An Effective Frequency Domain Equalization Scheme for Cyclic Prefix Assisted Cyclic Code Shift Keying Systems

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***Abstract*—In this paper we propose a frequency domain equalization scheme for the cyclic prefix assisted (CPA) cyclic code shift keying (CCSK) system. In particular, the proposed scheme performs both correlation and decision in the frequency domain, which helps to reduce the computational complexity. Simulation results show that the proposed system achieves the best bit error rate (BER) performance with the maximum ratio combining (MRC).**

I. Introduction

Together with the development of the advanced communication systems for the civil applications, the military communication has also achieved great steps to migrate from the analogue to the digital technology. In contrast to the civil communication, the military communication systems require more advanced technologies for both signal quality and interference mitigation. Moreover, the military communication systems also need to avoid the signal interception from the counter-partners. In order to achieve this goal, Dillard et al. [1] have proposed the cyclic code shift keying (CCSK) system, which was demonstrated to have lower probability of interception compared to the conventional orthogonal modulation systems. Due to this advantage, the CCSK system was selected as the baseband modulation for the Joint Tactical Information Distribution System (JTIDS) [2],[3]. Following the work in [1] various attempts have been made to evaluate the performance of the CCSK system [2]–[4]. These evaluations, however, were limited only to either the case of additive white Gaussian noise or flat fading channel. It is well known that the wireless channel in the tactical environment is tedious and affected by both the frequency and time selectivity. Therefore, in order to apply CCSK to the real-world environment, it is clear that methods for mitigate the selectivity of the channel is of importance. In order to cope with the problem of time selectivity, recently we have proposed a chip-level differential scheme for CCSK to exploit the time diversity of the fast fading channel [5].

In this paper, we focus our interest in mitigating the frequency selectivity of the channel. Specifically, we proposed a frequency domain equalization scheme for the cyclic prefix-assisted single-carrier (SC) CCSK transmission based on zero forcing (ZF), equal gain combining (EGC), maximal ratio combining (EGC), orthogonal restoring combining (ORC), and the minimum mean square error (MMSE). Different from previous SC-FDE schemes, our proposal performs correlation and decision in the frequency domain which helps to reduce the computational complexity while still providing the same performance. It is shown that the proposed scheme with the MRC equalization achieves the best bit error rate performance (BER) performance.

The remainder of the paper is organized as follows. Section II describes the proposed system configuration. Equalization methods are shown in Section III. Simulation evaluation is carried in Section IV and conclusions are drawn in Section V.

II. System Description

The proposed system is shown in Fig. 1. At the transmitter, the CCSK system uses a base sequence $\boldsymbol{c} = [c_1, c_2, \ldots, c_M]^T$ and its cyclic shifted version to represent the transmit B-bit symbols. The elements $c_m \in \{+1, -1\}$ of the base sequence $\boldsymbol{c}$ are also referred to as "chips". Each chip has the duration of T_c and the symbol duration is $T_s = MT_c$. The bandwidth of the chip sequence $W_c = 1/T_c$ is defined such that $W_c T_c = 1$. The cyclic shifted versions of the base chip sequence $\boldsymbol{c}$ is denoted as $\boldsymbol{c}_0, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_{M-1}$ with $\boldsymbol{c}_0 = \boldsymbol{c}$ and $\boldsymbol{c}_n$ being the $n-th$ shifted version. In order to implement CCSK modulation, the data bit sequence b_i is grouped into combinations with the length of $k = \log_2 M$ bit. These combinations of the transmit k bits correspond to the integer s_m. The M integer symbols s_m will be mapped into one of the shifted sequence $\boldsymbol{c}_n$. The output chip sequence $\{c_\ell\}$ will be appended with a cyclic prefix (CP) of length L_c similar in the orthogonal frequency division multiplexing (OFDM) system. The newly spread chip sequence has length of $M + L_c$. The transmit sequence will have the form

$$\boldsymbol{x} = [c_{M-L_c+1}, \cdots, c_M, c_1, c_2, \cdots, c_M] \tag{1}$$

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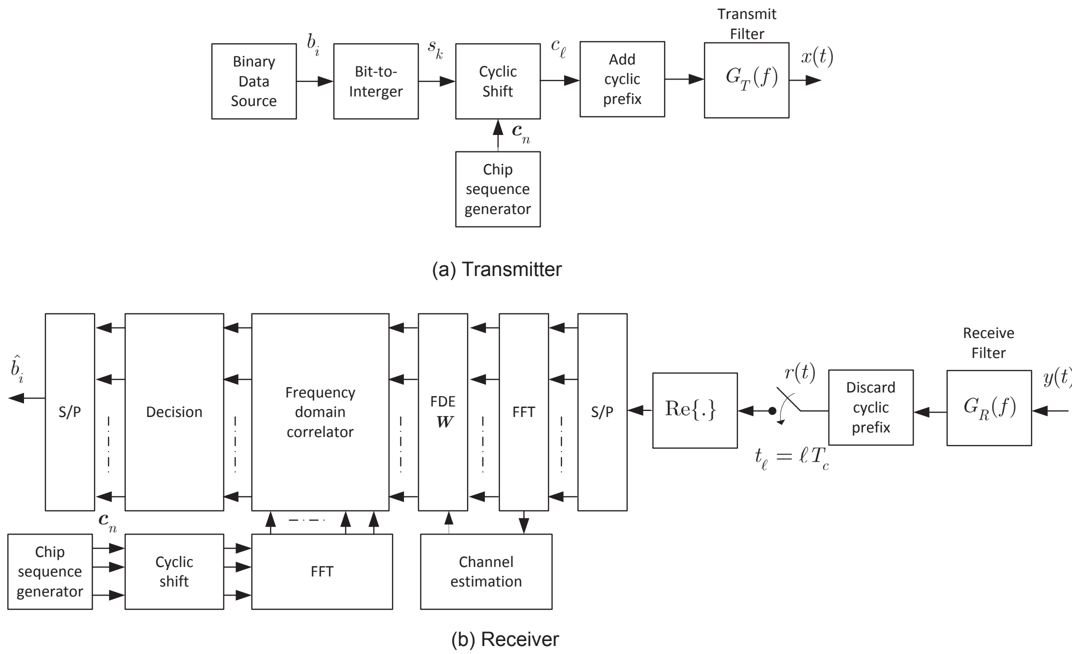

Fig. 1. Configuration of the proposed SC-CPA transmission system for CCSK.

with the transmit signal satisfying the following relation

$$c_l = c_{l+M}, l \in \{1, 0, \cdots, L_c\}. \tag{2}$$

Similar to the OFDM system, the number of chips in the CP is selected such that its duration is not shorter than the channel maximum delay spread τ_{max}. The difference between a single carrier frequency domain equalization (SC-FDE) with an OFDM system is that the transmitter does not use the inverse fast Fourier transform (IFFT). After being added with a CP, the transmit sequence is put through a transmit filter $G_T(f)$ and sent through the channel.

Assume that the channel is affected by the multipath propagation with the maximum number of resolvable paths L_c. The channel can be described by a finite impulse response (FIR) filter with the order of L_c and the weight vector given by

$$\boldsymbol{h} = [h_0, h_1, \cdots, h_{L_c\ \ 1}]^T. \tag{3}$$

At the receiver, the received signal $y(t)$ is passed through a receive filter $G_R(f)$ and then the cyclic prefix is discarded. It is then sampled at an interval of T_c to have the discrete receive samples

$$\boldsymbol{y} = [y_1, y_2, \cdots, y_M]^T. \tag{4}$$

Using the assumption that the cyclic prefix is longer than the channel maximum delay spread τ_{max} the linear convolution between the transmit signal and the channel impulse response becomes the circulant convolution [6]

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{c_n} + \boldsymbol{z}, \tag{5}$$

where $\boldsymbol{H}$ is a $M \times M$ circulant Toeplitz matrix given by

$$\boldsymbol{H} = \begin{bmatrix} h_0 & 0 & \cdots & h_{L_c-1} & & & h_1 \\ h_1 & h_0 & 0 & & & & h_2 \\ \vdots & \vdots & h_0 & \vdots & \vdots & \vdots & \vdots \\ h_{L_c-1} & & & \ddots & & & h_{L_c-1} \\ 0 & h_{L_c-1} & & h_1 & h_0 & & \vdots \\ \vdots & & & & h_1 & h_0 & 0 \\ 0 & 0 & h_{L_c-1} & \cdots & \cdots & h_1 & h_0 \end{bmatrix}$$

and $\boldsymbol{z} = [z_1, z_2, \cdots, z_M]^T$ is the noise vector. Each non-zero element of $\boldsymbol{H}$ is modeled by a complex Gaussian random variable with mean zero and unit variance, which implies the multipath Rayleigh fading channel. Each element z_m denotes a sample of an additive white Gaussian noise (AWGN). The channel matrix $\boldsymbol{H}$ can be decomposed into the matrix $\boldsymbol{D}$ via the Fourier transform as [6]

$$\boldsymbol{H} = \boldsymbol{F}^H \boldsymbol{D} \boldsymbol{F} \tag{6}$$

where $\boldsymbol{F}$ denotes the fast Fourier transform (FFT) and is represented as

$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & & {}^{2} & {}^{3} & \cdots & {}^{M-1} \\ 1 & {}^{2} & {}^{4} & {}^{6} & \cdots & {}^{2(M-1)} \\ 1 & {}^{3} & {}^{6} & {}^{9} & \cdots & {}^{3(M-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & {}^{M-1} & {}^{2(M-1)} & {}^{3(M-1)} & \cdots & {}^{M(M-1)} \end{bmatrix}$$

with $\omega = e^{j\frac{2\pi}{N}}$.

The diagonal matrix $\boldsymbol{D} = diag\{H_0, H_1, \cdots, H_{M \ 1}\}$ has the m-th element in the diagonal given by

$$H_m = \sum_{n=0}^{M \ 1} h_n e^{\ j2\pi\frac{mn}{M}}. \tag{7}$$

In order to perform equalization in the frequency domain, the receiver uses FFT to convert the receive sequence into the frequency domain to obtain

$$\boldsymbol{y} = \text{FFT}(y) = \boldsymbol{F}\boldsymbol{y} = \boldsymbol{F}\boldsymbol{H}\boldsymbol{c_n} + \boldsymbol{F}\boldsymbol{z}. \tag{8}$$

Using and the FFT property $\boldsymbol{F}\boldsymbol{F}^H = \boldsymbol{I_M}$ we have

$$\boldsymbol{y} = \boldsymbol{D}\boldsymbol{c_n} + \boldsymbol{z} \tag{9}$$

where $\boldsymbol{c_n} = \boldsymbol{F}\boldsymbol{c}_n$ and $\boldsymbol{z} = \boldsymbol{F}\boldsymbol{z}$. It is noted that the serial-to-parallel (S/P) operation and FFT transform illustrated in Fig. 1 is equivalent to dividing the large signal bandwidth of W_c into M smaller ones with the bandwidth of W_c/M. This facilitates the equalization and, as a result, the equalizer only needs a single tap for each channel. The equalizer now requires parallel equalization taps in the frequency domain as in an OFDM system instead of the serial ones in the time domain.

The receiver uses an weight matrix $\boldsymbol{W}$ to equalize the receive signal. The weight matrix $\boldsymbol{W}$ is an $M \times M$ diagonal matrix with elements $W_m, m = 0, 1, \cdots, M-1$ in the diagonal. Each weight coefficient W_m is obtained using a specific criterion. Detail of the criteria will be given in the following section. The equalized samples in the frequency domain are given by

$$\boldsymbol{y} = \boldsymbol{W}\boldsymbol{y}. \tag{10}$$

In the conventional frequency domain equalization this equalized vector $\boldsymbol{y}$ will be converted back into the time domain using an invesre FFT (IFFT) and then, once again, back into the serial sequence using a parallel-to-serial (P/S) converter [6]–[8]. For the direct sequence spread spectrum (DSSS) the receiver then despreads the received chip sequence to get the transmitted bit sequence [8]. For the CCSK system the despreading is equivalent to the correlation and decision operation in the time domain. Different from the previous DSSS system, in this work we propose the use of correlation and decision making (quantization) in the frequency domain as shown in Fig. 1. The operation of calculating the correlation in the frequency domain allows reduction in computational complexity compared to that in the time domain. This observation was noted in [1]. However, the authors of [1] have not indicated the system implementation. For calculating the frequency-domain correlation, we propose to convert the CCSK sequence into the frequency domain via the FFT transform. This means that the cyclic shifted sequences will be performed FFT successively to obtain the frequency domain spreading sequences $\boldsymbol{c_n}$. The frequency-domain spreading sequence will be correlated with the equalized chip sequence $\boldsymbol{y}$ and quantized to find the sequence index k corresponding to the transmitted binary sequence

$$k = \arg\max_k \{\tilde{s}_k\} = \arg\max_k \left\{ \sum_{k=0}^{M \ 1} \boldsymbol{y}^H \boldsymbol{c_k} \right\}. \tag{11}$$

Thanks to the use of frequency-domain correlation and quantization the receiver does not require an IFFT to transform the equalized chip sequence back to the time domain

III. Frequency Domain Equalization

There have been numerous works in the literature for the frequency domain equalization. Some related examples are the use of zero-forcing (ZF) and minimum mean squared error (MMSE) for the powerline communication [6]. Others are the maximum ratio combining (MRC), the equal gain combining (EGC), the orthogonal restoring combining (ORC) [8]. In the current work these methods will be applied to the proposed FDE scheme to find the best one.

A. ZF Equalization

The ZF equalization assumes that the received frequency-domain samples $\boldsymbol{y}$ do not contain noise and tries to compensate for the effect of the channel frequency transfer function by using the following coefficient for the subband n [6]

$$W_n^{\text{ZF}} = \frac{1}{H_n}. \tag{12}$$

B. MMSE Equalization

The MMSE equalization aims at minimizing the MSE between the transmitted and equalized sequence by using the weight coefficient W_n which is calculated taking into consideration the effect the noise. The equalization coefficients in the frequency domain are given by the following equation [6],[8]

$$W_n^{\text{MMSE}} = \frac{H_n^*}{|H_n|^2 + \frac{\sigma_z^2}{\sigma_c^2}} \tag{13}$$

where σ_z^2 and σ_c^2 are the noise and the CCSK chip sequence variance respectively. The notation $*$ represents the complex conjugation.

C. Maximum Ratio Combining (MRC)

The MRC equalization is developed based on the space diversity combining [9]. The MRC method considers the subband channels as the space diversity branches and multiplies them with the combining coefficients [8]

$$W_n^{\text{MRC}} = H_n^*. \tag{14}$$

D. Equal Gain Combining (EGC)

EGC is a simplified version of the MRC due to the fact that the combining weights are not multiplied proportionally with the signals in the subbands. The combining weight for the n-th subband is given by [8]

$$W_n^{\mathrm{EGC}} = \frac{H_n^*}{|H_n^*|}. \tag{15}$$

The main drawback of EGC is that the low level subband signals are multiplied with a larger gain. The result is that the method is affected by the noise amplification problem.

E. Orthogonal Restoring Combining (ORC)

The purpose of the ORC scheme is to restore the orthogonality of the spreading codes. Its equalization coefficients are similar to those of EGC [8]

$$W_n^{\mathrm{ORC}} = \frac{H_n^*}{|H_n^*|^2}. \tag{16}$$

Comparing (15) with (16) we can see that the EGC is similar with ORC in facing the noise amplification problem. It is even much more affected than the EGC for the low level subband channels due to the square in the denominator.

IV. Simulation and Performance Evaluation

A. Simulation parameters

In order evaluate the performance of the proposed single-carrier FDE scheme, Monte-Carlo simulation has been done using Matlab. The transmit chip sequences have the same length M of the FFT. The channel is assumed frequency selective fading with each resolvable path undergoing i.i.d. Rayleigh fading. Two power delay profiles used for simulations are the typical urban (TU) and the hilly terrain (HT) [10]. It is assumed that the channel is quasi-static such that the channel gain does not vary within a transmit sequence but different from sequence to sequence. The normalized Doppler frequency used for simulation is $f_D T_c = 0.001$. The cyclic prefix length L_c is assumed larger than the maximum channel delay spread so that inter-symbol interference (ISI) is perfectly mitigated. In the simulation BPSK modulation has been used for sake of simplicity.

B. Simulation results

In the first simulation, BER performance of the proposed FDE scheme is evaluated using different equalization methods, namely, ZF, MMSE, MRC, EGC and ORC. In order to compare performance of the proposed FDE scheme with that of the conventional one, results are shown in pair in each figure. Figure. 2 shows BER performance obtained for the case of using $M = 8$ chips under the HT channel and Fig. 3 for the TU channel. The first observation from the two figures is that the proposed FDE scheme achieves the same BER performance with that of the conventional one for all cases of investigation. This proves the advantage of the proposal. As the second observation, the MRC equalizer shows the best performance, followed by the MMSE and EGC, and the ORC and ZF in the last. This observation matches with the theoretical discussion in the above section. The MRC equalizer is more superior as it can achieve the maximum frequency diversity gain. The MMSE equalizer has the minimum MSE but does not achieve the maximum diversity gain. It can be realized that the difference in performance is about 1dB at the high SNR region. Meanwhile, the equalizer based on EGC is affected by the noise amplification for the low-level subband channels as it uses the equalization coefficients with the same magnitude for all subband channels. As a result, the achieved BER performance is equivalent to that of the MMSE. The two equalizers using ZF and ORC have poor performance due to the effect of noise amplification. The effect to the ZF is the same for all the subbands, whereas stronger to the ORC for those subbands with low-level signal.

Another observation that can be realized is that the BER performance of the proposed equalizers is better achieved in the TU than in the HT channel. It can be seen in the figures that with the equalizer using MRC at $E_b/N_0 = 18$dB the average BER is $4.5 \times 10^{\ 4}$ in the TU, and $5 \times 10^{\ 3}$ in the HT channel. This is due to the different frequency selectivity of the two channels. The maximum delay spread of the HT channel is 20μs while that of the TU is only 7μs. Although the length of CP is assumed larger than the channel maximum delay spread it is still clear that the effectiveness of the equalizer is better in the channel with less selectivity.

Figure 4 compares BER performance for the case of using the CCSK sequences with different length (M = 8, 16, 32). It can be realized that for both the TU and HT channels it is possible to achieve better performance with the longer chip sequence. This is due to the fact that the longer sequence will have larger processing gain.

V. Conclusion

In this paper, we have proposed a frequency domain equalization scheme for the single carrier cyclic-prefix assisted CCSK system. The proposed system uses a frequency domain correlator and quantizer to detect the transmit signal. This allows the implementation of the receiver with low complexity. The proposed system with the MRC equalization has shown to be the most suitable candidate for the CCSK system.

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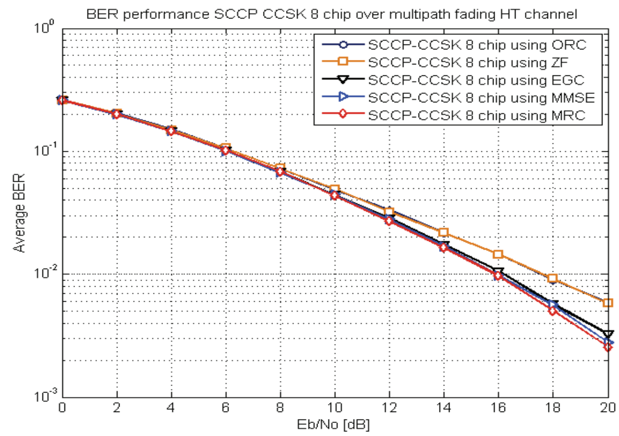
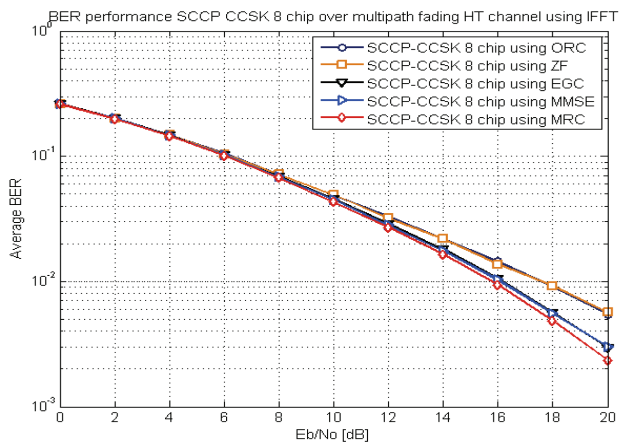


(a) Frequency domain correlation and decision.



(b) Time domain correlation and decision.

Fig. 2. BER performance of CP Assisted-FDE for SC-CCSK; HT channel, $M = 8$, $f_D T_c = 0.001$.

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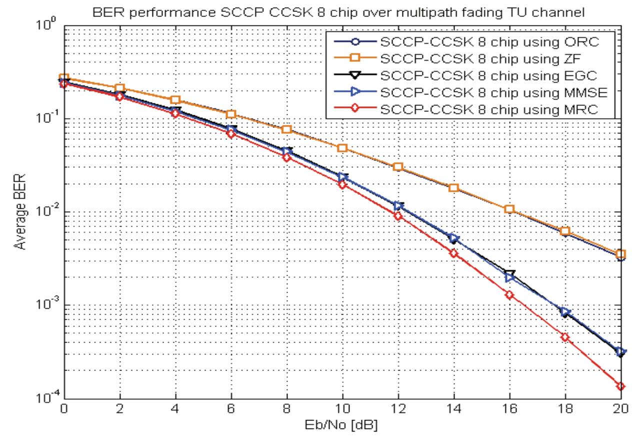
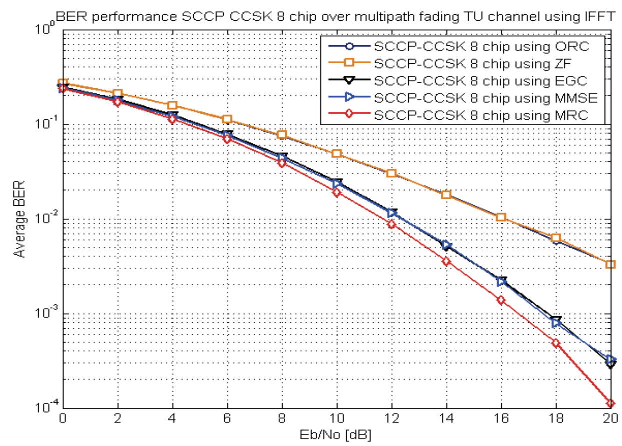


(a) Frequency domain correlation and decision.



(b) Time domain correlation and decision.

Fig. 3. BER performance of CP Assisted-FDE for SC-CCSK; TU channel, $M = 8$, $f_D T_c = 0.001$.

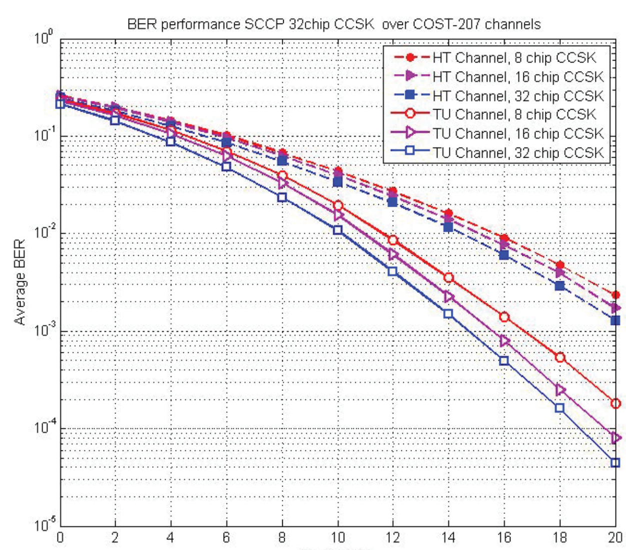


Fig. 4. BER performance of CP Assisted-FDE for SC-CCSK; TU and HT channel, $M = 8, 16, 32$, $f_D T_c = 0.001$.