Full Diversity Space-Time Block Coding For Linear Receiver with Low Peak-To-Average Ratio

Pham Van Bien [1], Pham Truong Son [2]

[1] Faculty of Radio-Electronics, Le Quy Don Technical University, Hanoi, Vietnam.
[1] Email: bienngaxanh@gmail.com
[2] Department of Electrical Engineering & Inf. Technology, University of the German Federal Armed Forces Munich, Germany

***Abstract*—Recently, Shang and Xia (*IEEE Transactions on Information Theory*, 54(10), 4528-4547, 2008) introduced Overlapped Alamouti codes (OAC) and showed that OAC codes achieve full diversity when a linear receiver, zero-forcing (ZF) or minimum mean square error (MMSE) receiver, is used. Unfortunately, OAC codes suffer from transmitting "zero-symbol" on all transmit antenna. The "zero-symbol" in the design results in high peak-to-average power ratio (PAPR) and also imposes a severe constraint on hardware implementation of the code when turning off some of the transmitting antennas whenever a "zero-symbol" is transmitted. In this paper, we propose a new space-time block code (STBC) for linear receivers. The proposed STBC achieves all desirable properties of OAC codes as full diversity, high rate and group orthogonality. In addition, transmitting "zero-symbol" only occurs on half of transmit antenna instead of all transmit antenna as for OAC code. As a result, the proposed STBCs not only have lower PAPR, but also easier implementation than OAC codes. Moreover, simulation results also show that our codes outperform OAC codes under peak power constraint.**

I. INTRODUCTION

In a practical multiple-input multiple-output (MIMO) system, decoding complexity is an important concern and a decoding scheme with low complexity is always desired. The linear receivers, such as zero-forcing (ZF) or minimum mean square error (MMSE) receiver, have addressed this concern. Space-time codes to achieve full diversity with linear receivers have been recently studied [1-5], design criteria have been proposed in [2-3]. Five known families for such codes are orthogonal OSTBCs [6-10], Toeplitz codes [1], overlapped Alamouti codes (OACs) [3], embedded Alamouti codes (EAC) [4], and group-orthogonal Toeplitz codes (GOTCs) [5]. For OSTBCs, due to the orthogonality, their maximum likelihood and linear receivers are the same. However, their symbol rates are upper-bounded by 3/4 for more than 2 transmit antennas and a tight upper bound was conjectured to be $(k+1)/(2k)$ for $2k$ or $2k-1$ transmit antennas in [10]. Although the symbol rates of Toeplitz codes can approach 1, their performance is not as good as OSTBCs due to Toeplitz codes do not have group orthogonality. In [3], OACs have been proposed and it has been shown via simulations that OACs outperform Toeplitz codes for any number of transmit antennas due to their group orthogonality, and furthermore, they outperform OSTBCs for over 4 transmit antennas. GOTCs [5] are good tradeoff between rate and orthogonality. It has been shown via simulations that GOTCs outperform OACs for MISO but worse for MIMO. However, a disadvantage of OACs is their codeword matrix still contains many zero entries. In theory, their symbols rates can approach one as the block sizes go to infinity for any number of transmit antennas, i.e., the effect of zero entries on BER performance and practical implementation may be ignored. Nevertheless, this is impossible in reality because as the block sizes tend to infinity, the inverse matrix becomes larger, resulting in increased decoding complexity and decoding delay. Avoiding the transmitting "zero-symbol" in codeword matrix is important for many reasons [11]. The first, the regular transmission of "zeros" implies turning off the transmit antennas at regular intervals, leading to undesirable low-frequency interference. The second, the zero entries result in high PAPR and increased difficulty in the front-end power amplifier design.

In this paper, by using no-zero-entry (NZE) Toeplitz matrix which presented in [12] we propose a new full diversity STBC for linear receiver which named as low PAPR space-time block code (LP-STBC). The proposed LP-STBC transmits "zero-symbol" only on half of transmit antennas instead of all transmit antennas as for OAC code. Thus, the advantage of the proposed LP-STBC over OAC codes is lower PAPR and easier implementation in hardware design. Moreover, simulation results also show that our LP-STBC outperforms OAC codes under peak power constraint while comparable performing under average power constraint. The remainder of this paper is organized as follows. In Section II, we describe the channel model and a brief review of the NZE Toeplitz matrix. In Section III, we present the systematic construction of our proposed LP-STBC. Section IV includes simulation results and performance comparisons. Finally, our conclusions and direction for further research are presented in Section V.

Notations: $\boldsymbol{A}^*$, $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^{\mathrm{H}}$ denote the conjugate, transpose and conjugate transpose of $\boldsymbol{A}$, respectively.

II. BACKGROUNDS

A. Chanel model

A linear STBC achieves full diversity with ZF/MMSE receivers for QAM, PAM, and PSK signals for multiple receive

978-1-4673-4352-7/12/$31.00 ©2012 IEEE

antennas if only if it can do the same for the case of one receive antenna [3, Corollary 1]. So, without loss of generality, we only need to focus on the channel model for a single receive antenna, i.e., a multiple-input single-output (MISO) system.

Consider a MISO system with M transmit antennas and one receive antenna transmitting the symbols $\{s_k\}$, $k = 1,..,L$ which are selected from a given constellation $\mathcal{A}$ with unity average energy such as QAM or PSK. To be transmitted from the M antennas, the L symbols $\boldsymbol{s} = (s_1,s_2,..,s_L)^{\mathrm{T}}$ are encoded into a space-time block codeword matrix $\boldsymbol{X}(\boldsymbol{s})$ of size $T\times M$, where T is the block length (coding delay) of the codeword. The (t,m)-th entry of $\boldsymbol{X}(\boldsymbol{s})$ will be transmitted to the receiver from the m^{th} antenna during the t^{th} symbol period through flat fading channels. The received signal model can be written as

$$\boldsymbol{y}' = \sqrt{\rho/\mu}\,\boldsymbol{X}(\boldsymbol{s})\boldsymbol{h} + \boldsymbol{n}' \tag{1}$$

where, $\boldsymbol{y}' = (y_1,y_2,..,y_T)^{\mathrm{T}}$ is received signal vector, $\boldsymbol{n}' = (n_1,n_2,..,n_T)^{\mathrm{T}}$ is the noise vector whose elements are of independently, identically distributed (*iid*) $\mathcal{CN}(0,1)$. $\boldsymbol{h} = (h_1,h_2,..,h_M)^{\mathrm{T}}$, in which h_i, $i = 1,..,M$, denote the channel coefficients of the link from the i-th transmit antenna to the receive antenna, is the channel vector whose entries are also *iid* $\mathcal{CN}(0,1)$. ρ denotes the average signal-to-noise ratio (SNR) per receive antenna. μ is the normalization factor such that the average energy of the coded symbols transmitted from all antennas during one symbol period is one.

To decode the transmitted sequence **s** with a linear receiver, we need to extract $\boldsymbol{s}$ from $\boldsymbol{X}(\boldsymbol{s})$. Through a number of operations, we can get an equivalent signal model from (1) as:

$$\boldsymbol{y} = \sqrt{\rho/\mu}\,\boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \tag{2}$$

where, $\boldsymbol{y}$ denotes a signal vector of length T, $\boldsymbol{H}$ is an equivalent channel matrix of size $T\times L$, and $\boldsymbol{n}$ is the noise vector of length T. If $(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}$ exists, the estimate of the transmitted symbol sequence $\boldsymbol{s}$ for ZF receiver is,

$$\boldsymbol{s}_{ZF} = \sqrt{\rho/\mu}\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}\boldsymbol{y} \tag{3}$$

and for MMSE receiver is

$$\boldsymbol{s}_{MMSE} = \sqrt{\rho/\mu}\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \frac{\mu}{\rho}\boldsymbol{I}_L\right)^{-1}\boldsymbol{H}\boldsymbol{y} \tag{4}$$

Design criterion for full-fiversity STBC for a MISO system with linear receivers

Theorem 1 [2]**:** For a MISO system employing a square QAM, a PAM, or a PSK signalling scheme of cardinality **s** in the transmission, a linear receiver achieves full diversity for the system if $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ is non-singular for any nonzero $\boldsymbol{h}$.

B. NZE-Toeplitz matrices

Let $\boldsymbol{v} = (v_1,v_2,..,v_L)^{\mathrm{T}}$, then a Toeplitz matrix of size $(K+L-1)\times K$ generated by vector **v** and positive integer K, denoted by $\mathcal{T}(\boldsymbol{v},L,K)$, is defined as [1]

$$\mathcal{T}\left[(\boldsymbol{v},L,K)\right]_{i,j} = \begin{cases} v_{i-j+1}, & \text{if } i \ge j \text{ and } i-j<L \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

The NZE-Toeplitz matrices of size $(L+K-1)\times K$ generated by vector $\boldsymbol{v}$ and positive integer K, denoted by $\mathcal{B}(\boldsymbol{v},L,K)$ and $\mathcal{C}(\boldsymbol{v},L,K)$, where $\mathcal{B}(\boldsymbol{v},L,K)$ is defined as [12]

$$\left[\mathcal{B}(\boldsymbol{v},L,K)\right]_{i,j} = \begin{cases} \left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i,j} & \text{if } i \ge j \text{ and } i-j<L \\ -\left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i-L,j} & \text{if } i \ge j \text{ and } i-j \ge L \\ \left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i+L,j} & \text{if } i<j \end{cases} \tag{6}$$

And $\mathcal{C}(\boldsymbol{v},L,K)$ is proposed as

$$\left[\mathcal{C}(\boldsymbol{v},L,K)\right]_{i,j} = \begin{cases} \left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i,j} & \text{if } i \ge j \text{ and } i-j<L \\ \left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i-L,j} & \text{if } i \ge j \text{ and } i-j \ge L \\ -\left[\mathcal{T}(\boldsymbol{v},L,K)\right]_{i+L,j} & \text{if } i<j \end{cases} \tag{7}$$

Lemma 1: There exists a positive constant c such that for any nonzero vector $\boldsymbol{v}$, the following inequalities hold,

$$c\|\boldsymbol{v}\|^{2K} \le \det\left(\mathcal{B}^H(\boldsymbol{v},L,K)\mathcal{B}(\boldsymbol{v},L,K)\right) \tag{8}$$

$$c\|\boldsymbol{v}\|^{2K} \le \det\left(\mathcal{C}^H(\boldsymbol{v},L,K)\mathcal{C}(\boldsymbol{v},L,K)\right) \tag{9}$$

Proof: the proof of the inequality (8) given in [12] and the proof of the inequality (9) is similar.

III. Our Proposed LP-STBCs

A. Construction of the proposed LP-STBC

A block of $2L$ data (information) symbols $\boldsymbol{s} = (s_1,s_2,\ldots,s_{2L})$ are divided into two symbol vectors $\boldsymbol{s}_1 = (s_1,s_2,\ldots,s_L)$ and $\boldsymbol{s}_2 = (s_{1+L},s_{2+L},\ldots,s_{2L})$. Then the proposed LP-STBC for $2M$ transmit antennas is formulated as

$$\boldsymbol{S}(\boldsymbol{s},2L,2M) = \begin{bmatrix} \mathcal{B}(\boldsymbol{s}_1,L,M) & \mathcal{T}(\boldsymbol{s}_2,L,M) \\ \mathcal{C}^*(\boldsymbol{s}_2,L,M)\times\boldsymbol{P}(M) & -\mathcal{T}^*(\boldsymbol{s}_1,L,M)\times\boldsymbol{P}(M) \end{bmatrix} \tag{10}$$

where, $\boldsymbol{P}(n)$ is an $n\times n$ permutation matrix given as

$$\boldsymbol{P}(n) = \left[\boldsymbol{e}_n \quad \boldsymbol{e}_{n-1} \quad \ldots \quad \boldsymbol{e}_1\right] \tag{11}$$

with $\boldsymbol{e}_i$ is the i-th column of the $n\times n$ identity matrix.

The LP-STBC code for $2M$-1 transmit antennas $\boldsymbol{S}(\boldsymbol{s},2L,2M-1)$ is taken from the LP-STBC code for $2M$ -1 transmit antennas (4.21) by deleting the last column in its codeword matrix. It is not hard to check that the symbol rate of LP-STBC code is

$$R = \frac{2L}{2L+2M-2}\text{; for } 2M \text{ or } 2M\text{ -1 transmit antennas} \tag{12}$$

If we partition all the columns of the code matrix (10) into 2 groups as

$$\boldsymbol{S}(\boldsymbol{s},2L,2N_{\mathrm{T}})=[\boldsymbol{S}_1\ \ \boldsymbol{S}_2] \tag{13}$$

where $\boldsymbol{S}_1=\left[\boldsymbol{\mathcal{B}}(\boldsymbol{s}_1,L,M)\quad -\boldsymbol{\mathcal{C}}^*(\boldsymbol{s}_2,L,M)\times\boldsymbol{P}(M)\right]^T$ and $\boldsymbol{S}_1=\left[\boldsymbol{\mathcal{T}}(\boldsymbol{s}_2,L,M)\quad \boldsymbol{\mathcal{T}}^*(\boldsymbol{s}_1,L,M)\times\boldsymbol{P}(M)\right]^T$, then it is not hard to verify that we have $\boldsymbol{S}_1^H\boldsymbol{S}_2=\boldsymbol{0}_{M\times M}$, i.e., the codeword matrix has the group orthogonality property. The following few examples to illustrate the construction of the proposed LP-STBC code and the OAC code.

Example 1: Constructing of the LP-STBC code for 8 transmit antennas with rate of 3/4 ($2M$ = 8, L=9).

Consider total $2L$ = 18 independent information symbols $\boldsymbol{s}$ = (s_1 s_2… s_{18}) and these 12 symbols are split into 2 information symbol vectors $\boldsymbol{s}_1$ = (s_1 s_2 … s_9) and $\boldsymbol{s}_2$ = (s_{10} s_{11} … s_{18}). Permutation matrix $\boldsymbol{P}$(4) is given as

$$\boldsymbol{P}(4)=[\boldsymbol{e}_4\quad \boldsymbol{e}_3\quad \boldsymbol{e}_2\quad \boldsymbol{e}_1]=\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix} \tag{14}$$

Substituting $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ and $\boldsymbol{P}$(4) into (10) we obtain the LP-STBC code $\boldsymbol{S}(\boldsymbol{s}$,18,8) for 8 transmit with rate of 3/4 as

$$\boldsymbol{S}(\boldsymbol{s},18,8)=\begin{bmatrix}
s_1 & s_9 & s_8 & s_7 & s_{10} & 0 & 0 & 0\\
s_2 & s_1 & s_9 & s_8 & s_{11} & s_{10} & 0 & 0\\
s_3 & s_2 & s_1 & s_9 & s_{12} & s_{11} & s_{10} & 0\\
s_4 & s_3 & s_2 & s_1 & s_{13} & s_{12} & s_{11} & s_{10}\\
s_5 & s_4 & s_3 & s_2 & s_{14} & s_{13} & s_{12} & s_{11}\\
s_6 & s_5 & s_4 & s_3 & s_{15} & s_{14} & s_{13} & s_{12}\\
s_7 & s_6 & s_5 & s_4 & s_{16} & s_{15} & s_{14} & s_{13}\\
s_8 & s_7 & s_6 & s_5 & s_{17} & s_{16} & s_{15} & s_{14}\\
s_9 & s_8 & s_7 & s_6 & s_{18} & s_{17} & s_{16} & s_{15}\\
-s_1 & s_9 & s_8 & s_7 & 0 & s_{18} & s_{17} & s_{16}\\
-s_2 & -s_1 & s_9 & s_8 & 0 & 0 & s_{18} & s_{17}\\
-s_3 & -s_2 & -s_1 & s_9 & 0 & 0 & 0 & s_{18}\\
-s_{16}^* & -s_{17}^* & -s_{18}^* & s_{10}^* & 0 & 0 & 0 & -s_1^*\\
-s_{17}^* & -s_{18}^* & s_{10}^* & s_{11}^* & 0 & 0 & -s_1^* & -s_2^*\\
-s_{18}^* & s_{10}^* & s_{11}^* & s_{12}^* & 0 & -s_1^* & -s_2^* & -s_3^*\\
s_{10}^* & s_{11}^* & s_{12}^* & s_{13}^* & -s_1^* & -s_2^* & -s_3^* & -s_4^*\\
s_{11}^* & s_{12}^* & s_{13}^* & s_{14}^* & -s_2^* & -s_3^* & -s_4^* & -s_5^*\\
s_{12}^* & s_{13}^* & s_{14}^* & s_{15}^* & -s_3^* & -s_4^* & -s_5^* & -s_6^*\\
s_{13}^* & s_{14}^* & s_{15}^* & s_{16}^* & -s_4^* & -s_5^* & -s_6^* & -s_7^*\\
s_{14}^* & s_{15}^* & s_{16}^* & s_{17}^* & -s_5^* & -s_6^* & -s_7^* & -s_8^*\\
s_{15}^* & s_{16}^* & s_{17}^* & s_{18}^* & -s_6^* & -s_7^* & -s_8^* & -s_9^*\\
s_{16}^* & s_{17}^* & s_{18}^* & s_{10}^* & -s_7^* & -s_8^* & -s_9^* & 0\\
s_{17}^* & s_{18}^* & s_{10}^* & s_{11}^* & -s_8^* & -s_9^* & 0 & 0\\
s_{18}^* & s_{10}^* & s_{11}^* & s_{11}^* & -s_9^* & 0 & 0 & 0
\end{bmatrix} \tag{15}$$

By deleting the last column in codeword matrix of $\boldsymbol{S}(\boldsymbol{s}$,18,8) we obtain the proposed LP-STBC code for 7 transmit antennas with the same rate of 3/4.

Example 2: The OAC code for 8 transmit antennas with rate of 3/4 given in [3]

$$\boldsymbol{S}_{OAC}=\begin{bmatrix}
s_1 & 0 & 0 & 0 & 0 & 0 & 0 & s_2\\
0 & s_1^* & 0 & 0 & 0 & 0 & -s_3^* & 0\\
s_3 & 0 & s_1 & 0 & 0 & s_2 & 0 & s_4\\
0 & s_3^* & 0 & s_1^* & -s_2^* & 0 & -s_4^* & 0\\
s_5 & 0 & s_3 & s_2 & s_1 & s_4 & 0 & s_6\\
0 & s_5^* & -s_2^* & s_3^* & -s_4^* & s_1^* & -s_6^* & 0\\
s_7 & s_2 & s_5 & s_4 & s_3 & s_6 & s_1 & s_8\\
-s_2^* & s_7^* & -s_4^* & s_5^* & -s_6^* & s_3^* & -s_8^* & s_1^*\\
s_9 & s_4 & s_7 & s_6 & s_5 & s_8 & s_3 & s_{10}\\
-s_4^* & s_9^* & -s_6^* & s_7^* & -s_8^* & s_5^* & -s_{10}^* & s_3^*\\
s_{11} & s_6 & s_9 & s_8 & s_7 & s_{10} & s_5 & s_{12}\\
-s_6^* & s_{11}^* & -s_8^* & s_9^* & -s_{10}^* & s_7^* & -s_{12}^* & s_5^*\\
s_{13} & s_8 & s_{11} & s_{10} & s_9 & s_{12} & s_7 & s_{14}\\
-s_8^* & s_{13}^* & -s_8^* & s_{11}^* & -s_{10}^* & s_9^* & -s_{14}^* & s_7^*\\
s_{15} & s_{10} & s_{13} & s_{12} & s_{11} & s_{14} & s_9 & s_{16}\\
-s_{10}^* & s_{15}^* & -s_{12}^* & s_{13}^* & -s_{14}^* & s_{11}^* & -s_{16}^* & s_9^*\\
s_{17} & s_{12} & s_{15} & s_{14} & s_{13} & s_{16} & s_{11} & s_{18}\\
-s_{12}^* & s_{17}^* & -s_{14}^* & s_{15}^* & -s_{16}^* & s_{13}^* & -s_{18}^* & s_{11}^*\\
0 & s_{14} & s_{17} & s_{16} & s_{15} & s_{18} & s_{13} & 0\\
-s_{14}^* & 0 & -s_{16}^* & s_{17}^* & -s_{18}^* & s_{15}^* & 0 & s_{13}^*\\
0 & s_{16} & 0 & s_{18} & s_{17} & 0 & s_{15} & 0\\
-s_{16}^* & 0 & -s_{18}^* & 0 & 0 & s_{17}^* & 0 & s_{15}^*\\
0 & s_{18} & 0 & 0 & 0 & 0 & s_{17} & 0\\
-s_{18}^* & 0 & 0 & 0 & 0 & 0 & 0 & s_{17}^*
\end{bmatrix} \tag{16}$$

From (15) and (16) we can see that the main advantage of the proposed LP-STBC code over the OAC code is avoiding transmitting "zero-symbol" at first M antennas in $2M$ or $2M$-1 antenna MIMO system.

B. Full diversity property of the proposed LP-STBC

From (1), the received signals can be written as:

$$\boldsymbol{y'}=\sqrt{\rho/\mu}\,\boldsymbol{S}(\boldsymbol{s},2L,2M)\boldsymbol{h}+\boldsymbol{n}' \tag{17}$$

where $\boldsymbol{h}=[h_1\ h_2\ \cdots\ h_{2M}]^{\mathrm{T}}$ with h_m (m = 1, 2, ⋯ , $2M$) denoting the channel coefficient of the link from the m-th transmit antenna to the single antenna receiver.

For all elements in L+M-1 last rows of $\boldsymbol{y'}$ in (17), we take the conjugate. Then, we can obtain

$$\boldsymbol{y}=\sqrt{\rho/\mu}\,\boldsymbol{\mathcal{H}}\boldsymbol{s}+\boldsymbol{n} \tag{18}$$

where $\boldsymbol{\mathcal{H}}$ is equivalent channel matrix of the proposed LP-STBC code $\boldsymbol{S}(\boldsymbol{s},2L,2M)$ and has form

$$\mathcal{H}=\begin{bmatrix}\mathcal{B}(\boldsymbol{h}_1,M,L) & \mathcal{T}(\boldsymbol{h}_2,M,L)\\ -\mathcal{T}^*(\tilde{\boldsymbol{h}}_2,M,L) & \mathcal{C}^*(\tilde{\boldsymbol{h}}_1,M,L)\end{bmatrix} \quad (19)$$

where $\boldsymbol{h}_1=[h_1\ \ h_2\ \ \dots\ \ h_M]^T$, $\boldsymbol{h}_2=[h_{M+1}\ \ h_{M+2}\ \ \dots\ \ h_{2M}]^T$ and $\tilde{\boldsymbol{h}}_1$, $\tilde{\boldsymbol{h}}_2$ are is the index reversed version of $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$, respectively (e.g., $\tilde{\boldsymbol{h}}_1=[h_M\ \ h_{M-1}\ \ \dots\ \ h_1]^T$). For the proposed LP-STBC code for $2M$-1 antennas, $\boldsymbol{S}(\boldsymbol{s},2L,\ 2M\ -1)$, its equivalent channel matrix is similar as that of the proposed LP-STBC code for $2M$ antennas, $\boldsymbol{S}(\boldsymbol{s},2L,\ 2M)$, with $h_{2M}=0$.

Similar to the code matrix in (10), the equivalent channel matrix in (19) also has the group orthogonality property. Partitioning the columns of $\mathcal{H}$ into two groups as

$$\mathcal{H}=[\boldsymbol{H}_1\ \boldsymbol{H}_2] \quad (20)$$

where $\boldsymbol{H}_1=\left[\mathcal{B}(\boldsymbol{h}_1,M,L)\ \ -\mathcal{T}^*(\tilde{\boldsymbol{h}}_2,M,L)\right]^T$ and $\boldsymbol{H}_2=\left[\mathcal{T}(\boldsymbol{h}_2,M,L)\ \ \mathcal{C}^*(\tilde{\boldsymbol{h}}_1,M,L)\right]^T$, then it is not hard to verify that they are orthogonal with each other, i.e., $\boldsymbol{H}_1^H\boldsymbol{H}_2=\boldsymbol{0}_{L\times L}$. Meanwhile, due to the Toeplitz structure, we have:

$$\begin{aligned}\mathcal{T}^H(\boldsymbol{h},M,L)\times\mathcal{T}(\boldsymbol{h},M,L)&=\mathcal{T}^T(\boldsymbol{h}^*,M,L)\times\mathcal{T}(\boldsymbol{h},M,L)\\&=\mathcal{T}^T(\boldsymbol{h},M,L)\times\mathcal{T}(\boldsymbol{h}^*,M,L)\end{aligned} \quad (21)$$

$$\begin{aligned}\mathcal{B}^H(\boldsymbol{h},M,L)\times\mathcal{B}(\boldsymbol{h},M,L)&=\mathcal{B}^T(\boldsymbol{h}^*,M,L)\times\mathcal{B}(\boldsymbol{h},M,L)\\&=\mathcal{B}^T(\boldsymbol{h},M,L)\times\mathcal{B}(\boldsymbol{h}^*,M,L)\end{aligned} \quad (22)$$

Based on these observations, we have:

$$\det(\mathcal{H}^H\mathcal{H})=\det\begin{pmatrix}\boldsymbol{H}_1^H\boldsymbol{H}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{H}_2^H\boldsymbol{H}_2\end{pmatrix}=\det(\boldsymbol{G}_1)\times\det(\boldsymbol{G}_2) \quad (23)$$

where

$$\boldsymbol{G}_1=\mathcal{B}^H(\boldsymbol{h}_1,M,L)\times\mathcal{B}(\boldsymbol{h}_1,M,L)+\mathcal{T}^H(\tilde{\boldsymbol{h}}_2,M,L)\times\mathcal{T}(\tilde{\boldsymbol{h}}_2,M,L) \quad (24)$$

$$\boldsymbol{G}_2=\mathcal{C}^H(\tilde{\boldsymbol{h}}_1,M,L)\times\mathcal{C}(\tilde{\boldsymbol{h}}_1,M,L)+\mathcal{T}^H(\boldsymbol{h}_2,M,L)\times\mathcal{T}(\boldsymbol{h}_2,M,L) \quad (25)$$

For any nonzero $\boldsymbol{h}$ any, we assume that $\boldsymbol{h}_1\neq 0$. Then,

$$\begin{aligned}\det(\boldsymbol{G}_1)\geq&\det\left(\mathcal{B}^H(\boldsymbol{h}_1,M,L)\times\mathcal{B}(\boldsymbol{h}_1,M,L)\right)\\&+\det\left(\mathcal{T}^H(\tilde{\boldsymbol{h}}_2,M,L)\times\mathcal{T}(\tilde{\boldsymbol{h}}_2,M,L)\right)\end{aligned} \quad (26a)$$

$$\geq\det\left(\mathcal{B}^H(\boldsymbol{h}_1,M,L)\mathcal{B}(\boldsymbol{h}_1,M,L)\right)>0 \quad (26b)$$

$$\begin{aligned}\det(\boldsymbol{G}_2)\geq&\det\left(\mathcal{C}^H(\tilde{\boldsymbol{h}}_1,M,L)\times\mathcal{C}(\tilde{\boldsymbol{h}}_1,M,L)\right)\\&+\det\left(\mathcal{T}^H(\boldsymbol{h}_2,M,L)\times\mathcal{T}(\boldsymbol{h}_2,M,L)\right)\end{aligned} \quad (27a)$$

$$\geq\det\left(\mathcal{C}^H(\tilde{\boldsymbol{h}}_1,M,L)\times\mathcal{C}(\tilde{\boldsymbol{h}}_1,M,L)\right)>0 \quad (27b)$$

where inequalities (26a) and (27a) hold because $\mathcal{B}^H(\boldsymbol{h}_1,M,L)\times\mathcal{B}(\boldsymbol{h}_1,M,L)$, $\mathcal{T}^H(\tilde{\boldsymbol{h}}_2,M,L)\times\mathcal{T}(\tilde{\boldsymbol{h}}_2,M,L)$, $\mathcal{C}^H(\tilde{\boldsymbol{h}}_1,M,L)\times\mathcal{C}(\tilde{\boldsymbol{h}}_1,M,L)$ and $\mathcal{T}^H(\boldsymbol{h}_2,M,L)\times\mathcal{T}(\boldsymbol{h}_2,M,L)$ are positive semi-definite matrices, and inequalities (26b) and (27b) comes from the result in Lemma 1. For any nonzero $\boldsymbol{h}$, if $\boldsymbol{h}_1=0$, then $\boldsymbol{h}_2\neq 0$. Using Lemma 1 and following the similar proof in (26) and (27), we can get $\det(\mathcal{H}^H\mathcal{H})>0$. Therefore, for any nonzero $\boldsymbol{h}$ in MISO systems, our proposed LP-STBC in (10) achieves full diversity with linear receivers following Theorem 1. This means that our codes also achieve full diversity for MIMO systems.

IV. COMPARISON RESULTS

Our proposed LP-STBC codes have some good properties that will be investigated and described in this subsection, and their performance comparison with other STBC is also carried out. We now make the following remarks for LP-STBC codes and the main counterpart in the comparison is OAC codes [3]. Why do we choose OAC codes? The first, OAC codes have higher rate than OSTBC, Toeplitz codes and GOTC codes. The second, OAC codes have group othogonality while Toeplitz codes do not, so OAC codes have lower decoding complexity. Finally, OAC codes outperform OSTBC, Toeplitz codes and GOTC codes in MIMO systems when number of transmit antennas is greater than four [3], [5].

A. Symbol rate

The proposed LP-STBC code and the OAC code have symbol rate $R=\dfrac{2L}{2L+2M-2}$ for $2M$ or $2M$-1 transmit antennas.

B. Othogonality

Structure (10) ensures that the proposed LP-STBC codes have group othogonality. Codeword matrix can divide to two groups. The first group includes the first M columns. The second group includes remaining columns. Each column of the first group is orthogonal to all columns of the second group and opposite. This group orthogonality property results in low decoding complexity

C. Number of zero-symbols and PAPR

A main advantage of proposed LP-STBC codes over OAC codes is eliminating completely transmitting "zero-symbols" in first M antennas. This makes hardware implementation becomes easier, we reduce number of turn-off switcher for antennas, reduce low-frequency interference due to switching turn-off when transmitting zeros. Moreover, eliminating zeros leads to decreasing PAPR at first M antennas. For example, for 8 transmit antennas, the proposed LP-STBC code rate of 3/4 given in (15) have lower PAPR of 1.25 dB than the same rate OAC code given in (16) at 4 first antennas.

D. Decoding performance

We perform some Monte-Carlo simulations to compare the performance of the proposed LP-STBC codes with known OAC codes. In all the simulations, the channel model follows that described in Section II.A. The proposed LP-STBC code and OAC code are chosen with the same parameter as: code rate of 3/4, 8 transmit antennas, 1 receive antenna, 4QAM modulation and ZF receiver.

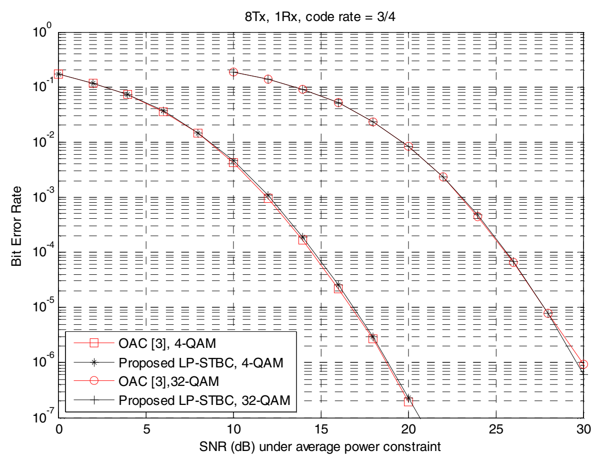

Figure 1: Comparisons of performance between the proposed LP-STBC code and the OAC code under average power constraint.

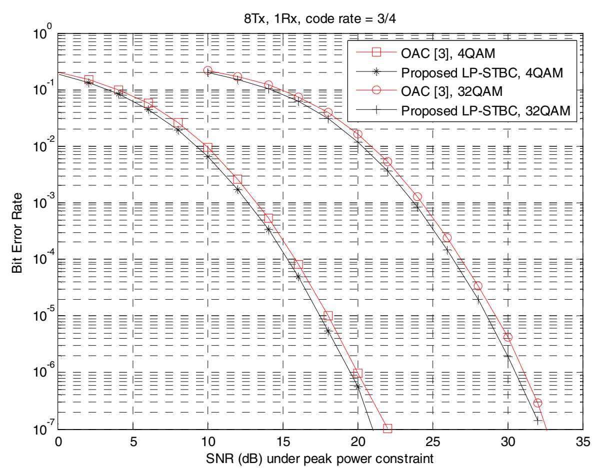

Figure 2: Comparisons of performance between the proposed LP-STBC code and the OAC code under peak power constraint.

Figure 1 presents simulation results under average power constraint. Figure 2 presents simulation results under peak power constraint. From the figures, it is observed that the bit error rate (BER) curves of the proposed LP-STBC code and OAC code with the same antenna configuration have the same slope, suggesting that they have the same diversity order. This confirms that the proposed LP-STBC code is full diversity code for linear receiver. The simulation results in Fig.1 and Fig.2 also show that the BER performance of the proposed LP-STBC is better than that of OAC code under peak power constraint and is similar under average power constraint.

V. CONCLUSIONS

In this chapter, we focused on designing low PAPR full diversity STBCs for linear receivers. Based on the elementary NZE matrices, we proposed LP-STBC codes which can achieve full diversity with linear receivers. Our proposed codes not only maintain the code properties of the OAC codes (as full diversity, high symbol rate, group orthogonality, and low decoding complexity), but also avoid transmitting “zero-symbol” on a half of transmit antennas. This not only helps the proposed codes to achieve better PAPR efficiency, but also to simplify hardware implementation. Therefore, our LP-STBC codes can become a good candidate for designing practical MIMO systems with linear receivers.

REFERENCES

[1] J.-K. Zhang, J. Liu, and K.M.Wong, “Linear Toeplitz space time block codes,” in Proc. IEEE Int. Symp. Inf. Theory (ISIT’05), Adelaide, Australia, Sep. 4–9, 2005, pp. 1942–1946.

[2] J. Liu, J.-K. Zhang, and K. M. Wong, “Full-diversity codes for MISO systems equipped with linear or ML detectors,” IEEE Trans. Inf. Theory, vol. 54, no. 10, pp. 4511–4527, Oct. 2008.

[3] Y. Shang and X.-G. Xia, “Space-time block codes achieving full diversity with linear receivers," IEEE Trans. Inf. Theory, vol. 54, pp. 4528-4547, Oct. 2008.

[4] W. Zhang and J. Yuan, “A simple design of Space-time block codes achieving full diversity with linear receivers,” ICASSP 2009, Page(s): 2729 – 2732, 2009.

[5] H.-M. Wang, X.-G Xia, Q.-Y. Yin and B. Li, “A Family of Space-Time Block Codes Achieving Full Diversity with Linear Receivers,” IEEE Trans. Wireless Commun., vol. 57, no. 12, pp. 3607–3617, Dec 2009.

[6] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” IEEE J. Sel. Areas Commun., vol. 16, pp.1451–1458, Oct. 1998.

[7] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” IEEE Trans. Inf. Theory, vol. 45, pp. 1456–1467, Jul. 1999.

[8] X.-B. Liang, “Orthogonal designs with maximal rates,” IEEE Trans. Inf. Theory, vol. 49, pp. 2468–2503, Oct. 2003.

[9] W. Su, X.-G. Xia, and K. J. R. Liu, “A systematic design of high-rate complex orthogonal space-time block codes,” IEEE Commun. Letter, vol. 8, pp. 380–382, Jun. 2004.

[10] K. Lu, S. Fu, and X.-G. Xia, “Closed-form designs of complex orthogonal space-time block codes of rates $(k+1)/2k$ for $(2k-1)$ or $2k$ transmit antennas,” IEEE Trans. Inf. Theory, vol. 51, pp. 4340–4347, Dec. 2005.

[11] C. Yuen, Y. L. Guan, and T. T. Tjhung, “Power-balanced orthogonal space-time block code,” IEEE Trans. Veh. Technol., vol. 57, no. 5, pp. 3304-3309, Sept. 2008.

[12] P.V. Bien, W.X. Sheng, X.F. Ma, and H. Wang, “No-Zero-Entry STBC Scheme of Achieving Full Diversity with Linear Receiver”, Proc. of The 2011 Int. Conf. on Advanced Technologies for Communications (ATC’11), Danang, Vietnam, pp. 212-215, August 2011.