LETTER

Cognitive Fixed-Gain Amplify-and-Forward Relay Networks under Interference Constraints

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SUMMARY In this work, we analyze the performance of cognitive amplify-and-forward (AF) relay networks under the spectrum sharing approach. In particular, by assuming that the AF relay operates in the semi-blind mode (fixed-gain), we derive the exact closed-form expressions of the outage probability for the cognitive relaying (no direct link) and cognitive cooperative (with direct link) systems. Simulation results are presented to verify the theoretical analysis.

key words: *spectrum sharing, cognitive radio, fixed-gain (FG), amplify-and-forward (AF), relay networks, selection diversity*

1. Introduction

Cognitive radio technology is a promising approach to improve the utilization of scarce radio frequency spectrum resources [1]. The concept of relaying communication in cognitive radio networks with cooperative spectrum sharing and amplify-and-forward (AF) relaying has attracted great attention [2]–[4]. Here, AF relaying is an important protocol, where the relay just simply forwards signals to the destination with no regeneration. The exact closed-form expression of outage probability (OP) for cognitive AF relaying has been derived in [2]. The asymptotic OP expressions have been reported in [3]. By considering the direct link, the selection combining has been included in [4]. These works only consider the channel state information (CSI)-assisted AF relaying, where full CSI knowledge is perfectly acquired by the secondary transmitter.

However, it is not always feasible to assume that the secondary networks can acquire the large amounts of CSI knowledge needed. Motivated by the above discussion, our paper presents for the first time the cooperative spectrum sharing with fixed-gain (FG) AF relay. In particular, the secondary user (SU) transmitter (SU-Tx) communicates with the SU receiver (SU-Rx) through the assistance of the AF SU relay (SU-Relay) under a strict power constraint on the primary user (PU). We consider both cases: with and without direct link. The FG AF scheme is also obtained by considering full FG or semi FG relaying. Assuming that the direct link exists, the SU-Rx can combine the two signals from the SU-Tx and the SU-Relay using selection combining (SC). As a result, the instantaneous received signal-to-noise ratio (SNR) is the maximum between the SNRs of the relaying and direct links. In the spectrum sharing model, the existence of a common random variable, i.e., the channel fading coefficient from SU-Tx to PU, results in dependence between the two SNR terms, which is cumbersome for the analysis. By taking into account the conditioned statistics on the fading coefficient from SU-Tx to PU, the exact OP is obtained in a tractable closed-form expression. This result readily allows us to investigate the advantage of deploying AF relay in a cognitive spectrum sharing environment. In fact, the cognitive cooperation considered herein significantly outperforms both direct transmission (DT) and AF relaying transmission without direct link.

2. System and Channel Model

Consider a dual-hop spectrum-sharing system with the coexistence of PU and SUs. The secondary relay network can operate in the same spectrum licensed to the PU as long as the SU transmission does not cause any harmful interference on the PU. For the first hop transmission, the SU-Tx broadcasts signal s to both SU-Relay and SU-Rx with the maximum transmitted power P_{S} given as $P_{\mathrm{S}} = I_{\mathrm{p}}/|h_{\mathrm{S,P}}|^2$, where I_{p} is the maximum tolerable interference power at PU and $h_{\mathrm{S,P}}$ is the channel coefficient of the link from SU-Tx to PU. As a result, the received signals at the SU-Relay and the SU-Rx are written as, respectively

$$y_{\mathrm{R}} = h_{\mathrm{S,R}}s + n_{\mathrm{R}}, \quad y_{\mathrm{D}}^{(1)} = h_{\mathrm{S,D}}s + n_{\mathrm{D}}^{(1)}, \tag{1}$$

where $h_{\mathrm{S,R}}$ and $h_{\mathrm{S,D}}$ are the channel coefficients for the link from SU-Tx to SU-Relay and to SU-Rx; n_{R} and $n_{\mathrm{D}}^{(1)}$ are additive white Gaussian noise (AWGN) components at SU-Relay and SU-Rx, respectively. Then, the received signal at the SU-Relay is amplified with variable gain G and forwarded to the SU-Rx. The received signal at the SU-Rx from SU-Relay is therefore given by

$$y_{\mathrm{D}}^{(2)} = Gh_{\mathrm{R,D}}h_{\mathrm{S,R}}s + Gh_{\mathrm{R,D}}n_{\mathrm{R}} + n_{\mathrm{D}}^{(2)}, \tag{2}$$

where $n_{\mathrm{D}}^{(2)}$ is AWGN at the SU-Rx. In this paper, we assume that non-identical Rayleigh fading for all links in which the channel power gain $|h_{\mathrm{A,B}}|^2$ is exponentially distributed with $\mathbb{E}\left\{|h_{\mathrm{A,B}}|^2\right\} = \Omega_{\mathrm{A,B}}$, where $\mathrm{A} \in \{\mathrm{s,r}\}$, $\mathrm{B} \in \{\mathrm{r,p,d}\}$, and $\mathbb{E}\{\cdot\}$

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denotes the expectation. Additionally, all AWGN components have zero mean and variance N_0. Then, the equivalent end-to-end SNR at the destination is

$$\gamma_{\mathrm{AF}} = \frac{\frac{I_{\mathrm{p}}}{N_0}\frac{|h_{\mathrm{S,R}}|^2}{|h_{\mathrm{S,P}}|^2}|h_{\mathrm{R,D}}|^2}{|h_{\mathrm{R,D}}|^2 + 1/G^2}. \quad (3)$$

Due to the channel knowledge of AF relay, the amplifying gain can be differently decided. Here, we consider for two different cases.

- First, when SU-Relay has only the full knowledge on the instantaneous CSI of the link to PU whereas only the statistical channel knowledge, i.e., the fading power, of the link to SU-Tx is required. We consider the relay as semi FG AF relaying operation. In this particular case, the amplifying gain is selected as

$$G^2 = \frac{I_{\mathrm{p}}}{|h_{\mathrm{R,P}}|^2 N_0}\,\mathbb{E}\left\{\frac{1}{|h_{\mathrm{S,R}}|^2 P_{\mathsf{S}}/N_0 + 1}\right\}. \quad (4)$$

By substituting (4) into (3), the instantaneous received SNR at SU-Rx for semi FG AF relaying can be written as

$$\gamma_{\mathrm{AF}_1} = \frac{\gamma_1\gamma_2}{\gamma_2 + \mathsf{c}_1}, \quad (5)$$

where $\gamma_1 = \frac{I_{\mathrm{p}}}{N_0}\frac{|h_{\mathrm{S,R}}|^2}{|h_{\mathrm{S,P}}|^2}$, $\gamma_2 = \frac{I_{\mathrm{p}}}{N_0}\frac{|h_{\mathrm{R,D}}|^2}{|h_{\mathrm{R,P}}|^2}$, and $\mathsf{c}_1 = \frac{\lambda_1(\lambda_1-1-\ln\lambda_1)}{(\lambda_1-1)^2}$ with $\lambda_1 = \frac{N_0\Omega_{\mathrm{SP}}}{I_{\mathrm{p}}\Omega_{\mathrm{SR}}}$.

- Second, SU-Relay cannot exploit the instantaneous realization of all links but only the fading powers. This operation can be considered as full FG AF relaying. The amplifying gain is computed as

$$G^2 = \frac{I_{\mathrm{p}}}{N_0}\frac{\mathbb{E}\left\{\frac{1}{|h_{\mathrm{S,R}}|^2 P_{\mathsf{S}}/N_0+1}\right\}}{\mathbb{E}\{|h_{\mathrm{R,P}}|^2\}}. \quad (6)$$

From (3) and (6), we have

$$\gamma_{\mathrm{AF}_2} = \frac{\gamma_1\gamma_3}{\gamma_3 + \mathsf{c}_2}, \quad (7)$$

where $\gamma_3 = \frac{I_{\mathrm{p}}}{N_0}|h_{\mathrm{R,D}}|^2$ and $\mathsf{c}_2 = \mathsf{c}_1\Omega_{\mathrm{R,P}}$.

3. Exact Outage Probability Analysis

3.1 Cognitive Relaying Networks - Without Direct Link

When there exists no direct link between SU-Tx and SU-Rx, the instantaneous received SNR at SU-Rx is given by $\gamma_{\mathrm{Rx}} = \gamma_{\mathrm{AF}_n}$, where $n \in \{1, 2\}$.

Theorem 1. *The OP of cognitive relaying networks with semi FG AF relay is given by*

$$P_{\mathrm{out}}^{\mathsf{semi}} = 1 + \frac{\lambda_1^{-1}}{-\lambda_1^{-1} + \mathsf{c}_1\lambda_2\gamma_{\mathrm{th}} - \gamma_{\mathrm{th}}} - \frac{\mathsf{c}_1\lambda_2\lambda_1^{-1}\gamma_{\mathrm{th}}}{(-\lambda_1^{-1} + \mathsf{c}_1\lambda_2\gamma_{\mathrm{th}} - \gamma_{\mathrm{th}})^2}\ln\left(\frac{\mathsf{c}_1\lambda_2\gamma_{\mathrm{th}}}{\lambda_1^{-1} + \gamma_{\mathrm{th}}}\right). \quad (8)$$

Proof. We start the proof by rewriting the cumulative distribution function (CDF) of γ_{Rx} as

$$F_{\gamma_{\mathrm{Rx}}}(\gamma) = F_{\gamma_1}(\gamma) + \int_{\gamma}^{\infty} \Pr\left(\gamma_2 \le \frac{\gamma\mathsf{c}_1}{\gamma_1-\gamma}\right) f_{\gamma_1}(\gamma_1)\, d\gamma_1. \quad (9)$$

The CDF and probability density function (PDF) of γ_n, for $n \in \{1, 2\}$, can be easily given as $F_{\gamma_n}(z) = 1-(1+\lambda_n z)^{-1}$ and $f_{\gamma_n}(z) = \lambda_n(1+\lambda_n z)^{-2}$, where $\lambda_1 = \frac{N_0\Omega_{\mathrm{SP}}}{I_{\mathrm{p}}\Omega_{\mathrm{SR}}}$ and $\lambda_2 = \frac{N_0\Omega_{\mathrm{RP}}}{I_{\mathrm{p}}\Omega_{\mathrm{RD}}}$. Then, by applying the change of variable $x = \gamma_1 - \gamma$, we can express (9) as

$$F_{\gamma_{\mathrm{Rx}}}(\gamma) = 1 - \frac{1}{\lambda_1}\int_0^{\infty} \frac{x\mathrm{d}x}{(x + \mathsf{c}_1\lambda_2\gamma)(x + \lambda_1^{-1} + \gamma)^2}. \quad (10)$$

By expanding the integrand of (10), i.e., $\mathcal{I}_1 = \frac{x}{(x+a)(x+b)^2}$, in the form of partial fractions as

$$\mathcal{I}_1 = \frac{A}{x+a} - \frac{A}{x+b} + \frac{B}{(x+b)^2}, \quad (11)$$

where $a = \mathsf{c}_1\lambda_1\gamma$, $b = \lambda_2^{-1} + \gamma$, and the three expanding coefficients are given by $A = -\frac{a}{(a-b)^2}$ and $B = -\frac{b}{a-b}$, which then yield

$$\int_0^{\infty}\mathcal{I}_1\mathrm{d}x = \int_0^{\infty}\left(\frac{A}{x+a} - \frac{A}{x+b}\right)\mathrm{d}x + \int_0^{\infty}\frac{B}{(x+b)^2}\mathrm{d}x. \quad (12)$$

It is important to note that the two above integrals in the right-hand-side of (12) converges. By plugging (12) into (10), we obtain (8), which completes the proof. □

Theorem 2. *The OP of cognitive relaying networks with full FG AF relay is given by*

$$P_{\mathrm{out}}^{\mathrm{full}} = 1 - \frac{1}{1+\lambda_1\gamma_{\mathrm{th}}} - \frac{\mathsf{c}_2\lambda_1\lambda_3\gamma_{\mathrm{th}}e^{\frac{\mathsf{c}_2\lambda_1\lambda_3\gamma_{\mathrm{th}}}{1+\lambda_1\gamma_{\mathrm{th}}}}}{(1+\lambda_1\gamma_{\mathrm{th}})^2} \times\left[\mathsf{Chi}\left(\frac{\mathsf{c}_2\lambda_1\lambda_3\gamma_{\mathrm{th}}}{1+\lambda_1\gamma_{\mathrm{th}}}\right) - \mathsf{Shi}\left(\frac{\mathsf{c}_2\lambda_1\lambda_3\gamma_{\mathrm{th}}}{1+\lambda_1\gamma_{\mathrm{th}}}\right)\right], \quad (13)$$

where $\mathsf{Chi}(z)$ *and* $\mathsf{Shi}(z)$ *is hyperbolic cosine integral and hyperbolic sine integral functions, respectively [5, Eq. (8.221.1-2)].*

Proof. Similarly as in (9), we obtain

$$F_{\gamma_{\mathrm{Rx}}}(\gamma) = F_{\gamma_1}(\gamma) + \int_{\gamma}^{\infty} F_{\gamma_3}\left(\frac{\gamma\mathsf{c}_2}{\gamma_1-\gamma}\right) f_{\gamma_1}(\gamma_1)\, d\gamma_1. \quad (14)$$

Since γ_3 is an exponential distributed random variable with parameter $\lambda_3 = N_0/(I_{\mathrm{p}}\Omega_{\mathrm{RD}})$, where the PDF is given as $f_{\gamma_3}(z) = \lambda_3 e^{-\lambda_3 z}$, by applying the change of variable $x = \gamma_1 - \gamma$, (14) is rewritten as

$$F_{\gamma_{\mathrm{Rx}}}(\gamma) = 1 - \int_0^{\infty}\frac{e^{-\lambda_3\mathsf{c}_2\gamma/x}}{\lambda_1\left(\lambda_1^{-1} + \gamma + x\right)^2}\mathrm{d}x. \quad (15)$$

By expanding and computing the integrand of (15), we can obtain (13). □

3.2 Cooperative Cognitive Networks - With Direct Link

In this case, SU-Rx receives two signals from SU-Tx and SU-Relay to apply the selection combining, which results in $\gamma_{\text{Rx}} = \max(\gamma_{\text{AF}}, \gamma_{\text{DT}})$, where γ_{DT} is the SNR from the direct link. We also investigate for two cases of semi- and full-FG AF relay.

Theorem 3. *The OP for cooperative cognitive with selection combining and semi-FG AF relay is given by*

$$\begin{aligned} P_{\text{out}}^{\text{semi}} &= 1 - \frac{\lambda_{\text{SP}}}{\lambda_{\text{SP}} + \lambda_{\text{SD}}\gamma_{\text{th}}} - \frac{\lambda_{\text{SP}}}{\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}}} \\ &+ \frac{\lambda_{\text{SP}}}{\lambda_{\text{SP}} + (\lambda_{\text{SD}} + \lambda_{\text{SR}})\gamma_{\text{th}}} + \frac{\lambda_{\text{SP}}\lambda_{\text{SR}}\lambda_2 \mathtt{c}_1 \gamma_{\text{th}}}{2(\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}})^2} \\ &\times {}_2F_1\left(1, 2; 3; \frac{\lambda_{\text{SP}} + (\lambda_{\text{SR}} - \lambda_{\text{SR}}\lambda_2\mathtt{c}_1)\gamma_{\text{th}}}{\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}}}\right) \\ &- \frac{\lambda_{\text{SP}}\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma_{\text{th}}}{2[\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}})\gamma_{\text{th}}]^2} \\ &\times {}_2F_1\left(1, 2; 3; \frac{\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}} - \lambda_{\text{SR}}\lambda_2\mathtt{c}_1)\gamma_{\text{th}}}{\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}})\gamma_{\text{th}}}\right), \end{aligned} \tag{16}$$

where $\lambda_{\text{SP}} = 1/\Omega_{\text{SP}}$, $\lambda_{\text{SR}} = N_0/(I_{\text{p}}\Omega_{\text{SR}})$, $\lambda_{\text{SD}} = N_0/(I_{\text{p}}\Omega_{\text{SD}})$ *and* ${}_2F_1(.)$ *is the Gauss hypergeometric function [5, Eq. (9.111)].*

Proof. The CDF of $F_{\gamma_{\text{Rx}}}(\gamma) = \Pr(\max\{\gamma_{\text{AF}_1}, \gamma_{\text{DT}}\} < \gamma)$, where $\gamma_{\text{DT}} = \frac{V}{|h_{\text{S,P}}|^2}$ with $V = \frac{I_{\text{p}}}{N_0}|h_{\text{S,D}}|^2$ is the SNR for the direct link and γ_{AF_1} can be rewritten from (5) as

$$\gamma_{\text{AF}_1} = \frac{U\gamma_2}{|h_{\text{S,P}}|^2(\gamma_2 + \mathtt{c}_1)}, \tag{17}$$

where $U = \frac{I_{\text{p}}}{N_0}|h_{\text{S,R}}|^2$. It is clearly to see that γ_{AF_1} and γ_{DT} has a common random variable $X = |h_{\text{S,P}}|^2$, which produces a correlation among them. In other words, we have $F_{\gamma_{\text{Rx}}}(\gamma) \neq F_{\gamma_{\text{AF}_1}}(\gamma) F_{\gamma_{\text{DT}}}(\gamma)$. It is interesting to observe that conditioned on X, γ_{AF_1} and γ_{DT} are statistically independent since U, V, and γ_2 are independent. Taking this observation into account, we have

$$F_{\gamma_{\text{Rx}}}(\gamma|X) = F_{\gamma_{\text{AF}_1}}(\gamma|X) F_{\gamma_{\text{DT}}}(\gamma|X). \tag{18}$$

Now we aim at deriving the two conditional CDF, i.e., $F_{\gamma_{\text{AF}_1}}(\gamma|X)$ and $F_{\gamma_{\text{DT}}}(\gamma|X)$. It is easy to see that

$$\begin{aligned} F_{\gamma_{\text{AF}_1}}(\gamma|X) &= \Pr\left(\frac{U\gamma_2}{X(\gamma_2 + \mathtt{c}_1)} < \gamma \middle| X\right) \\ &= 1 - e^{-\lambda_{\text{SR}}X\gamma} \int_0^\infty \lambda_2 e^{-\lambda_{\text{SR}}\mathtt{c}_1\gamma X/\gamma_2}(1 + \lambda_2\gamma_2)^{-2} d\gamma_2, \end{aligned} \tag{19}$$

where (19) is obtained from the fact that U is an exponential distributed RV with parameter $\lambda_{\text{SR}} = I_{\text{p}}/(N_0\Omega_{\text{SR}})$. By applying [5, Eq. (3.353.3)] for (19), we obtain

$$\begin{aligned} F_{\gamma_{\text{AF}_1}}(\gamma|X) = 1 - e^{-\lambda_{\text{SR}}\gamma X} - \lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma X \\ \times e^{-\lambda_{\text{SR}}\gamma X} e^{\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma X} \text{Ei}(-\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma X). \end{aligned} \tag{20}$$

where Ei(.) is the exponential integral function [5, Eq. (8.211)]. For the direct link, we get

$$F_{\gamma_{\text{DT}}}(\gamma|X) = 1 - e^{-\lambda_{\text{SD}}\gamma X}, \tag{21}$$

where $\lambda_{\text{SD}} = N_0/(I_{\text{p}}\Omega_{\text{SD}})$. By substituting (20), (21) into (18) and taking the expectation over RV X, the unconditional CDF of γ_{Rx} can be given in the form

$$F_{\gamma_{\text{Rx}}}(\gamma) = 1 - \lambda_{\text{SP}}I_1 + \lambda_{\text{SP}}\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma(I_2 - I_3), \tag{22}$$

where

$$I_1 = \frac{1}{\lambda_{\text{SP}} + \lambda_{\text{SD}}\gamma} + \frac{1}{\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma} - \frac{1}{\lambda_{\text{SP}} + (\lambda_{\text{SD}} + \lambda_{\text{SR}})\gamma}$$

and I_2 and I_3 are respectively, given by

$$\begin{aligned} I_2 &= -\int_0^\infty X e^{[(\lambda_{\text{SR}}\lambda_2\mathtt{c}_1 - \lambda_{\text{SR}})\gamma - \lambda_{\text{SP}}]X} \text{Ei}(-\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma X) dX \\ I_3 &= -\int_0^\infty X e^{[(\lambda_{\text{SR}}\lambda_2\mathtt{c}_1 - \lambda_{\text{SR}} - \lambda_{\text{SD}})\gamma - \lambda_{\text{SP}}]X} \text{Ei}(-\lambda_{\text{SR}}\lambda_2\mathtt{c}_1\gamma X) dX. \end{aligned}$$

By utilizing the help of [5, Eq. (6.228.2)] for I_2 and I_3, we reach (16), which finalizes the proof. □

Theorem 4. *The OP for cooperative spectrum sharing with full-FG relay can be given by*

$$\begin{aligned} P_{\text{out}}^{\text{full}} &= 1 - \frac{\lambda_{\text{SP}}}{\lambda_{\text{SP}} + \lambda_{\text{SD}}\gamma_{\text{th}}} - \frac{\lambda_{\text{SP}} e^{\frac{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma_{\text{th}}}{2(\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}})}}}{\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}}} \\ &\times W_{-1,\frac{1}{2}}\left(\frac{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma_{\text{th}}}{\lambda_{\text{SP}} + \lambda_{\text{SR}}\gamma_{\text{th}}}\right) + \frac{\lambda_{\text{SP}} e^{\frac{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma_{\text{th}}}{2[\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}})\gamma_{\text{th}}]}}}{\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}})\gamma_{\text{th}}} \\ &\times W_{-1,\frac{1}{2}}\left(\frac{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma_{\text{th}}}{\lambda_{\text{SP}} + (\lambda_{\text{SR}} + \lambda_{\text{SD}})\gamma_{\text{th}}}\right), \end{aligned} \tag{23}$$

where $W_{\mu,\kappa}(\cdot)$ *is the Whittaker function [5, Eq. (9.220.4)].*

Proof. We start from

$$F_{\gamma_{\text{Rx}}}(\gamma) = \Pr(\max\{\gamma_{\text{AF}_2}, \gamma_{\text{DT}}\} < \gamma). \tag{24}$$

Here, γ_{AF_2} can be rewritten from (7) as

$$\gamma_{\text{AF}_2} = \frac{U\gamma_3}{|h_{\text{S,P}}|^2(\gamma_3 + \mathtt{c}_2)}, \tag{25}$$

which then leads to

$$\begin{aligned} F_{\gamma_{\text{AF}_2}}(\gamma|X) &= \Pr\left(\frac{U\gamma_3}{X(\gamma_3 + \mathtt{c}_2)} < \gamma \middle| X\right) \\ &= 1 - e^{-\lambda_{\text{SR}}X\gamma} \int_0^\infty \lambda_3 e^{-\lambda_{\text{SR}}\mathtt{c}_2\gamma X/\gamma_3} e^{-\lambda_3\gamma_3} d\gamma_3 \\ &= 1 - e^{-\lambda_{\text{SR}}\gamma X} 2\sqrt{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma X}\, \mathcal{K}_1\left(2\sqrt{\lambda_{\text{SR}}\lambda_3\mathtt{c}_2\gamma X}\right), \end{aligned} \tag{26}$$

where (26) is followed from [5, Eq. (6.228.2)] and $\mathcal{K}_1(.)$ denotes the modified Bessel function of the second kind [6,

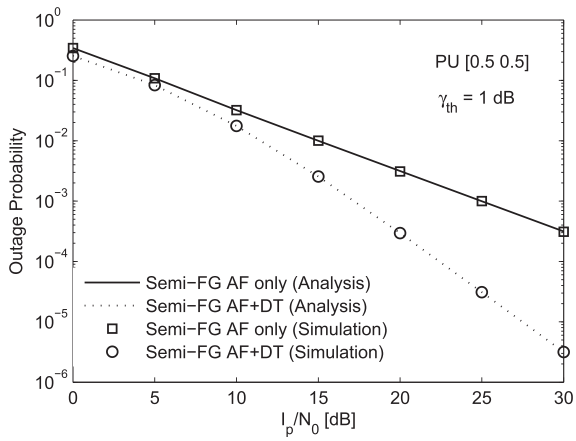

Fig. 1 OP for fixed-gain AF relay networks: Semi-FG.

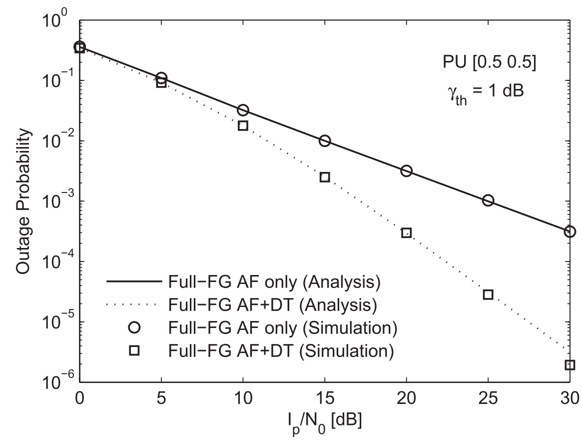

Fig. 2 OP for fixed-gain AF relay networks: Full-FG.

Eq. (9.6.22)]. From (21) and (26), we can obtain the conditional CDF of γ_{Rx}, i.e., $F_{\gamma_{\mathrm{Rx}}}(\gamma|X)$, by utilizing the fact that $F_{\gamma_{\mathrm{Rx}}}(\gamma|X) = F_{\gamma_{\mathrm{AF}_2}}(\gamma|X)\, F_{\gamma_{\mathrm{DT}}}(\gamma|X)$. Then, by taking the expectation over RV X, the unconditional CDF of γ_{Rx} can be expressed as

$$F_{\gamma_{\mathrm{Rx}}}(\gamma) = 1 - \frac{\lambda_{\mathrm{SP}}}{\lambda_{\mathrm{SP}} + \lambda_{\mathrm{SD}}\gamma} - \lambda_{\mathrm{SP}}(I_4 - I_5), \tag{27}$$

where I_4 and I_5 are, respectively, written as

$$I_4 = \int_0^\infty 2e^{-aX}\sqrt{bX}\mathcal{K}_1\left(2\sqrt{bX}\right)\mathrm{d}X,$$
$$I_5 = \int_0^\infty 2e^{-cX}\sqrt{bX}\mathcal{K}_1\left(2\sqrt{bX}\right)\mathrm{d}X,$$

where $a = \lambda_{\mathrm{SP}} + \lambda_{\mathrm{SR}}\gamma$, $b = \lambda_{\mathrm{SR}}\lambda_3 c_2\gamma$, and $c = \lambda_{\mathrm{SP}} + (\lambda_{\mathrm{SR}} + \lambda_{\mathrm{SD}})\gamma$. The two integrals can be obtained in closed-form expression by using [5, Eq. (6.631.3)], which results in (23). □

4. Numerical Results

In this section, we examine the performance of cognitive FG AF relay networks under interference constraints based on the OP. The system topology of primary and secondary networks are similar to those of [2], where the pathloss follows the exponential decay model, i.e. $\Omega_{\mathrm{A,B}} = d_{\mathrm{A,B}}^{-\eta}$. Here, $d_{\mathrm{A,B}}$ denotes the distance between node A and node B and η is the path loss exponent. The positions of SU-Tx, SU-Relay, and SU-Rx are respectively given as [0 0], [0.5 0], and [1 0]. We assume the PU position as [0.5 0.5] and $\eta = 4$. Simulations were conducted to verify the derived OP in (8), (13), (16), and (23), and the results closely match the analysis, as shown in Fig. 1 and Fig. 2. In addition, as can be clearly observed from these two figures, the use of selection combining significantly increases the system performance compared to the relaying link only.

5. Conclusions

In this paper, the OP of cognitive fixed-gain AF relay networks under interference constraints is studied mathematically. The exact closed-form expressions of the OP are derived for both semi FG and full FG relaying as well as with and without direct link. It has been shown that the simulation results perfectly match those of the analysis. This result verifies the promising perspective of deploying the semi-blind AF relay in a cognitive spectrum sharing environment.

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