A Novel Computation for Supplementing Interference Analytical Model in 802.11-based Wireless Mesh Networks

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***Abstract*—Interference impact problem in multi-hop networks such as Wireless Mesh Networks (WMNs) has been attracted many studies due to its challenges in recent years. Three-Markov chain model was a favorable model to analyze interference quantity around nodes. In this paper, we adopt a simple computation method to calculate steady states of node behavior's in three-state Markov chain and propose a novel computation for supplementing interference analytical model in 802.11-based wireless mesh networks.**

Keywords: Interference, analytical model, Markov chain, 802.11 DCF, CSMA/CA.

I. Introduction

In recent years, Wireless Mesh Networks (WMNs) have been considered to be a key solution for next generation wireless networks. The primary advantages of a WMN lie in its inherent fault tolerance again network failures, simplicity of setting up a network, and broadband capability. However, it is pointed out main challenges related with performance downgrade of WMNs are originated from inherit multi-hop networks as shared medium, dynamic configuration, varying network traffic, natural environment and fluctuation of link quality, etc [1]. Among these challenges, the interference issue has been attracted many researchers due to its complexity and grave impacts on the network performance. Some models and interference aware metrics in [2]-[6] are probably the most notable examples to overcome this challenge. With our best knowledge, the analytical model based on three-Markov chain proposed by [6] [7] is a good way to reflect interference on link quality in 802.11-based multi-hop wireless networks such as WMNs. These approaches allow predicting link quality through local information around a node. So interference aware link quality can be considered not only time manner but also state manner. However, the computation of steady states of nodes was not clarified and the successful transmission probability was not covered fully in these proposals. This paper presents a clearly analytical method to determine steady states of nodes. Specifically, the proposed complementary computation is solved by a new generic problem to overcome the previous limitation. The rest of this paper is organized as follows: Section 2 briefly discusses the existing approaches that are directly related to our development. We adopt a simple calculation method based on mathematical analysis to calculate steady states of the nodes model in section 3. Section 4 presents our detailed analytical solution to compute winning contention probability of nodes in the back-off state. Finally, the conclusion is drawn in Section 5 with the indication of our future work.

II. Related Work

Performance on wireless mesh networks gravely suffers from losses due to wireless link deterioration and contention stemming from interference characteristics naturally. The authors in [8] have proven throughput capacity in multi-hop wireless networks reduce of the order $\Theta(W/\sqrt{n})$. They proposed two interference models: *Protocol model* and *physical model*. On the time aspect of a communication session, the physical model related to physical phenomenona during a transmission session. The protocol model implied the logical aspect of interference impact which is related to events occurring before a transmission session. Otherwise, focusing into the CSMA/CA mechanism in IEEE 802.11-based wireless networks, the authors in [2] used mathematical analysis to analyze the behaviors of a node in the back-off process. In which, a two-dimensional Markov chain to model the evolution of the back-off process is proposed. The supplementary unsaturated traffic condition and evaluating throughput as a function of the collision probability were proposed in [9]. They found that a relatively low value of the collision probability maximizes the throughput. These studied results have shown that interference impacts are closely related with throughput, MAC protocols and transmission session time, etc. In the fact that link quality can be seen as an input parameter of routing algorithms in link state routing protocols. In WMNs context, interference aware metrics could be constructed by prediction based methods or measurement based methods. Based on multi-area interference model, author in [3] used a prediction based method to estimate interference degree and interpret quality of link through packet loss ratio. The authors in [4] [5] used measurement based method to estimate interference level through busy/idle time ratio. However, these proposed metrics were interpreted as a partial aspect of interference impacts. Link quality must be considered in both time manner and state manner for more accurate expressions. This can be seen as the key factor to enhance network performance of WMNs.

From our point of view, a favorable analytical model proposed by [6] [7] is very suitable to exploit the behaviors of a node related with link quality due to its accuracy. It is clear resemblance to the back-off process of the 802.11 DCF protocol. So that it can give a good way to improve interference aware metrics in 802.11-based wireless mesh networks. However, these proposals lack of a detailed computation for steady states of a node in these models. Moreover, winning contention probability should be added to successful transmission probability to reflect the back-off process more completely. We will deal with these issues as our contributions in next sections.

III. Steady States Computation

To estimate interference quantity around a node in 802.11-based wireless mesh networks, our approach uses the Markov chain to model the behaviors of a node similar to previous proposals [6] [7]. The behaviors of a node *i* can be modeled by three - state Markov chain, as shown in Fig 1. The three states of this Markov chain are: *Wait* is state when node *i* defers or stays in the back-off proccess, *Success* is state when node *i* can complete a successful transmission to its target node *j* with an acknowledgement, *Failure* is state when node *i* initiates but cannot complete its transmission session. Let π_w, π_s, π_f denote the steady state probabilities of *Wait*, *Success*, and *Failure* states, respectively. The transition probabilities (*Wait*

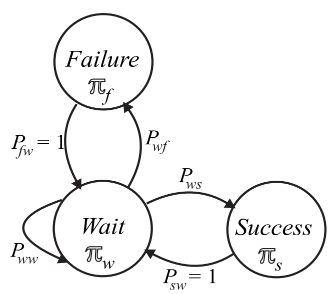

Fig. 1. Markov chain for Node state model

to *Success*, *Wait* to *Wait*, *Wait* to *Failure*) of this Markov are denoted as P_{ws}, P_{ww}, P_{wf} and satisfied Eq.(1) below.

$$P_{ws} + P_{ww} + P_{wf} = 1 \quad (1)$$

The transition matrix *P* of this Markov chain can be presented as a matrix in (2), in which $(a = P_{ws}, b = P_{ww}, c = P_{wf})$.

$$P = \begin{pmatrix} 0 & 1 & 0 \\ a & b & c \\ 0 & 1 & 0 \end{pmatrix} \quad (2)$$

The relationship between the steady-state probability $\pi = (\pi_s, \pi_w, \pi_f)$ and the transition matrix *P* is $\pi = \pi \times P$. So that $\pi = \pi \times P = \pi \times P^2 = ... = \pi \times P^n, \forall n \in N$. To determine π, we need to find $\lim\limits_{n\to\infty} P^n$. Let $C = A^{-1}PA$ is the diagonal matrix of *P*, we have

$$C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & b-1 \end{pmatrix}, A = \begin{pmatrix} c & 1 & 1 \\ 0 & 1 & b-1 \\ -a & 1 & 1 \end{pmatrix}$$

Due to $C = A^{-1}PA$ then we have $P = ACA^{-1}$ and $P^n = (ACA^{-1})^n = AC^nA^{-1}$.

$$A^{-1} = \begin{bmatrix} \frac{1}{1-b} & 0 & \frac{1}{1-b} \\ \frac{a}{2-b} & \frac{1}{2-b} & \frac{c}{2-b} \\ \frac{a}{(1-b)(2-b)} & \frac{1}{2-b} & \frac{c}{(1-b)(2-b)} \end{bmatrix}$$

$$P^n = \begin{bmatrix} \frac{a[1-(b-1)^{n-1}]}{2-b} & \frac{[1-(b-1)^n]}{2-b} & \frac{c[1-(b-1)^{n-1}]}{2-b} \\ \frac{a[1-(b-1)^n]}{2-b} & \frac{[1-(b-1)^{n+1}]}{2-b} & \frac{c[1-(b-1)^n]}{2-b} \\ \frac{a[1-(b-1)^{n-1}]}{2-b} & \frac{[1-(b-1)^n]}{2-b} & \frac{c[1-(b-1)^{n-1}]}{2-b} \end{bmatrix} \quad (3)$$

From Eq.(1) we have got $0 < b \leq 1 \Rightarrow 0 \leq |b-1| < 1$. When $n \to \infty$, we have $(b-1)^{n-1} \to 0; (b-1)^n \to 0; (b-1)^{n+1} \to 0$. Implying that

$$\lim_{n\to\infty} \mathrm{P}^n = \begin{pmatrix} \frac{a}{2-b} & \frac{1}{2-b} & \frac{c}{2-b} \\ \frac{a}{2-b} & \frac{1}{2-b} & \frac{c}{2-b} \\ \frac{a}{2-b} & \frac{1}{2-b} & \frac{c}{2-b} \end{pmatrix} \quad (4)$$

The steady vector of this Markov chain is

$$\pi = \left[\frac{a}{2-b}\,\frac{1}{2-b}\,\frac{c}{2-b}\right] = \left[\frac{P_{ws}}{2-P_{ww}}\,\frac{1}{2-P_{ww}}\,\frac{P_{wf}}{2-P_{ww}}\right] \quad (5)$$

Therefore, steady state probabilities of states are

$$\pi_w = \frac{1}{2-P_{ww}}; \pi_s = \frac{P_{ws}}{2-P_{ww}}; \pi_f = 1 - \pi_w - \pi_s. \quad (6)$$

IV. Contention probability

The steady state probability of node states are determined when transition probabilities are identified. We handle these issues by considering the communication between a transmitter *i* and a receiver *j* as an illustration in Fig 2. Assuming that all nodes in the analytical network model have the same transmission range r_t, carrier sense range r_{cs} and transmission probability p_t (probability of each node transmits in a time slot). Nodes are distributed as a two-dimensional Poisson process over a 2D plane with mean λ, i.e., the probability of finding *n* nodes in an area of *S* is given by $\frac{(\lambda S)^n}{n!}e^{-\lambda S}$. The average number of nodes within the carrier sense area of each node is $M = \lambda \pi r_{cs}^2$. So that, λ or *M* parameter reflects the density of the network. Therefore, the probability of finding *n* nodes in this area is $\frac{M^n}{n!}e^{-M}$.

Following the result in Eq.(6), we need to determine the transition probability of a node *i* from *wait* state to *wait* state P_{ww} and from *wait* state to *success* state P_{ws}.

The probability P_{ww} is probability that node *i* continues to stay in *wait* state in a given time slot, i.e., none of the nodes in the carrier sense area of node *i* transmits in this time slot.

$$P_{ww} = \sum_{n=1}^{\infty}(1-p_t)^n \frac{M^n}{n!}e^{-M} = e^{-Mp_t} - e^{-M} \quad (7)$$

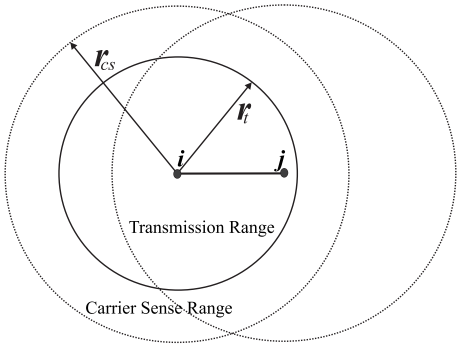

Fig. 2. Illustration of interference range for $i \rightarrow j$ communication

The P_{ws} can express as a formula below:

$$P_{ws} = P_{st} \times P_{sr} \tag{8}$$

In which:
P_{st}: Probability of node *i* transmits a packet successfully;
P_{sr}: Probability of node *j* receives a packet successfully.
So that, P_{st} composes of two component probabilities: p_t is the probability of node *i* transmits in a given slot and p_o is the probability of successful outgoing packets sending.

$$P_{st} = p_t \times p_o \tag{9}$$

On the previous proposals [2] [6] [7], p_o is defined as the probability of none of nodes within the carrier sense area of node *i* transmits in the same slot. However, in the CSMA/CA mechanism, if some nodes in the carrier sense area of node *i* transmit simultaneously, they could fall into the back-off process. The back-off counter of these nodes are assigned to random values drawn from a uniform distribution over the interval [0,*CW*]. The value *CW* is called *Contention Window*, and depends on the number of retransmissions failed for the packet. *CW* is initiated by CW_{min} at the first attempt. When the counter decreases one by one to *zero*, a node is allowed to transmit a packet. If some back-off counters of nodes reach to zero at the same time, these nodes need another attempts. Every time a packet requires a retransmission, the contention window *CW* is increased exponentially until it reaches the maximum value CW_{max}. The contention window at the i^{th} attempt is CW_i satisfied $\mathrm{CW}_i + 1 = 2^{i-1} \times (\mathrm{CW}_{\min} + 1)$. The contention window is reset to CW_{min} after either a successful transmission or a packet dropping. The back-off process will be finished when only one counter of the contention nodes has smallest and unique back-off counter value. Therefore, we need to add a winning contention probability to reflect successful transmission probability completely. Before computing the winning contention probability P_{win} in the CSMA/CA mechanism, we establish a new generic problem to approach our purpose.

Problem:
Let a function $f : \{1, 2, ..., n\} \rightarrow \{1, 2, ..., k\}$ with $k, n \in N, 0 < n < k$. Determining the probability (P_*) is satisfied $P_* = P\left\{\exists! i \in \{1, 2, ..., n\} \,|f(i) = \min_{1 \le j \le n} \{f\,(j)\}\right\}$.
Solution:
From the suppose, the total number of functions $f : \{1, 2, ..., n\} \rightarrow \{1, 2, ..., k\}$ is k^n. Let the smallest value of function $f(i) = \min\limits_{1 \le j \le n} \{f(j)\} = m, \forall j \in \overline{1, n}; m \in \{1, 2, ..., k\}$.
If *m*=*k* then$f(j) = k, \forall j = \overline{1, n}$ leading to assumption contradistinction $(\exists! i) \Rightarrow 1 \le m \le k - 1$.
We have $f(j) \in \{m+1, m+2, ..., k\}, \forall j \in \overline{1, n}, j \ne i$. Therefore, there are $(k - m)^{n-1}$ satisfied functions corresponding with an *m* value. The total number of satisfied functions can be expressed as

$$a(n, k) = \sum_{m=1}^{k-1} (k-m)^{n-1} = 1 + 2^{n-1} + ... + (k-1)^{n-1}. \tag{10}$$

That is

$$P_* = P\left\{\exists! i \in \{1, 2, ..., n\} \,|f(i) = \min\nolimits_{1 \le j \le n} \{f\,(j)\}\right\}$$

$$P_* = \frac{\sum\limits_{m=1}^{k-1} (k - m)^{n-1}}{k^n} \tag{11}$$

In our case, the back-off process in the CSMA/CA mechanism can be presented with some terms:
n: the number of contention nodes (within the carrier sense range of each node).
i: i^{th} attempt of the back-off process $1 \le i \le m$ (*m*: the maximum number of attempts).
k: the number of random values over the interval [0, CW_i] at the i^{th} attempt, $k = CW_i + 1$.
The smallest and unique value of function *f(i)* indicates that the back-off process comes to finish at the i^{th} attempt. The probability of system finished at the i^{th} attempt is $p(n, i)$ expressed as

$$p\,(n, i) = \frac{a\,(n, \mathrm{CW}_i + 1)}{(\mathrm{CW}_i + 1)^n} = \frac{1 + 2^{n-1} + ... + (\mathrm{CW}_i)^{n-1}}{(\mathrm{CW}_i + 1)^n} \tag{12}$$

The transition probability of system from the i^{th} attempt to the $(i + 1)^{th}$ attempt is $(1 - p\,(n, i))$. So, the probability of each node has right to send out a packet as formula below.

$$P^*(n) = \frac{1}{n}\left(p(n, 1) + \sum_{i=2}^{m}\left(p\,(n, i)\prod_{j=1}^{i-1}(1 - p\,(n, j))\right)\right) \tag{13}$$

The winning contention probability of *M* contention nodes expressed as a formula below:

$$P_{win}(n) = p_t \sum_{n=2}^{\infty} \sum_{k=1}^{n-1} C_{n-1}^{k} {p_t}^{k} (1 - p_t)^{n-1-k} P^*(k+1) \frac{M^n}{n!} e^{-M} \tag{14}$$

As shown in (9), the successful transmission probability P_{st} of node *i* depends on p_t and p_o. In this case, P_{win} is ignored in this model, $P_{st} = P'_{st}$ in [6] [7] can be rewritten as a formula.

$$P'_{st} = p_t \sum_{n=2}^{\infty} (1 - p_t)^{n-1} \frac{M^n}{n!} e^{-M} \tag{15}$$

After supplementing P_{win} to P_{st}, we obtain Eq. (16).
We used the Matlab tool to verify our implementation by

$$P_{st} = p_t \sum_{n=2}^{\infty} \left(\left((1-p_t)^{n-1} + \sum_{k=1}^{n-1} C_{n-1}^{k} {p_t}^{k} (1-p_t)^{n-1-k} P^{*}(k+1) \right) \frac{M^n}{n!} e^{-M} \right) \tag{16}$$

comparing with previous proposal in [6] [7]. The relationship of ${P_{st}}'$ vice versa $M\&p_t$ in Eq.(15) illustrated in Fig 3. From that, we recognize that the maximum value of ${P_{st}}'$ can be reached around $M = 3$ or 4 and $p_t \in [0.3, 0.5]$. The shape of the graph can be explained through the correlation between p_t and the infinite sum component. When M's value is small, ${P_{st}}'$ is dominated by p_t . When M comes to larger values, ${P_{st}}'$ is dominated by the infinite sum factor and its value decreases.

In order to examine our proposal result in Eq.(16), we surveyed the functions ${P_{st}}'$ and P_{st} simultaneously with the varying of M and p_t. As shown in Fig 4, the smaller M is (small contending nodes), the larger difference between two plots becomes. It is reasonable because the winning chance of each node increases when M' value reduces. In the Fig 5, we illustrate the effect of winning probability P_{win} on successful transmission probability P_{st} comparing with ${P_{st}}'$. With p_t's value is small, the value of P_{win} can be neglected because the transmission probability is also small. But the difference between ${P_{st}}'$ and P_{st} becomes clear when p_t increases to large values. That mean, P_{win} becomes to the important factor of successful transmission probability in the strong interference cases.

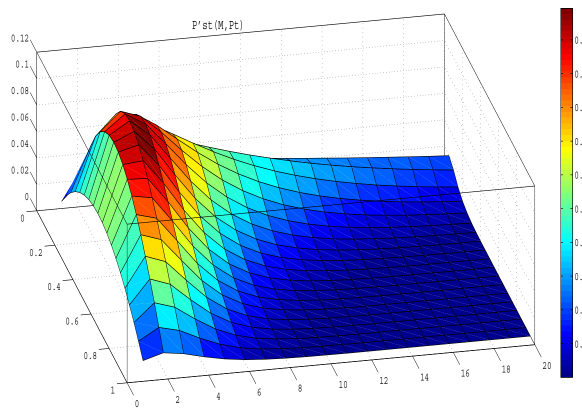

Fig. 3. Illustration of ${P_{st}}'$ vice versa M and p_t

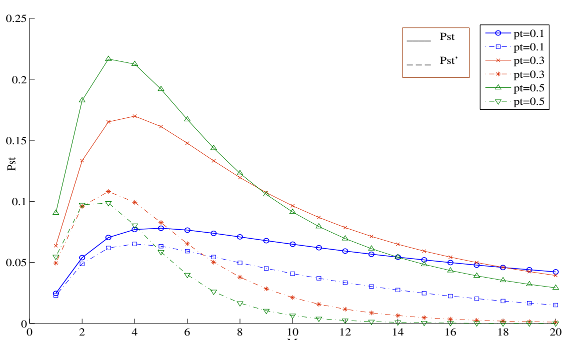

Fig. 4. P_{st} and ${P_{st}}'$ vice versa M

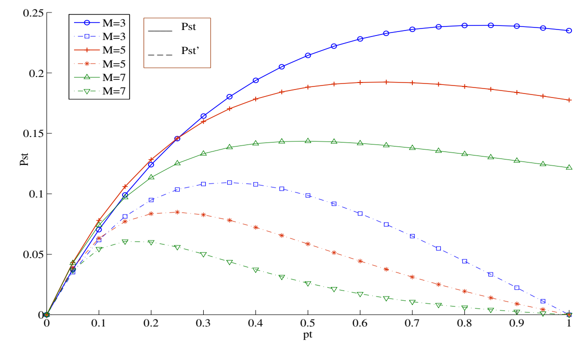

Fig. 5. P_{st} and ${P_{st}}'$ vice versa p_t

V. Conclusion

In this paper, we adopt a simple computation based on mathematical analysis to calculate steady state probabilities of the three-state Markov model. We presented the new generic problem to determine the winning contention probability. It is the first step to toward to more analytical evaluations of 802.11-based wireless mesh network scenarios with multi services. Our ongoing research will extend the Markov chain model to get close to practical environments and apply interferece impact's prediction into routing metrics.

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