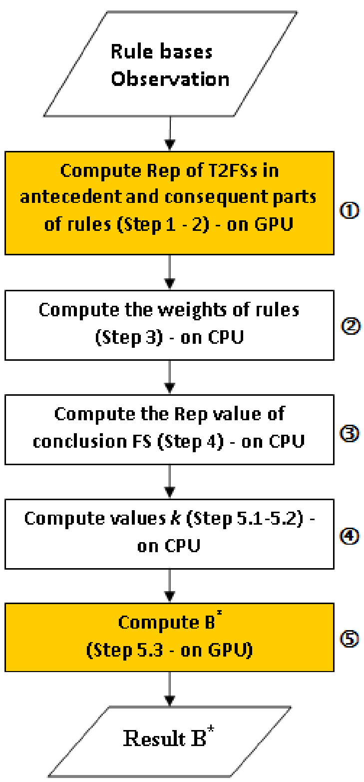

Fig. 4. Diagram of interpolative reasoning schema.

IV. Experimental Results

In this section, we mention some examples of fuzzy interpolation schemas with various shaped functions as Gaussian functions, Trapezoid functions.

Experiment 4.1: Suppose that we have fuzzy rule schema as the Fig. 5 with four rules as follows:

Rule 1: **If** X_1 is $\tilde{A}_{1,1}$ and X_2 is $\tilde{A}_{1,2}$ **Then** Y is $\tilde{B}_1$

Rule 2: **If** X_1 is $\tilde{A}_{1,1}$ and X_2 is $\tilde{A}_{2,2}$ **Then** Y is $\tilde{B}_2$

Rule 3: **If** X_1 is $\tilde{A}_{2,1}$ and X_2 is $\tilde{A}_{1,2}$ **Then** Y is $\tilde{B}_3$

Rule 4: **If** X_1 is $\tilde{A}_{2,1}$ and X_2 is $\tilde{A}_{2,2}$ **Then** Y is $\tilde{B}_4$

Observation: X_1 is $\tilde{A}_1^*$ and X_2 is $\tilde{A}_2^*$

Conclusion: Y is B^*.

in which $\tilde{A}_{11}$, $\tilde{A}_{12}$, $\tilde{A}_1^*$, $\tilde{A}_{21}$, $\tilde{A}_{22}$, $\tilde{A}_2^*$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$, $\tilde{B}^*$ are reduced grid general T2FSs with their membership functions in Fig. 5.

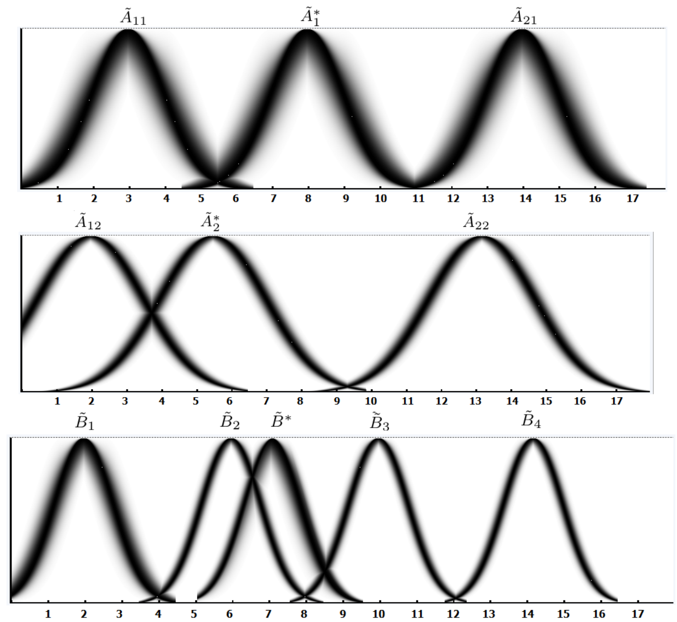

Fig. 5. Fuzzy rule schema and the fuzzy interpolated results of the proposed method.

Now, we apply the above proposed method to find the conclusion T2FS $\tilde{B}^*$. The result is described as follows:

Step 1: The representative values of the consequent T2FSs with $M \times N = 1024 \times 256$: $Re(\tilde{B}_1) = 2.045$, $Re(\tilde{B}_2) = 6.000$, $Re(\tilde{B}_3) = 9.999$ and $Re(\tilde{B}_4) = 14.193$.

Step 2: The representative values of the antecedent T2FSs with $M \times N = 1024 \times 256$: $Re(\tilde{A}_{11}) = 3.036$, $Re(\tilde{A}_{12}) = 2.727$, $Re(\tilde{A}_{21}) = 14.038$ and $Re(\tilde{A}_{22}) = 13.189$.

Step 3: The weights of rules: $W_1 = 0.362$, $W_2 = 0.172$, $W_3 = 0.294$ and $W_4 = 0.172$.

Step 4: The representative value of conclusion: $Re(\tilde{B}^*) = \sum_{i=1}^{4} W_i \times Re(\tilde{B}_i) = 7.153$.

Step 5:

1. $k_0 = \left[\dfrac{Re(\tilde{B}^*) - x_{min}}{dx} \right] = 406 \in [0, M] \equiv [0, 1024]$.

2. $k_1 = 116$, $k_2 = 341$, $k_3 = 568$ and $k_4 = 806 \in [0, M] \equiv [0, 1024]$.

3. Compute membership grade at vertices of $\tilde{B}^*$. The fuzzy interpolated result of $\tilde{B}^*$ is depicted as Fig. 5. Fig. 6 shows the interpolated result for interval type-2 fuzzy sets of the Chen's method [19]. These two results are similar with respect to $\tilde{B}^*$'s FOU.

Experiment 4.2: ***GPU implementation.***

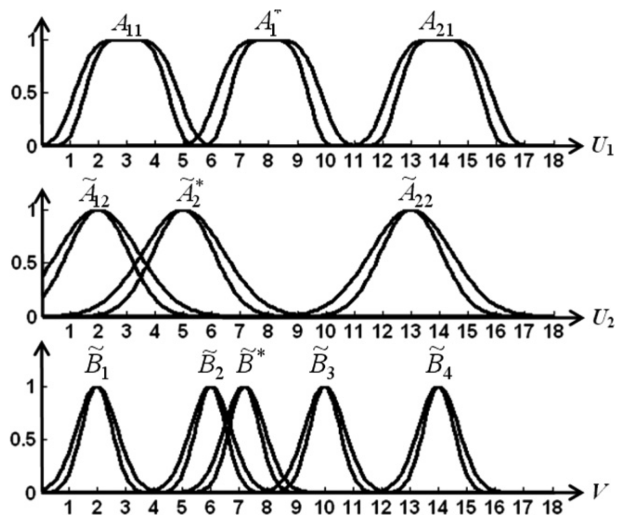

Fig. 6. The fuzzy interpolated results of the Chen's method [19].

The proposed method is implemented on the GPU in comparison with the one on the CPU. The experimental platform is on the computer with operating system Windows 7 64bit and nVIDIA CUDA support with specifications: CPU Core i7-460M 3.6 GHz; 8 Gb RAM (DDR3); GPU is nVIDIA Gerforce GTX 680 with 1536 CUDA cores, 2GB of texture memory.

We summarised the computational time of two operations in which the one is the single operation for computing representative value, the one is the process of interpolative reasoning. The operation for computing representative value is for T2FS $\tilde{A}_1^*$ with various size of the grid. The computational time is summarised in the table II in which the one on GPU platform takes few time even thought in the case of large size of grid. Secondly, all process of interpolative reasoning as the experiment 4.1 are implemented. The summarized data in the table III show that the speed-up rate are much larger if the size of grid is much larger. The fuzzy interpolated results are similar the ones on CPU in Fig. 5.

TABLE II
COMPUTING REPRESENTATIVE VALUE

M × N	GPU (ms)	CPU (ms)	Rate
512 × 128	0.298	0.430	1.444
1024 × 256	0.435	1.578	3.629
2048 × 512	0.751	6.320	8.417
4096 × 1024	1.572	25.147	15.996
8192 × 2048	4.088	100.596	24.607

V. CONCLUSION

The paper introduces an approach to interpolative reasoning for sparse general type-2 fuzzy rules based on reduced grid representation of general T2FS. The fuzzy interpolated results are computed using the weights of rules. The interpolative reasoning processes are described on sequence algorithms

TABLE III
COMPUTATIONAL TIME OF INTERPOLATIVE REASONING

M × N	GPU (ms)	CPU (ms)	Rate
256 × 64	56.982	14.749	0.258
512 × 128	58.895	85.342	1.449
1024 × 256	68.575	555.017	8.094
2048 × 512	112.907	3898.604	34.529
4096 × 1024	376.891	29158.566	77.366
8192 × 2048	2268.398	227748.063	100.400

on GPU and parallel algorithms of GPU. Experiments are implemented on both GPU/GPU platforms.

Further researches about interpolative reasoning for general type-2 fuzzy rules may be developed based on this representation with different approaches.

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