Intuitionistic Type-2 Fuzzy Set Approach to Image Thresholding

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***Abstract*—In this paper, an image thresholding method based on Intuitionistic Type-2 Fuzzy Sets (InT2FS) method is introduced for the segmentation problems. Besides, intuitionistic type-2 fuzzy set has been formed as an extension of intuitionistic fuzzy set for handling uncertainty. As we know, the image data which usually contains noises or uncertainty so then utilizing the advantages of the InT2FS, we have introduced a thresholding algorithm using InT2FS for image thresholding. Experimental results with different types of images show that the proposed algorithm is better than the traditional thresholding algorithms especially with noisy images.**

***Index Terms*—Image thresholding, Intuitionistic fuzzy set, intuitiuonistic type-2 fuzzy set, type-2 fuzzy set.**

I. Introduction

Thresholding is one of the important techniques for image segmentation because it is used to split an image into smaller regions or segments by establishing at least one threshold value to define their boundary.

One of the most commonly used strategy for segmenting images is global thresholding that divide the pixels in an image on the basis of their intensity levels of gray by establishing one threshold value. Extensive research has been conducted to introduce many types of segmentation techniques in general and thresholding techniques in particular. These approaches are mainly based on a certain methodology to threshold images [6], [9]–[11]. Sankura and Sezginb [18] also list over 40 different thresholding techniques.

With the framework of fuzzy theory, fuzzy techniques are suitable for the development of new thresholding algorithms because they are able to remove vagueness/imprecision in the data [3], [8], [12]. The most popular thresholding algorithms were introduced in [3], [17], [19] based on the concept of fuzzy entropy. The main problem of these approaches is to handle the uncertainty for the membership functions that assigns each pixel either to the background or to the object.

Recently, Type-2 fuzzy sets which are the extensions of original fuzzy sets, have the advantage of handling uncertainty, have been developed and applied to many different problems [13]–[15] including image segmentation problems. In addition, a new thresholding technique which processes the thresholds as type-2 fuzzy sets was introduced [21].

Besides, the intutionistic fuzzy set (IFS) was introduced [1], [2] and used for representing the hesitance of an expert on determining the membership functions that assigns each pixel either to the background or to the object. This capability has created a different direction research to handle the uncertainty based on IFS [7], [16]. IFSs also have been recently used for the thresholding techniques [4], [20].

In addition, an other improvement of the thresholding technique is the use of more than one threshold. This improvement derives from the real problems, the images usually include several objects. Thus, we often need more than one threshold in order to correctly segment the images. For this reason, the approach to multilevel thresholding is an attention of various researchers. Pinto, P. M. et al. presented a multilevel image segmentation using IFS which determines two or three threshold values segmenting the image into one background and two or three objects [5].

Through the overview of image thresholding techniques presented above, we found outstanding developments of Type-2 fuzzy sets and Intuition fuzzy sets. They are applied to handle the uncertainty. However, their uncertainty processing is not the same. While the type 2 fuzzy sets handle the uncertainty based on the membership function selections, the intuitionistic fuzzy sets assess the hesitance by using the identification of the membership function and the non-membership function. We can see that the difference here is the uncertainty and the hesitance. Many people mistakenly believe the uncertainty and the hesitance are the same, we can see that there is a little difference between them, sometime in the uncertainty, there is still hesitance and vice versa.

Therefore, in this paper, we will state intuitionistic type-2 fuzzy set on the basis of the extension of intuition fuzzy set. It has ability to handle both the hesitance and the uncertainty. Next, InT2FS is applied in image thresholding algorithm. The experimental results show that the proposed algorithm gives better results than the traditional thresholding method, especially for noisy images.

This paper is organized as follows: In Section II, some basic definitions about intuitionistic fuzzy sets as well as measures of fuzziness and image thresholding using IFS are reviewed. Section III proposes image thresholding using InT2FS based method. Section IV shows some experimental results to illustrate the effectiveness and usefulness of the proposed approach. Finally, the paper is summarized with some conclusions in Section V.

II. BACKGROUND

A. Intuitionistic fuzzy sets

1) Intuitionistic fuzzy sets: Intuitionistic fuzzy sets (IFSs) were introduced by Atanassov as an extension of the fuzzy set theory as follows:([1], [2]):

Let X be an ordinary finite non-empty set. An IFS in X is an expression $\tilde{A}$ given by:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ $v_{\tilde{A}} : X \to [0,1]$ satisfy the condition $\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$ for all $x \in X$. The numbers $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set $\tilde{A}$.

Considering IFSs(X) as the set of all the intuitionistic fuzzy sets in X. For each IFS $\tilde{A}$ in X, the value $\pi_{\tilde{A}(x)} = 1-\mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ called the degree of uncertainty of x to $\tilde{A}$, or the degree of hesitancy of x to $\tilde{A}$.

Note that for an IFS $\tilde{A}$, if $\mu_{\tilde{A}}(x) = 0$, then $v_{\tilde{A}}(x)+\pi_{\tilde{A}}(x) = 1$, and if $\mu_{\tilde{A}}(x) = 1$ then $v_{\tilde{A}}(x) = 0$ and $\pi_{\tilde{A}}(x) = 0$

2) Entropy on Intuition fuzzy sets: Most of the fuzzy algorithms select the best threshold t using the concept of fuzzy entropy. In this paper, we will focus on the definition and characterization of the intuitionistic fuzzy entropy. The entropy on IFSs is defined as a magnitude that measures the degree of IFS that a set is with respect to the fuzziness of this set which satisfy the following conditions:

1. The entropy will be null when the set is a FSs(X),
2. The entropy will be maximum if the set is an A-IFS; that is $\mu(x) = v(x) = 0$ for all $x \in X$.
3. As in fuzzy sets, the entropy of an IFS will be equal to its respective complement.
4. If the degree of membership and the degree of non-membership of each element increase, the sum will as well, and therefore, this set becomes more fuzzy, and therefore the entropy should decrease. One of the simplest expressions that satisfy the conditions previously mentioned in [16]

$$IE\left(\tilde{A}\right) = \frac{1}{n}\sum_{k=1}^{n}\pi_{\tilde{A}}(x_k) \tag{1}$$

B. Image thresholding using IFS

Considering an image I, denoted (x, y) is the coordinate of a pixel on the image I, and $q(x, y)$ is the gray level of the pixel (x, y) so that $0 \leq q(x, y) \leq L-1$ for each $(x, y) \in I$ where L is the image gray-scale. The aim of this problem is to determine the threshold t using IFSs in order to obtain a good segmentation of the considered image I.

With the image thresholding problem, the threshold $t(t = 0, 1.., L-1)$ of the gray scale L was found to divide the image into two regions: the background and the object. Each pixel belongs to the background or the object. Therefore, with a threshold t, we will have two regions corresponds to two datasets. In image thresholding techniques using IFS, each region with threshold t will be described as an intuitionistic fuzzy set. Here, we have two intuitionistic fuzzy sets $\tilde{I_{Bt}}$ and $\tilde{I_{Ot}}$ corresponding to the background and the object, respectively.

$\tilde{I_{Bt}} = \{q, \mu_{\tilde{I_{Bt}}}(q), v_{\tilde{I_{Bt}}}(q) | q = 0, 1, ..., L-1\}$
$\tilde{I_{Ot}} = \{q, \mu_{\tilde{I_{Ot}}}(q), v_{\tilde{I_{Ot}}}(q) | q = 0, 1, ..., L-1\}$

Each pixel $q(x, y)$ will have three numerical values which should be considered: a value for representing its membership to the background $\mu_{\tilde{I_{Bt}}}(q)$, a value for representing its membership to the object $\mu_{\tilde{I_{Ot}}}(q)$ and a value called intuitionistic fuzzy index denoted $\pi(q)$ for representing the hesitance of the expert in determining the membership functions $\mu_{\tilde{I_{Bt}}}$ and $\mu_{\tilde{I_{Ot}}}$.

Remark: if $\pi(q) = 0$ means that the expert is positively sure that the pixel belongs either to the background or to the object and if the expert does not know if the pixel belongs to the background or to the object means that $\mu_{\tilde{I_{Ot}}} = 0.5$, then $\pi(q) = 0.5$ means that the expert has the greatest hesitance in the construction of the functions of membership to the background and to the object respectively. Hence, the intuitionistic fuzzy index value increases with respect to the hesitance of the expert as to whether the pixel belongs to the background or the object.

In [5], [20], the algorithm was proposed for calculating the best threshold value t of an image I with L image gray-scale is made up of the following steps:

Step 1: Each membership function for a background fuzzy set is associated with a level of intensity $t, (t = 0, 1, ..L-1)$ of the gray-scale L, we construct L membership functions $\mu_{I_{Bt}}$ for L background fuzzy sets I_{Bt} associated with the image I. These sets represent the background of the image I. The membership functions are described as follows:

$$\mu_{I_{Bt}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{Bt}(q)}{L-1}\right)\right),$$
$$\mu_{I_{Ot}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{Ot}(q)}{L-1}\right)\right)$$

where

$m_O(t) = \frac{\sum_{q=t+1}^{L-1} qh(q)}{\sum_{q=t+1}^{L-1} h(q)}$, $m_B(t) = \frac{\sum_{q=0}^{t} qh(q)}{\sum_{q=0}^{t} h(q)}$

and $h(q)$ is the number of pixels of the image with the gray level q, $F(x) = 1 - 0.5x$, $d(x, y) = |x - y|$ (see [5], [20]).

Step 2: Each membership function for a object fuzzy set is associated with a level of intensity $t, (t = 0, 1, ..L-1)$ of the gray-scale L, we construct L membership functions $\mu_{I_{Ot}}$ for L object fuzzy sets I_{Ot} associated with the image I. These sets represent the object of the image I considered. The membership functions are described as in *Step 1*.

Step 3: Represent the hesitance of the expert in the construction of the sets corresponding to *Step 1* and *Step 2* by means of intuitionistic fuzzy index. We know that the choice of the membership functions is conditioned by the hesitance of the expert when constructing these membership functions.

In this approach, the hesitance of the expert is represented by means of intuitionistic fuzzy index (π) as follows:

$$\pi(q) = \wedge\left(1 - \mu_{\tilde{I}_{Bt}}(q), 1 - \mu_{\tilde{I}_{Ot}}(q)\right) \tag{2}$$

or

$$\pi(q) = \left(1 - \mu_{\tilde{I}_{Bt}}(q)\right) . \left(1 - \mu_{\tilde{I}_{Ot}}(q)\right) \tag{3}$$

Step 4: Construct the L intuitionistic fuzzy sets $\tilde{I_{Bt}}$ associated with the background of the image.

Construct IFSs $\tilde{I_{Bt}}$ and $\tilde{I_{Ot}}$ associated with π

$\tilde{I_{Bt}} = \{q, \mu_{\tilde{I_{Bt}}}(q), v_{\tilde{I_{Bt}}}(q) | q = 0, 1, ..., L-1\}$

where

$\mu_{\tilde{I}_{Bt}}(q) = \mu_{I_{Bt}}(q)$

$v_{\tilde{I_{Bt}}}(q) = 1 - \mu_{\tilde{I_{Bt}}}(q) - \pi(q)$

and

$\tilde{I_{Ot}} = \{q, \mu_{\tilde{I_{Ot}}}(q), v_{\tilde{I_{Ot}}}(q) | q = 0, 1, ..., L-1\}$

$\mu_{\tilde{I}_{Ot}}(q) = \mu_{I_{Ot}}(q)$

$v_{\tilde{I_{Ot}}}(q) = 1 - \mu_{\tilde{I_{Ot}}}(q) - \pi(q)$

Step 5: Calculate the entropy ε_T of each one of the L intuitionistic fuzzy sets $\tilde{I_{Bt}}$

At this step, the entropy ε_T of L intuitionistic fuzzy set (constructed in *step 4*) is calculated by: (see [20])

$$\varepsilon_T (I_{Bt}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q) \pi(q) \tag{4}$$

Step 6: Take as best threshold the value of t associated with the intuitionistic fuzzy set $\tilde{I}_{Bt}$ of lowest entropy ε_T.

Find out the gray level q that entropy ε_T reached the minimum value, then q would be best selected threshold t associated with the intuitionistic fuzzy set $\tilde{I_{Bt}}$

III. Image thresholding using InT2FSs

A. Basic concepts

We give some basic concepts which are used for thresholding algorithm using InT2FSs for image segmentation problems. These basic concepts are extended by combining of type-2 fuzzy sets and intuitionistic fuzzy sets.

1) Intuitionistic type-2 fuzzy sets: A intuitionistic type-2 fuzzy set in X is denoted $\tilde{A}^*$, and its membership grade of $x \in X$ is $\mu_{\tilde{A}^*}(x, u_1)$ with $u_1 \in J_x^1 \subseteq [0, 1]$, its non-membership grade of $x \in X$ is $v_{\tilde{A}^*}(x, u_2)$ with $u_2 \in J_x^2 \subseteq [0, 1]$. The elements of domain of $(x, u_1), (x, u_2)$ are called primary membership and primary non-membership of x in $\tilde{A}^*$, respectively, memberships of primary memberships $\mu_{\tilde{A}^*}(x, u_1)$ and non-memberships of primary memberships $v_{\tilde{A}^*}(x, u_2)$ are called secondary memberships and secondary non-memberships, respectively, of x in $\tilde{A}^*$, with $u_1 \in J_x^1 \subseteq [0, 1]$, $u_2 \in J_x^2 \subseteq [0, 1]$, which are intuitionistic fuzzy sets.

Type-2 intuitionistic fuzzy sets are called an Interval type-2 intuitionistic fuzzy sets if the secondary membership function $\mu_{\tilde{A}}(x, u_1) = 1$ and $\mu'_{\tilde{A}^*}(x, u_2) = 1$ $\forall u_1, u_2 \in J_x)$ i.e. an Interval type-2 intuitinistic fuzzy set are defined as follows:

Definition 3.1: A type-2 intuitionistic fuzzy set (InT2FS), denoted $\tilde{A}^*$, is characterized by two type-2 intuitionistic membership functions: $\mu_{\tilde{A}}(x, u_1)$, $\mu'_{\tilde{A}}(x, u_2)$ and two type-2 intuitionistic non-membership function $v_{\tilde{A}^*}(x, u_1)$, $v'_{\tilde{A}^*}(x, u_2)$ where $x \in X$ and $u_1 \in J_x^1 \subseteq [0, 1]$, $u_2 \in J_x^2 \subseteq [0, 1]$, i.e.,

$$\tilde{A}^* = \{((x, u_1), \mu_{\tilde{A}^*}(x, u_1), v_{\tilde{A}^*}(x, u_1)), ((x, u_2), \mu'_{\tilde{A}^*}(x, u_2), v'_{\tilde{A}^*}(x, u_2)) | \forall x \in X, \forall u_1 \in J_x^1 \subseteq [0, 1], \forall u_2 \in J_x^2 \subseteq [0, 1]\}$$

in which

$0 \leq \mu_{\tilde{A}^*}(x, u_1)), \mu'_{\tilde{A}^*}(x, u_2)), v_{\tilde{A}^*}(x, u_1)), v'_{\tilde{A}^*}(x, u_2)) \leq 1$

and $0 \leq v_{\tilde{A}^*}(x, u_1)) + \mu_{\tilde{A}^*}(x, u_1)) \leq 1$, $0 \leq v'_{\tilde{A}^*}(x, u_1)) + \mu'_{\tilde{A}^*}(x, u_1)) \leq 1$.

Intuitionistic type-2 fuzzy sets are called an interval InT2Fs if the secondary membership function $\mu_{\tilde{A}}(x, u_1) = 1$ and $\mu'_{\tilde{A}^*}(x, u_2) = 1$ $\forall u_1 \in J_x^1, u_2 \in J_x^2$ i.e. an interval type-2 intuitinistic fuzzy set is defined as follows:

Definition 3.2: An interval InT2FS $\tilde{A}^*$ is characterized by membership bounding functions $\overline{\mu}_{\tilde{A}^*}(x)$, $\underline{\mu}_{\tilde{A}^*}(x)$ and non-membership bounding functions $\overline{v}_{\tilde{A}^*}(x)$, $\underline{v}_{\tilde{A}^*}(x)$ where $x \in X$ in which

$$0 \leq \overline{\mu}_{\tilde{A}}(x) + \underline{v}_{\tilde{A}}(x) \leq 1 \tag{5}$$

$$0 \leq \underline{\mu}_{\tilde{A}}(x) + \overline{v}_{\tilde{A}}(x) \leq 1 \tag{6}$$

Thus, an Interval type 2 intuitionistic fuzzy set can be described through FOUs as follow:

$$\tilde{A} = \{x, \overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x), \overline{v}_{\tilde{A}}(x), \underline{v}_{\tilde{A}}(x) | \forall x \in X, \tag{7}$$

$$\forall \overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x), \overline{v}_{\tilde{A}}(x), \underline{v}_{\tilde{A}}(x) \in [0, 1]\} \tag{8}$$

2) Entropy on InT2FS: The entropy on InT2FS is defined as a magnitude that measures the degree of InT2FS that a set is with respect to the fuzziness of this set called ultra-fuzziness. Therefore, with the definition of InT2FS above, if we interpret regions of images or thresholds as InT2FSs, then ultra-fuzziness must be satisfied the following conditions:

1. If the degrees of membership and the degrees of non-membership can be defined without any uncertainty (ordinary or intuitionistic fuzzy sets), then obviously the ultra-fuzziness should be 0. Thus, the entropy will be zero when the set is a IFSs(X).

2. The entropy will be maximum if the set is an InT2FS; that is,$\mu(x) = v(x) = 0$ for all $x \in X$,

3. As in fuzzy sets, the entropy of an InT2FS will be equal to its respective complement

4. If the degree of membership or the degree of non-membership of each element increase, this set becomes more fuzziness, and therefore the entropy should decrease.

With respect to these conditions and the way defined an entropy on InT2FS based on the previously mentioned entropies in [16], [21] , for a $M \times N$ image I, subset $\tilde{A}^* \subseteq I$ with L gray levels $q \in [0, L-1]$ histogram $h(q)$ and the membership function $\mu_{\tilde{A}^*}$ can be defined as follows:

$$\varepsilon_T \left(\tilde{A}^*\right) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q) \left(\overline{\pi}(q) - \underline{\pi}(q)\right) \tag{9}$$

in which

$$\overline{\pi}(q) = \wedge \left(1 - \underline{\mu}_{\tilde{A}_1^*}(q), ..., 1 - \underline{\mu}_{\tilde{A}_n^*}(q)\right) \tag{10}$$

$$\underline{\pi}(q) = \wedge \left(1 - \overline{\mu}_{\tilde{A}_1^*}(q), ..., 1 - \overline{\mu}_{\tilde{A}_n^*}(q)\right) \tag{11}$$

or

$$\overline{\pi}(q) = \left(1-\underline{\mu}_{\tilde{A}_1^*}(q)\right)...\left(1-\underline{\mu}_{\tilde{A}_n^*}(q)\right) \quad (12)$$

$$\underline{\pi}(q) = \left(1-\overline{\mu}_{\tilde{A}_1^*}(q)\right)...\left(1-\overline{\mu}_{\tilde{A}_n^*}(q)\right) \quad (13)$$

with

$$\overline{\mu}_{\tilde{A}^*}(q) = [\mu_{\tilde{A}^*}(q)]^{1/\alpha}, \underline{\mu}_{\tilde{A}^*}(q) = [\mu_{\tilde{A}^*}(q)]^{\alpha}, \alpha \in (1,2] \quad (14)$$

Here, $\mu_{\tilde{A}^*}(q)$ is a membership degree of x belonging to the InT2FS $\tilde{A}^*$ followed a membership function preselected.

This basic definition relies on the assumption that the singletons sitting on the FOUs are all equal in height (which is the reason why the interval InT2FS is used). Hence, it can only measure the variation in the length of the FOUs.

B. Image thresholding using InT2FSs

In this section, image thresholding algorithm using the InT2FSs will be detail described.

With the same conditions as in the image thresholding using IFS algorithm, we consider an image I; denote (x, y) is the coordinate of a pixel on the image I, and $q(x, y)$ is the gray level of the pixel (x, y) so that $0 \leq q(x, y) \leq L-1$ for each $(x, y) \in I$ where L is the image gray-scale. The aim of this problem is to determine the threshold t using InT2FSs in order to obtain a good segmentation of the considered image I by handling the uncertainty better for the noisy images.

As we know, with the image thresholding problem, the threshold $t(t = 0, 1, .., L-1)$ of the gray scale L was found to divide the image into two regions: the background and the object. Each pixel belongs to the background or of the object. Therefore, with a threshold t, we will have two regions corresponds to two datasets. In image thresholding techniques using InT2FS, each region with threshold t will be described as an interval InT2FS. Here, we have two interval InT2FS $\tilde{I}_{Bt}^*$ and $\tilde{I}_{Ot}^*$ corresponding to the background and the object, respectively.

$$\tilde{I}_{Bt}^* = \{q, \overline{\mu}_{\tilde{I}_{Bt}^*}(q), \underline{\mu}_{\tilde{I}_{Bt}^*}(q), \overline{v}_{\tilde{I}_{Bt}^*}(q), \underline{v}_{\tilde{I}_{Bt}^*}(q) | q = 0, 1, ..., L-1\} \quad (15)$$

$$\tilde{I}_{Ot}^* = \{q, \overline{\mu}_{\tilde{I}_{Ot}^*}(q), \underline{\mu}_{\tilde{I}_{Ot}^*}(q), \overline{v}_{\tilde{I}_{Ot}^*}(q), \underline{v}_{\tilde{I}_{Ot}^*}(q) | q = 0, 1, ..., L-1\} \quad (16)$$

Each pixel $q(x, y)$ will have three numerical values which should be considered: a value for representing its membership to the background $\mu_{\tilde{I}_{Bt}^*}(q)$, a value for representing its membership to the object $\tilde{I}_{Ot}^*(q)$ and a value denoted $\pi(q)$ for representing the hesitance of the expert in determining the membership functions $\mu_{\tilde{I}_{Bt}^*}$ and $\mu_{\tilde{I}_{Ot}^*}$.

Similar to the algorithm is presented in the above section, the proposed algorithm for calculating the best threshold value t of an image I with L image gray-scale consists of the following six steps:

Step 1: Construct L membership functions $\mu_{I_{Bt}}$ for L interval type-2 fuzzy sets I_{Bt} associated with the image I. These sets represent the background of the image I. Each one is associated with a level of intensity $t, (t = 0, 1, ..L-1)$ of the gray-scale L used. The membership functions of these fuzzy sets are selected same as the above section:

$$\mu_{I_{Bt}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{Bt}(q)}{L-1}\right)\right),$$
$$\mu_{I_{Ot}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{Ot}(q)}{L-1}\right)\right)$$

where
$m_O(t) = \frac{\sum_{q=t+1}^{L-1} qh(q)}{\sum_{q=t+1}^{L-1} h(q)}$, $m_B(t) = \frac{\sum_{q=0}^{t} qh(q)}{\sum_{q=0}^{t} h(q)}$
and h(q) is the number of pixels of the image with the gray level q, $F(x) = 1 - 0.5x$, $d(x, y) = |x - y|$ (see [5]).

Step 2: Construct L membership functions $\mu_{I_{Ot}}$ for L interval type-2 fuzzy sets I_{Ot} associated with the image I. These sets represent the object of the considered image I. Each one is associated with a level of intensity $t, (t = 0, 1, ..L-1)$ of the gray-scale L used. The membership functions of these fuzzy sets are formed as the *Step 1*.

Step 3: Represent the hesitance of the expert in the construction of the sets corresponding to *Step 1* and *Step 2*. We know that the choice of the membership functions is conditioned by the hesitance of the expert when constructing these membership functions.

In this approach, the hesitance $\pi(q)$ of the expert is represented by an interval $[\underline{\pi}(q), \overline{\pi}(q)]$ because of an interval membership $[\underline{\mu}_{I_{Bt}}(q), \overline{\mu}_{I_{Bt}}(q)]$ as follows:

$$\overline{\pi}(q) = \wedge\left(1-\underline{\mu}_{I_{Bt}}(q), 1-\underline{\mu}_{I_{Ot}}(q)\right) \quad (17)$$

$$\underline{\pi}(q) = \wedge\left(1-\overline{\mu}_{I_{Bt}}(q), 1-\overline{\mu}_{I_{Ot}}(q)\right) \quad (18)$$

(19)

or

$$\overline{\pi}(q) = \left(1-\underline{\mu}_{I_{Bt}}(q)\right).\left(1-\underline{\mu}_{I_{Ot}}(q)\right) \quad (20)$$

$$\underline{\pi}(q) = \left(1-\overline{\mu}_{I_{Bt}}(q)\right).\left(1-\overline{\mu}_{I_{Bt}}(q)\right) \quad (21)$$

Step 4: Construct the L InT2FSs $\tilde{I}_{Bt}^*$ associated with the background of the image. Construct InT2FSs $\tilde{I}_{Bt}^*$ and $\tilde{I}_{Ot}^*$ associated with $\overline{\pi}(q)$ and $\underline{\pi}(q)$ as 16:

$\tilde{I}_{Bt}^* = \{q, \overline{\mu}_{\tilde{I}_{Bt}^*}(q), \underline{\mu}_{\tilde{I}_{Bt}^*}(q), \overline{v}_{\tilde{I}_{Bt}^*}(q), \underline{v}_{\tilde{I}_{Bt}^*}(q) | q = 0, 1, ..., L-1\}$

where

$$\overline{\mu}_{\tilde{I}_{Bt}^*}(q) = \overline{\mu}_{I_{Bt}}(q)$$
$$\underline{\mu}_{\tilde{I}_{Bt}^*}(q) = \underline{\mu}_{I_{Bt}}(q)$$
$$\overline{v}_{\tilde{I}_{Bt}^*}(q) = 1 - \underline{\mu}_{\tilde{I}_{Bt}^*}(q) - \underline{\pi}(q)$$
$$\underline{v}_{\tilde{I}_{Bt}^*}(q) = 1 - \overline{\mu}_{\tilde{I}_{Bt}^*}(q) - \overline{\pi}(q)$$

and

$\tilde{I}_{Ot}^* = \{q, \overline{\mu}_{\tilde{I}_{Ot}^*}(q), \underline{\mu}_{\tilde{I}_{Ot}^*}(q), \overline{v}_{\tilde{I}_{Ot}^*}(q), \underline{v}_{\tilde{I}_{Ot}^*}(q) | q = 0, 1, ..., L-1\}$

where

$$\overline{\mu}_{\tilde{I}^*_{Ot}}(q) = \overline{\mu}_{I_{Ot}}(q)$$
$$\underline{\mu}_{\tilde{I}^*_{Ot}}(q) = \underline{\mu}_{I_{Ot}}(q)$$
$$\overline{v}_{\tilde{I}^*_{Ot}}(q) = 1 - \underline{\mu}_{\tilde{I}^*_{Ot}}(q) - \underline{\pi}(q)$$
$$\underline{v}_{\tilde{I}^*_{Ot}}(q) = 1 - \overline{\mu}_{\tilde{I}^*_{Ot}}(q) - \overline{\pi}(q)$$

Step 5: Calculate the entropy ε_T of each one of the L InT2FSs $\tilde{I}^*_{Bt}$. At this step, the entropy ε_T of L InT2FSs (constructed in Step 4) is calculated followed by: (see equation 9)

$$\varepsilon_T\left(\tilde{I}^*\right) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h\left(q\right)\left(\overline{\pi}\left(q\right) - \underline{\pi}\left(q\right)\right) \tag{22}$$

Step 6: Take as best threshold the value of t associated with the InT2FSs $\tilde{I}^*_{Bt}$ of lowest entropy ε_T. Find out the gray level q that entropy ε_T reached the minimum value, then q would be best selected threshold t associated with the InT2FS $\tilde{I}^*_{Bt}$

There is a bit different than the algorithm based on IFS, each region in image I represented as an interval type 2 fuzzy set. Therefore, each membership function will be valid in an interval, we will calculated the upper and lower membership functions for two interval fuzzy sets I_{Bt}, I_{Ot} are $\overline{\mu}_{I_{Bt}}(q)$, $\underline{\mu}_{I_{Bt}}(q)$ and $\overline{\mu}_{I_{Ot}}(q)$, $\underline{\mu}_{I_{Ot}}(q)$ respectively followed by 14:

$\overline{\mu}_{I_{Bt}}(q) = [\mu_{I_{Bt}}(q)]^{1/\alpha}$
$\underline{\mu}_{I_{Bt}}(q) = [\mu_{I_{Bt}}(q)]^{\alpha}$
$\overline{\mu}_{I_{Ot}}(q) = [\mu_{I_{Ot}}(q)]^{1/\alpha}$
$\underline{\mu}_{I_{Ot}}(q) = [\mu_{I_{Ot}}(q)]^{\alpha}$
Here, $\alpha = 2$.

IV. Experiments

The effect and performance of image segmentation using InT2FSs was tested using 5 different images with small and large objects, text, objects with clear or fuzzy boundaries, and were noisy or smooth and each image contained two type of images: normal and Luminance noise. The proposed algorithm was also compared to other algorithms through their results.

The Figure IV below shows the test results. From left to right: original image, desired image results, image thresholding using type 1 fuzzy set, image thresholding using type 2 fuzzy set [21], image thresholding using intuitionistic fuzzy set and image thresholding by the proposed method.

In order to verify the performance of the thresholding, the optimal thresholded image was created manually and used as a ground-truth image.

Performance of thresholding methods based on comparison of their results with the ground-truth images was displayed in Table I.

Table I show that the proposed algorithm has a better performance or higher quality thresholding with noise images than the other typical algorithms such as image segmentation using type 1 fuzzy sets, image segmentation using type 2 fuzzy sets and image segmentation using intuitionistic fuzzy sets.

As the results from Table I, image thresholding using InT2FSs has the lowest average error of 8.11 % and the lowest standard deviation of 3.83%. This proves that the proposed algorithm is generally relative stable than the other algorithms

V. Conclusions

This paper presented an image thresholding algorithm for segmentation based on InT2FSs which improved the segmentation results and overcame the drawbacks of the conventional thresholding algorithms which have difficulties in handling the uncertainty. The proposed approach has solved the problem of combining between thresholding method based on InT2FSs for handling the uncertainty and extending it to image thresholding. The experiments are done based on many images with the statistics show that the algorithm generates better results than other existing methods. The next goal is to use GPU to speed up this proposed algorithm or applying this approach to multi-level image thresholding problems.

References

[1] K. Atanassov. Intuitionistic fuzzy sets.Fuzzy Sets and Systems, 20, 87-96, 1986.

[2] K. Atanassov. Intuitionistic fuzzy sets. VII ITKRs Session, Deposed in Central Sci. Techn. Library of Bulg. Acd. of Sci., Sofia, 1684-1697, 1986.

[3] J.Bezdek, J.Keller, R.Krisnapuram and N.Pal. Fuzzy Models and algorithms for pattern recognition and image processing. Kluwer Academic Publishers, 1999.

[4] H. Bustince,E. Barrenechea, M. Pagola, and R.Orduna, Image Thresholding Computation Using Atanassovs Intuitionistic Fuzzy Sets, Journal of Advanced Comput. Int. and Int. Informatics, Vol.11(2), 187-194, 2007.

[5] P. M. Pinto, P. Couto, H.Bustince, E. Barrenechea, M. Pagola, and J.,Fernandez , Image segmentation using Atanassovs intuitionistic fuzzy sets, Expert Systems with Applications 40, 1526, 2013.

[6] K.C. Lin, Fast image thresholding by finding the zero(s) of the first derivative of between-class variance, Mach. Vis. Appl.13 (56) (2003) 254-262.

[7] P. Burillo, and H. Bustince, Construction theorems for intuitionistic fuzzy sets.Fuzzy Sets and Systems, 84, 271281, 1996

[8] L. Caponetti, C. Castiello, and P. Grecki, Document page segmentation using neuro-fuzzy approach.Applied Soft Computing, 18, 118-126, 2008.

[9] P.L. Rosin, Unimodal thresholding, Pattern Recognition 34 (11), 2083-2096, 2001.

[10] L. Snidaro, G.L. Foresti, Real-time thresholding with Euler numbers, Pattern Recognition Lett. 24 (9-10) (2003) 1543-1554.

[11] H. Yan, Unifiedformulation of a class of image thresholding techniques, Pattern Recognition 29 (12) (1996) 2025-2032.

[12] Chi, Z., Yan, H. and Pham, T. (1998). Optimal image thresholding.Fuzzy algorithms: with application to image processing and pattern recognition, 45-84.

[13] J.M. Mendel and R.I. John, Type-2 fuzzy set made simple, IEEE Trans.Fuzzy Syst., vol. 10(2), 117 - 127, 2002.

[14] N. Karnik, J.M. Mendel, Operations on Type-2 Fuzzy Sets, Fuzzy Sets and Systems, 122, 327-348, 2001.

[15] Q. Liang , J.M. Mendel, Interval Type-2 Fuzzy Logic Systems: Theory and Design, IEEE Trans. on Fuzzy Systems, 8(5), 535-550, 2000.

[16] Burillo, P. and Bustince, H. (1996b). Entropy on intuitionistic fuzzy sets and on interval valued fuzzy sets. Fuzzy Sets and Systems, 78, 81-103.

[17] Forero, M. (2003).Fuzzy filters for image processing. Springer [Ch. Fuzzy thresholding and histogram analysis, pp. 129-152].

[18] B. Sankur, M. Sezgin, Survey over image thresholding techniques and quantitative performance evaluation, J.Electron. Imaging 13 (1) (2004) 146-165.

[19] Huang, L. and Wang, M. (1995). Image thresholding by minimizing the measure of fuzziness.Pattern Recognition, 28(1), 41-51.

[20] Bustince, H., Barrenechea, E. and Pagola, M. (2007). Image thresholding using restricted equivalence functions and maximizing the measures of similarity. Fuzzy Sets and Systems, 158(5), 496- 516.

[21] Tizhoosh, H. (2005). Image thresholding using type II fuzzy sets.Pattern Recognition, 38, 2363 - 2372.

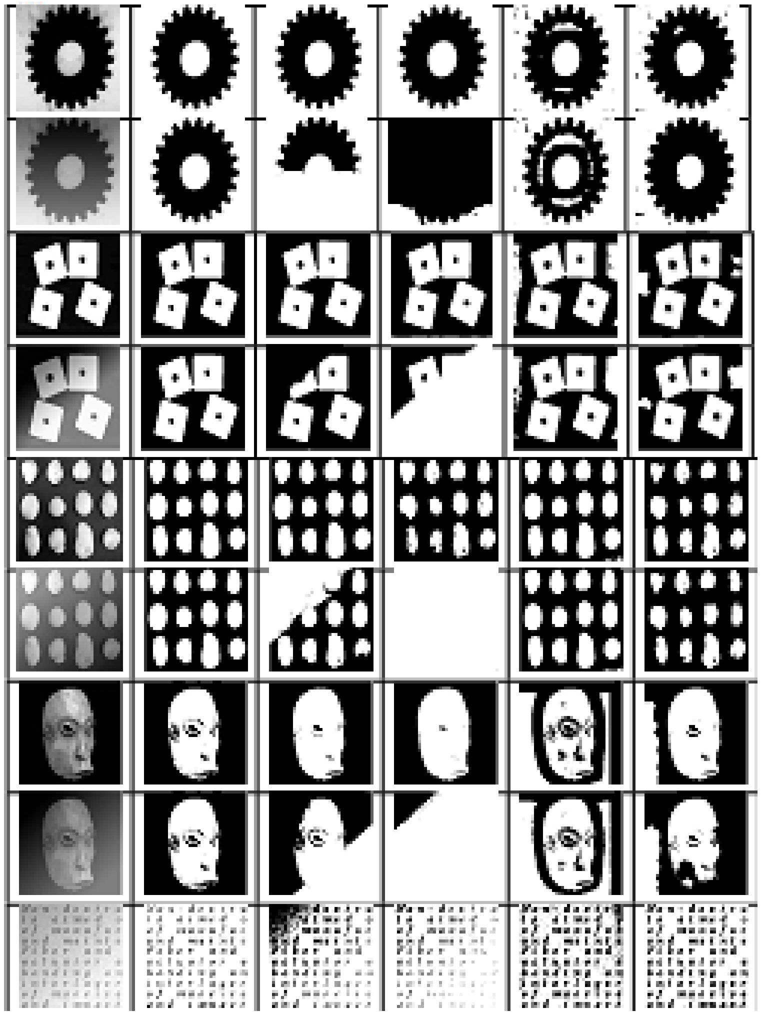

Fig. 1. The test results. From left to right: original image, desired image results, image thresholding using type 1 fuzzy sets (T1FS), image thresholding using type 2 fuzzy sets (T2FS), image thresholding using IFS (IFS), image thresholding using InT2FS (the proposed method)
(InT2FS).

TABLE I
PERFORMANCE OF THRESHOLDING METHODS BASED ON COMPARISON OF THEIR RESULTS WITH THE GROUND-TRUTH IMAGES

Image	**No of Pixels**	**T1FS**		**T2FS**		**IFS**		**InT2FS**	
		% False	**No**	**% False**	**No**	**% False**	**No**	**% False**	**No**
Wheel	42636	0.85	363	0.80	341	5.5	2343	5.09	2171
WheelNoise	42636	40.06	17081	25.16	10727	8.97	3826	6.35	2707
Dice	42432	0.67	286	1.19	504	8.36	3547	5.15	2187
DiceNoise	42432	42.21	17911	6.68	2836	8.07	3423	5.73	2432
Potato	42432	8.21	3482	0.32	135	1.33	564	8.11	3441
PotatoNoise	42432	61.94	26282	20.36	8638	1.22	517	10.21	4333
Mask	42640	4.79	2042	3.18	1357	22.78	9712	12.75	5437
MaskNoise	42640	57.8	24648	24.29	10359	26.41	11260	15.66	6676
Paper	42432	5.34	2265	12.87	5463	13.63	5782	8.61	3653
PaperNoise	42432	8.87	3763	51.37	21799	11.56	4907	3.41	1449
Average Error		23.07		14.62		10.78		8.11	
Standard Deviation		24.58		16.20		8.31		3.83	