The Image Reconstruction Principle in PET/CT Scanner

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Abstract — **Thanking to the ability of providing images of slices of organs with very high accuracy currently the PET / CT scanner is widely used for early diagnosis of many cancers and some other diseases. This paper examines the reconstruction algorithms in PET/CT scanner and builds mathematical model describing the operation of this device.**

Keywords — **PET / CT scanner, reconstruction algorithms, radiopharmaceuticals, attenuation correction factor.**

I. INTRODUCTION [1]

In radioactive emission computed tomography the heterogeneous structure, the hydrodynamics of biological subject and the radionuclide redistribution are determined by concentration distribution of the radiopharmaceuticals (FDG) injected onto patient's body or it's parts. Besides, the intensity of recorded γ- rays is under influence of tissue absorption that is characterized by the linear attenuation coefficient $\mu = \mu(\vec{r}) = \mu(x,y,z)$. Then, complete setting up the fundamental problem of radioactive emission computed tomography will include estimating both the distribution of FDG uptake $f(\vec{r}) = f(x,y,z)$ and the attenuation coefficient $\mu(\vec{r})$ based on intensity of γ- rays recorded from outside body.

In practice, the fundamental problem mentioned above is simplified by only calculating the distribution of FDG uptake uptake $f(\vec{r}) = f(x,y,z)$ with suggestion of negligible absorption, or estimating the attenuation coefficient $\mu(\vec{r})$ by another method such as CT -scanner . Just the latest approach is used nowadays in PET/CT equipment.

II. THE IMAGE RECONSTRUCTION ALGORITHM OF PET/CT SCANNER [2,3]

A. Operation Principles of PET/CT Scanner

Using the geometry depicted in Fig 1 the collected emission projection data can be represented mathematically by:

$$p(x',\phi)=\left[\exp\left(-\int_{-\infty}^{\infty}\mu(x,y)dy'\right)\right]\int_{-\infty}^{\infty}f(x,y)dy' \qquad (1)$$

where the exponential term (in square brackets) represents the attenuation along the line of response (LOR) at detector position x' and projection angle ϕ, and where f(x,y) represents the distribution of FDG or other positron tracer in the patient

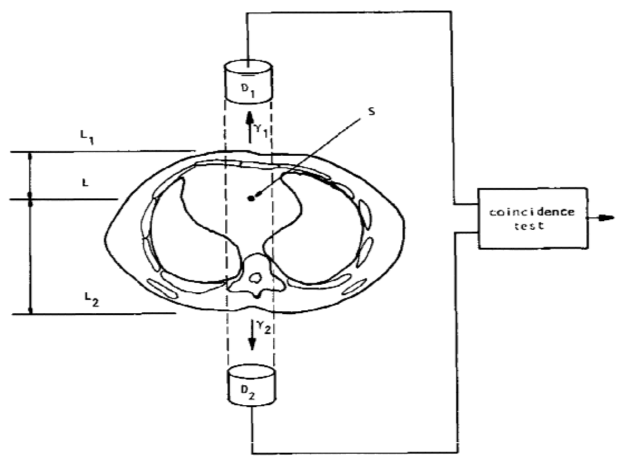

Fig. 1 A photon emitted at S and traveling toward the D1 detector is attenuated over a distance of L_1 – L. while a photon traveling toward the D2 detector undergoes an attenuation proportional to L – L2.

The form of Eq. 1 can be simplified by expressing the inverse of the exponential term as an attenuation correction factor

$$a(x',\phi)=\exp\left(\int_{-\infty}^{\infty}\mu(x,y)dy'\right) \qquad (2)$$

which represents inverse of the dual-photon attenuation along the LOR for detector position x' and projection angle ϕ. A multiplicative correction for photon attenuation is then given by

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$$p_{AC}(x',\phi)=a(x',\phi)p(x',\phi)=\int_{-\infty}^{\infty} f(x,y)dy' \qquad (3)$$

where $p_{AC}(x', \phi)$ are the emission data (sinograms) corrected for attenuation. The corrected projection values $p_{AC}(x', \phi)$ can be used in a tomographic image reconstruction algorithm, such as FBP, to estimate the two-dimensional distribution of radionuclide concentration represented by the function f(x,y). Alternatively, the acquired projection data $p(x, \phi)$ can be reconstructed directly with an iterative method, where the attenuation correction factors (ACFs) are used to provide proper statistical weighting to the data. An example of this approach is attenuation-weighted OSEM (AWOSEM), which has been implemented on most commercial PET scanners.

The Attenuation Compensation for positron emission tomography is easier in comparison with single photon emission tomography. This ease is explained as follows.

Let's say that the detectors D1 and D2 in Fig. 1 are being used to measure one ray in a projection and let's also assume that there is a source of positron emitters located at the point S. Suppose for a particular positron annihilation, the two annihilation gamma-ray photons labeled y1 and y2 in the figure are released toward D1 and D2, respectively. The probability of y1 reaching detector D, is given by

$$\exp(-\int_{L}^{L1} \mu(x)dx) \qquad (4)$$

where $\mu(x)$ is the attenuation coefficient at 511 keV as a function of distance along the line joining the two detectors. Similarly, the probability of the photon *γ2* reaching the detector D2 is given by

$$\exp(-\int_{L2}^{L} \mu(x)dx) \qquad (5)$$

Then the probability that this particular annihilation will be recorded by the detectors is given by the product of the above two probabilities

$$\exp(-\int_{L}^{L1} \mu(x)dx)\,.\exp(-\int_{L2}^{L} \mu(x)dx) \qquad (6)$$

which is equal to

$$\exp(-\int_{L1}^{L2} \mu(x)dx) \qquad (7)$$

This is a most remarkable result because, first, this attenuation factor is the same no matter where positron annihilation occurs on the line joining D1 and D2, and, second, the factor above is exactly the attenuation that a beam of monoenergetic photons at 511 keV would undergo in propagating from L1 at one side to L2 at the other. Therefore, one can readily compensate for attenuation by first doing a transmission study .

B. Determining Attenuation Correction Factors [4]

As shown in Eqs. 2 and 3, attenuation correction factors $a(x', \phi)$ must be derived from transmission data to correct the PET data for photon attenuation. With PET/CT scanners a 511 keV attenuation map can be generated from the CT image to correct the PET emission data for photon attenuation: Because the CT images contained attenuation information for effective photon energies of about 70 keV, given in HU, they had to be transformed to PET photon energies of 511 keV (μ-maps). For this purpose, different approaches (such as Scaling, Segmentation, Hybrid and Hybrid Dual-Energy methods) have been developed and implemented in commercially available tomographs. For example, the following established bilinear algorithm can be used:

$$\mu^{PET}(CT \le 0HU) = \mu^{PET}_{H_2O}(CT + 1,000)/1,000 \qquad (8)$$

$$\mu^{PET}(CT > 0HU) = \mu^{PET}_{H_2O} + CT\frac{\mu^{PET}_{H_2O}(\mu^{PET}_{Bone} - \mu^{PET}_{H_2O})}{1,000(\mu^{CT}_{H_2O} - \mu^{CT}_{H_2O})} \qquad (9)$$

where CT denotes the CT image value in HU and the different μ values represent the linear absorption coefficients of bones and water at CT and PET energies, respectively. The following values were used:

$$\begin{aligned} \mu^{PET}_{H_2O} &= 0.096cm^{-1} \\ \mu^{PET}_{Bone} &= 0.172cm^{-1} \\ \mu^{CT}_{H_2O} &= 0.184cm^{-1} \\ \mu^{CT}_{Bone} &= 0.428cm^{-1} \end{aligned} \qquad (10)$$

Thus, an analysis done above enables to define The Image Reconstruction Algorithm of PET/CT scanner as showed in Fig 2.

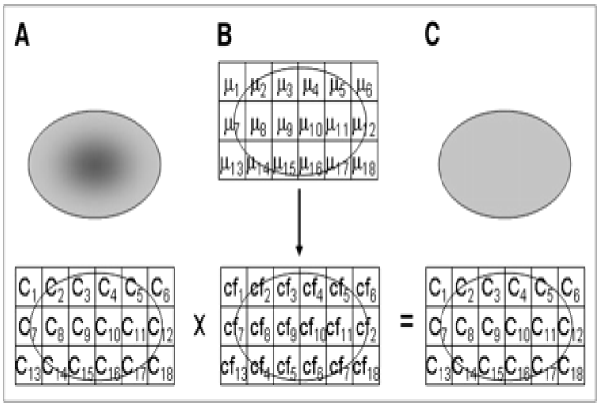

Fig. 2 PET images are corrected by Attenuation maps.

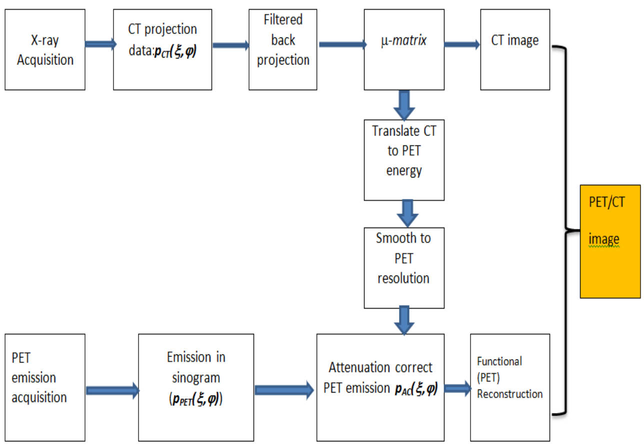

Fig. 3 Image Reconstruction algorithm in PET/CT scanner.

III. Mathematic modeling and simulation of PET/CT scanner' operation

To appreciate the correctness of Reconstruction algorithm and to illustrate the operation of PET/CT scanner it's mathematic simulation had been realized by software MATLAB.

Mathematic model is built from Reconstruction algorithm presented in Figure 3. Physical processes implemented in model of PET/CT scanner include next stages:

- CT and PET acquisions
- Forming CT projection data and PET sinogram
- Filtered back projection for receiving CT-image and μ-matrice
- Translating CT μ-matrice to PET energy
- Correcting Attenuation on PET emission:
- PET Functional Reconstruction
- Building PET/CT image

All these stages are imitated in software " PET/CT Simulator " with user's interface as follows:

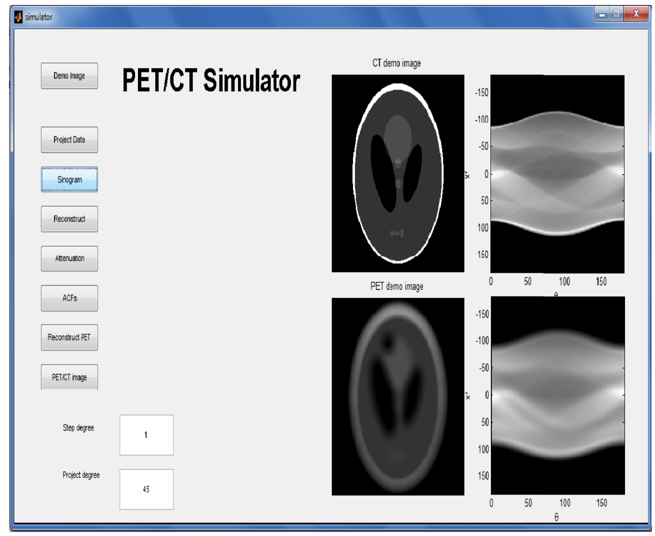

Fig. 4 The user's interface of software" PET/CT Simulator"

IV. Conclusions

- The algorithm and built upon it mathematic model describes properly the Image Reconstruction Principle in PET/CT scanner.
- The software " PET/CT Simulator " is useful for learning and reseaching Reconstruction principles of PET/CT scanner.

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