Learning Rule for TSK Fuzzy Logic Systems Using Interval Type-2 Fuzzy Subtractive Clustering

Binh Huy Pham, Hai Trung Ha, and Long Thanh Ngo

Department of Information Systems, Faculty of Information Technology,
Le Quy Don Technical University, No 100, Hoang Quoc Viet, Hanoi, Vietnam
{huybinhhvhc,hatrunghai1982,ngotlong}@gmail.com

Abstract. The paper deals with an approach to model TSK fuzzy logic systems (FLS), especially interval type-2 TSK FLS, using interval type-2 fuzzy subtractive clustering (IT2-SC). The IT2-SC algorithm is combined with least square estimation (LSE) algorithms to pre-identify a type-1 FLS form from input/output data. Then, an interval type-2 TSK FLS can be obtained by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centroids, standard deviation of Gaussian membership functions and consequence parameters. Results is shown in comparison with the approach based on type-1 subtractive clustering algorithm.

Keywords: subtractive clustering, type-2 fuzzy sets, fuzzy logic system, TSK model.

1 Introduction

TSK fuzzy logic systems (TSK FLSs) have widely been deployed in various real applications especially in model-based control and model-based fault diagnosis. TSK qualitative modelling, as known as TSK modelling, was proposed in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set [6,7]. When, the identification of a TSK FLS using clustering involves formation of clusters in the data space and translation of these clusters into TSK rules such that the model obtained is closer to the system to be identified [4,5]. However, in most real data exists uncertainty and vagueness which cannot be appropriately managed by type-1 fuzzy sets. Meanwhile, type-2 fuzzy sets allow us to obtain desirable results in designing and managing uncertainty. Mendel et al [1,2,3] extended previous studies and established a complete type-2 fuzzy logic theory with the handling of uncertainties. On the basis, type-2 TSK FLS was presented [16].

One of the important tasks to design a fuzzy system is how to determine the number of rules (structure identification). There are two approaches to generate initial fuzzy rules: manually and automatically. In the automatically approaches, the basic idea is to estimate fuzzy rules through learning process from input-output sample data. An automatic data-driven based method for generating

L.T. Bui et al. (Eds.): SEAL 2012, LNCS 7673, pp. 430–439, 2012.

the initial fuzzy rules is Chius subtractive clustering algorithm (SC) [6]. When, subtractive clustering algorithm is combined with least squares estimation algorithm to design TSK FLSs [8]. Then, an interval type-2 TSK FLS can be obtained by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centroids and consequence parameters [9]. In this way, clustering results of SC decides the structure of fuzzy systems. Interval type-2 fuzzy subtractive clustering (IT2-FSC) [15] is extension of SC algorithms to handle uncertainty.

In subtractive clustering algorithms, setting subtractive clustering parameters are very influential to the results of clustering. This paper deals with an approach to model type-2 TSK FLS from input/output dataset. Interval type-2 fuzzy subtractive clustering is used to determine the number of rules and to learn rule-base from dataset. IT2-FSC is also combined with LSE algorithm to estimate parameters for designing interval type-2 TSK FLS. Results on function approximation is shown that the proposed approach to obtain accuracy and simple TSK models.

The remainder of this paper is organized as follows. In Section 2 introduces briefly type-2 fuzzy sets, interval type-2 fuzzy subtractive clustering. In section 3, we discuss how to using interval type-2 fuzzy subtractive clustering algorithm to design TSK FLS and extend interval type-2 TSK FLS from type-1 TSK FLS. In section 4, we provide several experiments to show the validity of our proposed method. Finally, section 5 gives the summaries and conclusions.

2 Interval Type-2 Fuzzy Logic Systems

2.1 Type-2 Fuzzy Sets

A type-2 fuzzy set in X is denoted $\tilde{A}$, and its membership grade of $x \in X$ is $\mu_{\tilde{A}}(x,u)$, $u \in J_x \subseteq [0,1]$, which is a type-1 fuzzy set in [0, 1]. The elements of domain of $\mu_{\tilde{A}}(x,u)$ are called primary memberships of x in $\tilde{A}$ and memberships of primary memberships in $\mu_{\tilde{A}}(x,u)$ are called secondary memberships of x in $\tilde{A}$.

Definition 1. *A $type-2$ $fuzzy$ set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.,*

$$\tilde{A} = \{((x,u),\mu_{\tilde{A}}(x,u))|\forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{1}$$

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

At each value of x, say $x = x'$, the 2-D plane whose axes are u and $\mu_{\tilde{A}}(x',u)$ is called a *vertical slice* of $\mu_{\tilde{A}}(x,u)$. A *secondary membership function* is a vertical slice of $\mu_{\tilde{A}}(x,u)$. It is $\mu_{\tilde{A}}(x=x',u)$ for $x \in X$ and $\forall u \in J_{x'} \subseteq [0,1]$, i.e.

$$\mu_{\tilde{A}}(x=x',u) = \int_{u \in J_{x'}} f_{x'}(u)/u, J_{x'} \subseteq [0,1] \tag{2}$$

in which $0 \leq f_{x'}(u) \leq 1$.

Type-2 fuzzy sets are called an interval type-2 fuzzy sets if the secondary membership function $f_{x'}(u) = 1 \; \forall u \in J_x$ that are defined as follows:

Definition 2. *An interval type-2 fuzzy set $\tilde{A}$ is characterized by an interval type-2 membership function $\mu_{\tilde{A}}(x,u) = 1$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.,*

$$\tilde{A} = \{((x,u),1) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{3}$$

Uncertainty of $\tilde{A}$, denoted FOU, is union of primary functions i.e. $FOU(\tilde{A}) = \bigcup_{x \in X} J_x$. Upper/lower bounds of membership function (UMF/LMF), denoted $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, of $\tilde{A}$ are two type-1 membership function and bounds of FOU.

2.2 Type-1 TSK Fuzzy Logic Systems

A generalized type-1 TSK model is described by fuzzy IF-THEN rules which represent input-output relations of a system. For a multi-input-single-output (MISO) first order type-1 TSK model, its l^{th} rule can be expressed as follows:

R^l : IF x_1 is F_1^l AND x_2 is F_2^l AND ... AND x_n is F_n^l THEN

$$w^l = c_o^l + c_1^l x_1 + c_2^l x_2 + ... + c_n^l x_n \tag{4}$$

in which $x_i (i = 1, ..., n)$ are linguistic variables, $F_i^l (i = 1, ..., n)$ are type-1 fuzzy sets, w^l is output from the l^{th} IF-THEN rule, $c_i^l (i = 0, 1, ..., n)$ are consequent parameters.

The output of a TSK FLS is computed as following steps:

- Calculating degree of firing of l^{th} rule as:

$$f^l = \mu_1^l(x_1) \wedge \mu_2^l(x_2) \wedge \ldots \wedge \mu_n^l(x_n) \tag{5}$$

where $\wedge$ is a conjunction operator and a t-norm, can be minimum or product.

- Calculating the output from the l^{th} IF-THEN rule of M rules FLS:

$$w^l = c_o^l + c_1^l x_1 + c_2^l x_2 + ... + c_n^l x_n \tag{6}$$

- Calculating the output of FLS by weighted averaging:

$$\mathrm{W} = \frac{\sum_{i=1}^{k} f^i \mathrm{w}^i}{\sum_{i=1}^{k} f^i} \tag{7}$$

2.3 Interval Type-2 TSK Fuzzy Logic Systems

An interval type-2 TSK model includes M-rules, n-inputs, its l^{th} fuzzy IF-THEN rule can be expressed as bellow:

R^l : IF x_1 is $\tilde{F}_1^l$ AND IF x_2 is $\tilde{F}_2^l$ AND ... AND IF x_k is $\tilde{F}_k^l$ THEN

$$\tilde{w}l = \tilde{C}_0^l + \tilde{C}_1^l x_1 + \tilde{C}_2^l x_2 + ... + \tilde{C}_n^l x_n \tag{8}$$

in which $x_i(i = 1, ..., n)$ are linguistic variables, w^l is output from the l^{th} IF-THEN rule; $\tilde{C}_i^l(i = 0, 1, ..., n)$ type-1 fuzzy sets are consequent parameters and $\tilde{C}_i^l = [c_i^l - s_i^l, c_i^l + s_i^l]$ with c_i^l denotes the centroid of $\tilde{C}_i^l$ and s_i^l denotes the spread of $\tilde{C}_i^l$; $\tilde{F}_1^l(i = 0, 1, ..., n)$ are interval type-2 fuzzy sets and $\tilde{\mu}_i^l = [\underline{\mu}_{\mathrm{i}}^{\mathrm{l}}, \overline{\mu}_{\mathrm{i}}^{\mathrm{l}}]$.

Interval type-2 TSK FLS is computed as the following steps:

- Degree of firing of l^{th} rule $f^l = [\underline{f}^l, \overline{f}^{\mathrm{l}}]$ with

$$\begin{aligned} \underline{f}^l &= \underline{\mu}_1^l(x_1) \wedge \underline{\mu}_2^l(x_2) \wedge \ldots \wedge \underline{\mu}_n^l(x_n) \\ \overline{f}^l &= \overline{\mu}_1^l(x_1) \wedge \overline{\mu}_2^l(x_2) \wedge \ldots \wedge \overline{\mu}_n^l(x_n) \end{aligned} \tag{9}$$

- The output from the l^{th} IF-THEN rule of M rules: $\tilde{\mathrm{w}}^l = [\mathrm{w}_{\mathrm{L}}^{\mathrm{l}}, \mathrm{w}_{\mathrm{R}}^{\mathrm{l}}]$ with

$$\mathrm{w}_{\mathrm{L}}^{\mathrm{l}} = \sum_{j=1}^{n} c_j^l x_j + c_0^l - \sum_{j=1}^{n} s_j^l x_j - s_0^l \tag{10}$$

$$\mathrm{w}_{\mathrm{R}}^{\mathrm{l}} = \sum_{j=1}^{n} c_j^l x_j + c_0^l + \sum_{j=1}^{n} s_j^l x_j + s_0^l \tag{11}$$

- Calculating output of FLS by weighted averaging of individual rules contributions:

$$\mathrm{w}_{\mathrm{L}} = \frac{\sum_{j=1}^{k} \underline{f}^j * \mathrm{w}_{\mathrm{L}}^{\mathrm{j}}}{\sum_{j=1}^{k} \underline{f}^j} \ and\ \mathrm{w}_{\mathrm{R}} = \frac{\sum_{j=1}^{k} \overline{f}^j * \mathrm{w}_{\mathrm{R}}^{\mathrm{j}}}{\sum_{j=1}^{k} \overline{f}^j} \tag{12}$$

3 Rule Extraction for Interval Type-2 TSK FLS

The problem of identification of TSK model is divided into two sub-tasks: Learning the antecedent part of the model, which consists on the determination of centroids and spreads of membership functions by using IT2-FSC; and Learning the parameters of the linear subsystems of the consequent by using LSE algorithm.

3.1 Learning Rule Antecedents

Subtractive clustering estimated the potential of a data point as a cluster centroid based on the density of surrounding data points, which is actually based on the distance between the data point with the remaining data points. In addition, we must set four parameters: accept ratio $\overline{\varepsilon}$, reflect ratio $\underline{\varepsilon}$, cluster radius r_a and squash factor η (or r_b) [4,5]. The choice of parameters have greatly influences to results of clustering. SC includes various types of uncertainty as distance measure, initialization parameters... So we consider a fuzziness parameters that control the distribution of data points into clusters by making the parameter m in the density function to calculate the potential of a data point [15]. Membership degree of a point in the k^{th} cluster centroid is defined as following formula:

$$\mu_{ik} = e^{-\frac{4}{r_a^2}(x_i - x_k)^{\frac{2}{m-1}}} \tag{13}$$

where x_k is the k^{th} cluster centroid.

According to the formula (13), membership value of a data point in the k^{th} cluster centroid depends on the position of the k^{th} cluster and the fuzziness parameter m. Thus, the fuzziness parameter m is the most uncertainty element in the expanded subtractive clustering algorithm. Therefore, to design and manage the uncertainty for fuzziness parameter m, pattern set to interval type-2 fuzzy sets is extended using two fuzzifiers m_1 and m_2, which creates a footprint of uncertainty (FOU) for the fuzziness parameter m. Then the degree of membership of the k^{th} cluster centroid is defined as the following formula:

$$\begin{cases} \overline{\mu}_{ik} = e^{-\frac{4}{r_a^2}(x_i - x_k)^{\frac{2}{m_1-1}}} \\ \underline{\mu}_{ik} = e^{-\frac{4}{r_a^2}(x_i - x_k)^{\frac{2}{m_2-1}}} \end{cases} \tag{14}$$

Two density functions are computed the potential of each data point as follows:

$$\begin{cases} \overline{P}_i = \sum_{j=1}^{n} e^{-\frac{4}{r_a^2}(x_j - x_i)^{\frac{2}{m_1-1}}} \\ \underline{P}_i = \sum_{j=1}^{n} e^{-\frac{4}{r_a^2}(x_j - x_i)^{\frac{2}{m_2-1}}} \end{cases} \tag{15}$$

The centroids are identified by the formula (15) and type-reduction for centroids is done as bellows:

$$P_i = \frac{\overline{P}_i * m_1 + \underline{P}_i * m_2}{m_1 + m_2} \tag{16}$$

When the k^{th} cluster centroid is identified, the density of all data points is revised by using the following formula:

$$\begin{cases} \underline{P}_i^{sub} = P_k^* \sum_{j=1}^{n} e^{-\frac{4}{r_b^2} d_{\mathrm{ij}}^{\frac{2}{m_1-1}}} \\ \overline{P}_i^{sub} = P_k^* \sum_{j=1}^{n} e^{-\frac{4}{r_b^2} d_{\mathrm{ij}}^{\frac{2}{m_2-1}}} \\ P_i^{sub} = \frac{\underline{P}_i^{sub} * m_1 + \overline{P}_i^{sub} * m_2}{m_1 + m_2} \\ P_i = P_i - P_i^{sub} \end{cases} \tag{17}$$

Because, each cluster centroid is representative of a characteristic behaviour of the system, the resulting cluster centroids are used as parameters of the antecedent parts defining the focal points of the rules of the model. Then clustering results of IT2-SC decides the structure of fuzzy systems.

3.2 Learning Rule Consequent Using LSE Algorithm

The output of type-1 TSK model is determined by the formula (7).

Suppose that:

$$\delta^i = \frac{f^i}{\sum_{i=1}^{k} f^i} \tag{18}$$

When:

$$\mathrm{W} = \sum_{i=1}^{k} \delta^i w^i \tag{19}$$

For a given set of m input-output data points. The equations can be obtained as:

$$\begin{array}{l} W^1 = \sum_{i=1}^{k} \delta^i c_0^i + \sum_{i=1}^{k} x_i \delta^i c_1^i + ... + \sum_{i=1}^{k} x_i \delta^i c_n^i \\ W^2 = \sum_{i=1}^{k} \delta^i c_0^i + \sum_{i=1}^{k} x_i \delta^i c_1^i + ... + \sum_{i=1}^{k} x_i \delta^i c_n^i \\ ... \\ W^m = \sum_{i=1}^{k} \delta^i c_0^i + \sum_{i=1}^{k} x_i \delta^i c_1^i + ... + \sum_{i=1}^{k} x_i \delta^i c_n^i \end{array} \tag{20}$$

The formula (20) can be taken a standard form: $AP = W$, where A is a constant matrix (known), W is a matrix of the output and P is a matrix of parameters to be estimated. We use least square estimation problem to determine P as:

$$P = (A^T A)^{-1} A^T W \tag{21}$$

3.3 Building for Interval Type-2 TSK FLS

An interval type-2 TSK FLS can be obtained by considering the membership functions (MFs) of its existed type-1 counterpart as primary MFs and assigning uncertainty to cluster centroids, standard deviation of Gaussian MF and consequence parameters with membership functions of type-1 FLS is defined by

$$F_j^l = N(x_j, x_l^*, \sigma) = \exp\left[-\frac{1}{2}\left(\frac{x_j - x_l^*}{\sigma}\right)^2\right] \tag{22}$$

in which $\sigma = \frac{r_a}{2\sqrt{2}}$.

By doing that, cluster centroids are expanded from a certain point to a fuzzy number as follows:

$$\tilde{x}_l^* = [\mathrm{x}_l^*(1 - \mathrm{a}^l), \mathrm{x}_l^*(1 + \mathrm{a}^l)] = [\underline{\mathrm{x}}_l^*, \bar{\mathrm{x}}_l^*] \tag{23}$$

where a^l is the spread percentage of cluster centre x_l^* in Fig.1. Then, the upper membership function, $\overline{\mu}_j^l(\mathrm{x_j})$, is defined by

$$\overline{\mu}_j^l(x_j) = \begin{cases} N(x_j, \overline{x}_l^*, \sigma), x_j > \overline{x}_l^* \\ 1, \underline{x}_l^* <= x_j <= \overline{x}_l^* \\ N(x_j, \underline{x}_l^*, \sigma), x_j < \underline{x}_l^* \end{cases} \tag{24}$$

And the upper membership function, $\underline{\mu}_j^l(\mathrm{x_j})$, is defined by

$$\underline{\mu}_j^l(x_j) = \begin{cases} N(x_j, \underline{x}_l^*, \sigma), x_j >= \frac{\underline{x}_l^* + \overline{x}_l^*}{2} \\ N(x_j, \overline{x}_l^*, \sigma), x_j < \frac{\overline{x}_l^* + \underline{x}_l^*}{2} \end{cases} \tag{25}$$

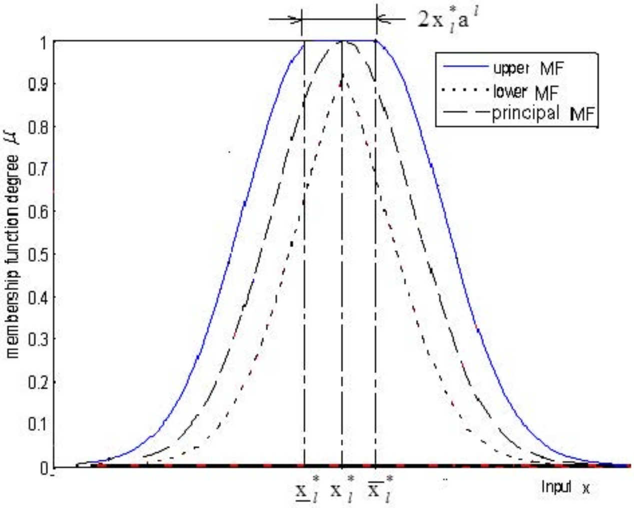

Fig. 1. Spread and centroid of Gaussian Type-2 FSs

Whereas, consequent parameters are obtained by expanding consequent parameters from its type-1 TSK model to fuzzy numbers by formula(26) where b_j^l is the spread percentage of fuzzy numbers $\tilde{c}_j^l$

$$\tilde{c}_j^l = c_j^l(1 \pm b_j^k) \tag{26}$$

The TSK FLS modelling algorithm be proposed as below:

Step 1: Use our proposed IT2-SC algorithm combined with least squares estimation algorithms to pre-identify a type-1 FLS form from input/output data.

Step 2: Calculate root-mean-square-error (RMSE), if RMSE is bigger than expected error limitation, go to Step 3. If not, go to Step 5, which means the model is acceptable, no need to use type-2 TSK model.

Step 3: Expand type-1 TSK model to type-2 TSK model:

- Spread cluster centroid to expanding premise membership functions from type-1 fuzzy sets to type-2 fuzzy sets using formulas (24) and (25)
- Spread the parameters of consequence to expanding parameters of consequences from certain value to fuzzy numbers below formula (26).

Step 4: Identify a type-2 TSK FLS

Step 5: Output the results of TSK FLS modelling.

The results of TSK FLS modeling algorithm are a type-1 or type-2 TSK FLS model.

4 Experimental Results

We consider the problem of type-1 TSK fuzzy model for approximating the following non-linear function:

$$y = (x - 2.5)^3 + x + 1 \tag{27}$$

where $x \in [0, 4]$, we used equally spaced values to generate 1001 data points. Here, we randomly selected 751 as training data and 250 as testing data. Table 1 describes four rules type-1 TSK model by using IT2-SC algorithm with initialization parameters, respectively,$\overline{\varepsilon} = 0.5$, $\underline{\varepsilon} = 0.15$, $r_a = 0.5$, $\eta = 1.25$ and two fuzzifiers: $m_1 = 1.85$ and $m_2 = 2.15$.

Table 1. Results of type-1 TSK model based on SC of Chiu and our proposed IT2-SC

Rules	If x then $y = p_1 * x + p_0$	
	TSK model based on SC of Chiu	TSK model based on our proposed IT2-SC
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.464}{0.70711}\right)^2)$ then $y = 4.57271x - 6.91789$	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.42}{0.70711}\right)^2)$ then $y = 6.4900x - 12.2933$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.9}{0.70711}\right)^2)$ then $y = 12.3745x - 24.1487$	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.824}{0.70711}\right)^2)$ then $y = 25.3741x - 49.8226$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x-3.724}{0.70711}\right)^2)$ then $y = 7.48193x - 21.9791$	If $x = \exp(-\frac{1}{2}\left(\frac{x-3.728}{0.70711}\right)^2)$ then $y = 8.04252x - 24.3523$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.188}{0.70711}\right)^2)$ then $y = 28.9666x - 10.2412$	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.148}{0.70711}\right)^2)$ then $y = 42.1097x + 3.72731$

In this case, the RMSE-training is 0.01534 and the RMSE-testing is 0.01511. Type-1 SC algorithm is also used for identification a type-1 TSK model with initialization parameters, respectively,$\overline{\varepsilon} = 0.5$, $\underline{\varepsilon} = 0.15$, $r_a = 0.5$, $\eta = 1.25$. Then, the RMSE-training is 0.02196 and the RMSE-testing is 0.02184. We can see that TSK model generated by our proposed method has result accuracy with smaller RMSE. In Fig. 2, both training data and testing data, TSK model generated by our proposed method has result as quite same as real system.

We can change value of two fuzzifiers to obtain better type-1 TSK model. In table 2, we see that type-1 TSK model based on our proposed IT2-SC with values of two fuzzifiers, $m_1 = 1.85$ and $m_2 = 2.15$, has the best result. RMSE on training data and RMSE on testing data is quite small. The figure 2 shows plots of obtained TSK model on training and testing data.

We consider two rules type-1 TSK model by using IT2-SC algorithm with two fuzzifiers: $m_1 = 1.3$ and $m_2 = 2.7$. The type-1 TSK model is described in table 3. In this model, the RMSE on training data is 0.46258 and RMSE on test data is 0.50296. Two rules of type-2 TSK model obtain from type-1 TSK model by using above described spread of fuzzy numbers. The system gains RMSE of training and testing data, respectively, are 0.84944 and 0.35684.

Table 2. Result of type-1 TSK model by different values of two fuzzifiers

m_1 and m_2	Number of rules	RMSE_training	RMSE_testing
1.95 and 2.05	4	0.02262	0.02113
1.9 and 2.1	4	0.01977	0.01811
1.85 and 2.15	4	0.01534	0.01511
1.8 and 2.2	3	0.3071	0.3031
1.7 and 2.3	3	0.3241	0.3204
1.6 and 2.4	2	0.4534	0.4531
1.5 and 2.5	2	0.42705	0.43769
1.4 and 2.6	2	0.42245	0.43887
1.3 and 2.7	2	0.4704	0.48038
1.2 and 2.8	1	2.1763	2.1285

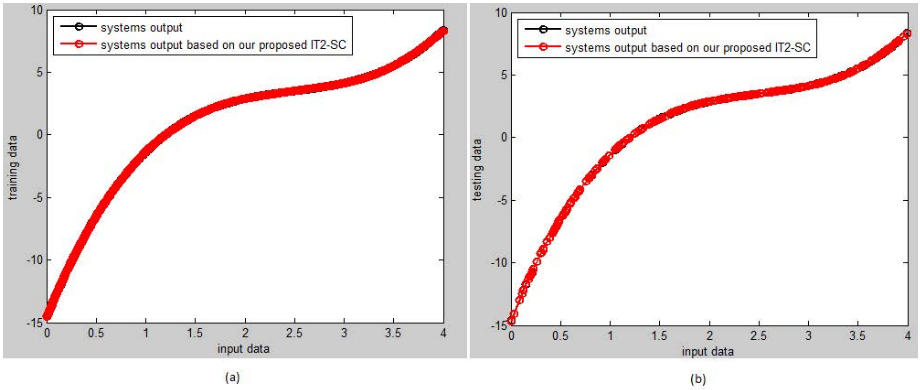

Fig. 2. Result of TSK model; (a): On training data; (b): On testing data

Table 3. Type-1 TSK model and type-2 TSK model

Rules	If x then $y = p_1 * x + p_0$	
	type-1 TSK model	type-2 TSK model
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.004}{0.7064}\right)^2)$ then $y = 1.72375x - 1.48008$	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.004*(1-10\%)}{0.7064}\right)^2)$ then $y = 1.72375 * (1-20\%)x - 1.48008 * (1-20\%)$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x}{0.7064}\right)^2)$ then $y = 12.3839x - 13.9905$	If $x = \exp(-\frac{1}{2}\left(\frac{x}{0.7064}\right)^2)$ then $y = 12.3839 * (1-20\%)x - 13.9905 * (1-20\%)$

5 Conclusion

The paper presents a new approach to design TSK model. Here, we used an our proposed IT2-SC combined with least squares estimation algorithm. The result of experiments is shown the validity of our proposed method.

For the future works, we will improve the computational performance by speeding up the algorithm using GPU.

References

1. Mendel, J., John, R.: Type-2 fuzzy set made simple. IEEE Trans. on Fuzzy Systems 10(2), 117–127 (2002)
2. Karnik, N., Mendel, J.M.: Operations on Type-2 Fuzzy Sets, Fuzzy Sets and Systems, vol. 122, pp. 327–348 (2001)
3. Mendel, J.M., John, R.I., Liu, F.: Interval Type-2 Fuzzy Logic Systems Made Simple. IEEE Trans. on Fuzzy Systems 14(6), 808–821 (2006)
4. Chiu, S.L.: Fuzzy Model Identification Based on Cluster Estimation. Journal on Intelligent Fuzzy Systems 2, 267–278 (1994)
5. Chiu, S.L.: Extracting Fuzzy Rules from Data for Function Approximation and Pattern Classification. In: Dubois, H., Prade, R., Yager, R. (eds.) Fuzzy Information Engineering: A Guided Tour of Applications. John Wiley & Sons (1997)
6. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. on Sys., Man, and Cyb. 15(1), 116–132 (1985)
7. Sugeno, M., Kang, G.: Structure identification of fuzzy model. Fuzzy Sets and Systems 28(1), 15–33 (1988)
8. Ren, Q., Baron, L., Balazinski, M., Jemielniak, K.: Tool condition monitoring using the TSK fuzzy approach based on subtractive clustering method. News Frontiers in Applied Artificial Intelligence, pp. 52–61. Springer, Berlin (2008)
9. Ren, Q., Baron, L., Balazinski, M.: Type-2 Takagi-Sugeno-Kang fuzzy logic modeling using subtractive clustering. In: Proceedings of the NAFIPS, pp. 1–6 (2006)
10. Ren, Q., Baron, L., Balazinski, M.: Uncertainty prediction for tool wear condition using type-2 TSK fuzzy approach. In: Proceeding of the 2009 IEEE Int' Conf. on Systems, Man, and Cybernetics (IEEE-SMC 2009), pp. 666–671 (2009)
11. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. on Sys., Man, and Cyb. 15(1), 116–132 (1985)
12. Zhang, W.B., Liu, W.J.: IFCM: Fuzzy Clustering for Rule Extraction of Interval Type-2 Fuzzy Logic System. In: The 46th IEEE Conf. on Decs. & Control, pp. 5318–5322 (2007)
13. Demirli, K., Muthukumaran, P.: Higher Order Fuzzy System identification Using Subtractive Clustering. J. of Intelligent and Fuzzy Systems 9, 129–158 (2000)
14. Demirli, K., Cheng, S.X., Muthukumaran, P.: Subtractive Clustering Based on Modeling of Job Sequencing with Parametric Search. Fuzzy Sets and Systems 137, 235–270 (2003)
15. Ngo, L.T., Pham, B.H.: Approach to Image Segmentation Based on Interval Type-2 Fuzzy Subtractive Clustering. In: Horng, M.-F. (ed.) ACIIDS 2012, Part II. LNCS, vol. 7197, pp. 1–10. Springer, Heidelberg (2012)
16. Liang, Q., Mendel, J.M.: An Introduction to Type-2 TSK Fuzzy Logic Systems. In: IEEE International Conference on Fuzzy Systems, vol. 3, pp. 1534–1539 (1999)