Full-Rate Full-Diversity Three-Time-Slot Space-Time Block Code with Single-Symbol ML Decoding

Pham Van Bien[1,2], Weixing Sheng[1], Tran Xuan Nam[2] and Dinh The Cuong[2]
[1] Department of Communication Engineering, Nanjing University of Science and Technology, China
[2] Department of Communication Engineering, Le Quy Don Technical University, Vietnam
Email: bienngaxanh@gmail.com, shengwx@mail.njust.edu.cn, namtx@mta.edu.vn and cuongdt@mta.edu.vn

***Abstract* —It is known that LTE systems do not fully support the Alamouti space-time code as the number of symbols per slot is not always an even number. Adapting to the case there is only three symbols per slot, Lei et al. proposed a class of quasi-orthogonal space time block codes (Q-STBC) for two transmit antennas and three time slots. This Q-STBC achieves some desirable properties of an STBC code such as full rate and full diversity. However, there are two drawbacks associated with it, namely, high decoding complexity due to pair-symbol maximum likelihood decoding and lack of maximum coding gain. Coping with these two issues we propose a class of STBC for three time slots and two transmit antennas with single-symbol maximum likelihood decoding. The proposed STBC also allows to achieve full-rate and full diversity. However, it is superior to Q-STBC in providing maximum coding gain while requiring lower decoding complexity.**

I. INTRODUCTION

Recently, the 3rd Generation Partnership Project (3GPP) standardized Long-Term Evolution (LTE) has been deployed worldwide to provide access rate up to 100Mbps. The 3GPP is now working on the next generation wireless system (4G) under the project LTE-Advanced [1]. It is known that user equipment (UE) will support minimum two antennas to achieve spatial diversity [2]. For this case the well-known Alamouti space-time block code (STBC) [3] would be most suitable candidate for uplink transmission. Unfortunately, it has been revealed that the LTE frame structure does not always contain an even number of time slots [5][6] and thus direct application of the Alamouti STBC is inappropriate. This unfortunate observation initiates an interesting STBC design problem for a system with three time slots and two transmit antennas. As in the case of the Alamouti STBC, the desired STBC should achieve full-rate and full-diversity transmission with low decoding complexity. However, to the best of our knowledge, there is no such STBC available at present.

In [4], a hybrid scheme of STBC for three time slots was proposed by repeating one more time slot after the Alamouti STBC (called H-STBC in this paper). The H-STBC achieves relatively good performance and requires only simple linear decoding at the receiver. However, H-STBC does not allow achieving full diversity. In a recent research [5][6], Lei *et al.* introduced a class of quasi-orthogonal STBCs (Q-STBC) for three time slots and two transmit antennas using pair-symbol maximum likelihood decoding. The Q-STBC achieves full-rate and full-diversity but does not obtain maximum coding gain. This leads to some error performance degradation compared to the Alamouti STBC. Moreover, complexity is also a problem as the Q-STBC uses pair-symbol rather than single symbol decoding. In this paper, we propose a new class of STBC for three-time-slot and two-antenna transmission. Our proposed STBC allow achieving full-rate and full-diversity transmission while requiring low complexity thanks to the proposed single symbol decoding. The proposed STBC is this superior to both H-STBC and Q-STBC in terms of error performance and also to Q-STBC in terms of computational complexity.

The remaining content of the paper is organized as follows. In Section II, we provide a brief review of the construction of three-time-slot STBCs. The proposed design of the new STBC is elaborated in Section III. Decoding complexity is analyzed in Section IV, followed by the simulation results in Section V. The paper is concluded in Section VI.

II. OVERVIEW OF THREE-TIME-SLOT STBCS

Consider a simple uplink transmission scheme in LTE-Advanced systems where user equipment (UE) has two transmit antennas and the base station (BS) one receive antenna. For the case there exists an even number of time slots, the Alamouti STBC is perfectly fitted for transmission to achieve the uplink spatial diversity. When the number of time slots is restricted to three as in the LTE frame there are two options for encoding the three transmit symbols. Denote three symbols to be transmitted as s_1, s_2, s_3. The first option is to use the full-rate alternative H-STBC proposed in [4]. The encoding matrix for the H-STBC is given by

$$\boldsymbol{X}_{\mathrm{H}} = \begin{bmatrix} s_1 & s_2 & s_3 \\ s_2^* & -s_1^* & s_3 \end{bmatrix}^{\mathrm{T}}, \tag{1}$$

where two rows represent symbols transmitted from transmit antennas while columns indicate transmit symbols at three time slots. The encoding scheme is simple as the

The work of the third and the fourth author was supported by NAFOSTED under project number 102.99.34.09..

first two time slots are still encoded as in the orthogonal STBC while the third time slot is simply repeated from two antennas. This simple encoding scheme allows for linear decoding with low complexity at the receiver. However, it is clear that the scheme is not able to provide full-diversity gain as spatial diversity is not achieved in the third time slot. As a result, it suffers from significant performance loss compared to the orthogonal STBC by Alamouti.

In order to achieve full diversity gain for the case with 3 time slots and 2 transmit antennas, Lei *et al.* proposed in [5][6] a class of Q-STBC whose encoding matrix is given by

$$\boldsymbol{X}_{\text{Q-STBC}} = \begin{bmatrix} s_1 & s_2 & s_3 \\ y_1 & y_2 & y_3 \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where

$$y_1 = \frac{s_1^* + 2e^{j\frac{2\pi}{5}} s_2^* + 2e^{-j\frac{2\pi}{5}} s_3^*}{3} \tag{3a}$$

$$y_2 = \frac{-2e^{j\frac{2\pi}{5}} s_1^* + e^{-j\frac{\pi}{5}} s_2^* + 2s_3^*}{3} \tag{3b}$$

$$y_3 = \frac{-2e^{-j\frac{2\pi}{5}} s_1^* + 2s_2^* + e^{j\frac{\pi}{5}} s_3^*}{3} \tag{3c}$$

In comparison to H-STBC, this Q-STBC achieves full-rate and full-diversity gain with bit error rate (BER) performance close to that of the Alamouti STBC. However, as the scheme uses pair-symbol maximum likelihood (ML) rather than single symbol decoding its complexity is still quite high. Moreover, although Q-STBC can achieve full diversity gain there is still an amount of E_b/N_0 loss due to not exploiting full coding gain.

In the next section, we will present a new class of STBC (abbreviated as TTS-STBC) which can achieve simultaneously four desirable properties, namely, full-rate, full-diversity gain, maximum coding gain and low complexity based on single-symbol ML decoding.

III. THE PROPOSED TTS-STBC

A. Structure of the proposed TTS-STBC

The encoder starts with an input symbol vector $\boldsymbol{s} = [s_1, s_2, s_3]^T$ of three information symbols chosen from a square QAM constellation $\mathcal{A}$, where $s_k = s_k^{\mathrm{I}} + js_k^{\mathrm{Q}}$ with s_k^{I} and s_k^{Q} denoting the in-phase and quardrature part of the complex symbol s_k, respectively. The proposed TTS-STBC is constructed in the following three steps.

Step 1: Rotating the input symbol vector $\boldsymbol{s}$ to generate the rotated symbol vector $\boldsymbol{x} = \boldsymbol{s}e^{j\theta}$. This is equivalent to rotating constellation $\mathcal{A}$ and choosing symbol x_k from the rotated constellation $\mathcal{A}e^{j\theta}$. The purpose of the constellation rotation is to ensure full diversity and maximum coding gain.

Step 2: Coordinating interleaved elements of vector $\boldsymbol{x}$ to generate vector $\boldsymbol{u} = [u_1\ u_2\ u_3]^{\mathrm{T}}$ based on the following interleaving rules: $u_1 = x_1^{\mathrm{I}} + jx_3^{\mathrm{Q}}, u_2 = x_2, u_3 = x_3^{\mathrm{I}} + jx_1^{\mathrm{Q}}$.

Step 3: Encoding u_k according to the following encoding matrix

$$\boldsymbol{S} = \begin{bmatrix} u_1 & -u_2^* & u_3 \\ u_2 & u_1^* & u_3 \end{bmatrix}^{\mathrm{T}}. \tag{4}$$

Clearly, the proposed encoding scheme transmits 3 symbols in 3 time slots, so it achieves *full rate*. Moreover, the transmit symbols from the two antennas have the same average power, i.e., $E\left(|u_1|^2\right) = E\left(|u_2|^2\right) = E\left(|u_3|^2\right)$. This means that our proposed code allows minimizing the signal *power fluctuation* at the transmitter side.

B. Diversity and coding gain analysis

In order to meet the full-diversity criterion, the codeword difference matrix $\boldsymbol{B} = \boldsymbol{S} - \boldsymbol{S}'$ should be of full-rank (i.e., rank 2) [7], or equivalently,

$$\det\left(\boldsymbol{B}^H \boldsymbol{B}\right) = \det\begin{bmatrix} \sum_{k=1}^{3} |u_k - u_k'|^2 & |u_3 - u_3'|^2 \\ |u_3 - u_3'|^2 & \sum_{k=1}^{3} |u_k - u_k'|^2 \end{bmatrix} \neq 0 \tag{5}$$

where $\boldsymbol{S}$ and $\boldsymbol{S}'$ are two distinct codeword matrices obtained from (4). In order to prove this fact, firstly we have to show that the minimum determinant $\delta_{\min} = \min\{\det(\boldsymbol{B}^{\mathrm{H}}\boldsymbol{B})\}$ is non-zero for any distinct codeword pairs $\boldsymbol{S}$ and $\boldsymbol{S}'$ of the proposed TTS-STBC. Let $\Delta x_k^{\mathrm{I}} = x_k^{\mathrm{I}} - x_k^{\mathrm{I}\prime}$ $\Delta x_k^{\mathrm{I}} = x_k^{\mathrm{I}} - x_k^{\mathrm{I}\prime}$ and $\Delta x_k^{\mathrm{Q}} = x_k^{\mathrm{Q}} - x_k^{\mathrm{Q}\prime}$, k = 1,2,3, denote the differences in real and imaginary parts of the transmitted and erroneously detected information symbols x_k and x_k', respectively, for any $x_k, x_k' \in \mathcal{A}e^{j\theta}$ and $x_k \neq x_k'$. We calculate the value $\delta_{\min}$ of the proposed TTS-STBC as follows

$$\delta_{\min} = \min\left\{\begin{aligned} &\left[\left(\Delta x_1^{\mathrm{I}}\right)^2 + \left(\Delta x_2^{\mathrm{I}}\right)^2 + \left(\Delta x_2^{\mathrm{Q}}\right)^2 + \left(\Delta x_3^{\mathrm{Q}}\right)^2\right] \\ &\times\left[\left(\Delta x_1^{\mathrm{I}}\right)^2 + 2\left(\Delta x_1^{\mathrm{Q}}\right)^2 + \left(\Delta x_2^{\mathrm{I}}\right)^2\right. \\ &\qquad \left. + \left(\Delta x_2^{\mathrm{Q}}\right)^2 + \left(\Delta x_3^{\mathrm{I}}\right)^2 + \left(\Delta x_3^{\mathrm{Q}}\right)^2\right] \end{aligned}\right\}. \tag{6}$$

It is obvious that (6) takes its minimum value when only one information symbol is erroneous. We have three cases as follows.

Case 1: if erroneous symbol is x_1 (i.e., $x_1 \neq x_1'$) then the resulting minimum determinant is given by

$$\delta_{\min,1} = \min\left\{\left(\Delta x_1^{\mathrm{I}}\right)^2\left[\left(\Delta x_1^{\mathrm{I}}\right)^2 + 2\left(\Delta x_1^{\mathrm{Q}}\right)^2\right]\right\} \quad (7)$$

Case 2: for the erroneous symbol x_2 (i.e., $x_2 \neq x'_2$) the minimum determinant is

$$\begin{aligned}\delta_{\min,2} &= \min\left\{\left(\Delta x_{2I}^2 + \Delta x_{2Q}^2\right)^2\right\} \\ \delta_{\min,2} &= \min\left\{\left[\left(\Delta x_2^{\mathrm{I}}\right)^2 + \left(\Delta x_2^{\mathrm{Q}}\right)^2\right]^2\right\}\end{aligned} \quad (8)$$

Case 3: similarly for the erroneous symbol x_3 (i.e., $x_3 \neq x_3$') we have the resulting minimum determinant

$$\begin{aligned}\delta_{\min,3} &= \min\left\{\Delta x_{3Q}^2\left(2\Delta x_{3I}^2 + \Delta x_{3Q}^2\right)\right\} \\ \delta_{\min,3} &= \min\left\{\left(\Delta x_3^{\mathrm{Q}}\right)^2\left[2\left(\Delta x_3^{\mathrm{I}}\right)^2 + \left(\Delta x_3^{\mathrm{Q}}\right)^2\right]\right\}\end{aligned} \quad (9)$$

It is clearly that $\delta_{\min,2}$ value is non-zero for any constellation $\mathcal{A}$, $\delta_{\min,1}$ value is non-zero with only constellation $\mathcal{A}$ where $\Delta x_k^{\mathrm{I}} = x_k^{\mathrm{I}} - x_k^{\mathrm{I}'} \neq 0$ (x_k, $x_k' \in \mathcal{A}$ and $x_k \neq x_k'$) and $\delta_{\min,3}$ value is non-zero with only constellation $\mathcal{A}$ where $\Delta x_k^{\mathrm{Q}} = x_k^{\mathrm{Q}} - x_k^{\mathrm{Q}'} \neq 0$. Thus value $\delta_{\min}$ in (6) is non-zero if only if the information symbols x_k is taken value from constellation $\mathcal{A}$ where $\Delta x_k^{\mathrm{I}} \Delta x_k^{\mathrm{Q}} \neq 0$.

From above observations, we can see that the proposed TTS-STBC cannot achieve full-diversity if the information symbols x_k take value from the conventional signal constellations $\mathcal{A}$ like the regular M-ary QAM or symmetric M-ary PSK. However, by rotating constellation in Step 1 (Section III.A), we can ensure $\Delta x_k^{\mathrm{I}} \Delta x_k^{\mathrm{Q}} \neq 0$ ($\forall x_k, x_k' \in \mathcal{A}e^{j\theta}$ and $x_k \neq x_k'$) for any square M-ary QAM or symmetric M-ary PSK constellation $\mathcal{A}$, i.e., it ensures a nonzero $\delta_{\min}$ value. This means that the proposed TTS-STBC can achieve full-diversity.

After having shown the full-diversity property, we have to choose the optimum CR angle to maximize the $\delta_{\min}$ value for the proposed TTS-STBC to achieve maximum coding gain. It is not difficult to demonstrate that $\delta_{\min,2}$ is always greater or equal than $\delta_{\min,1}$ and $\delta_{\min,3}$. Thus, the optimum CR angle is chosen to maximize value of below function

$$F = \max\min\left(\max\delta_{\min,1}, \max\delta_{\min,3}\right) \quad (10)$$

The analytical derivation of the optimum CR angle is unfortunately not as tractable hence we rely on computer search to find the optimum CR angle for the proposed TTS-STBC. For 4-QAM constellation $\mathcal{A}$ where signal points are $s = (2n\text{-}3) + j(2m\text{-}3)$ with n, $m \in [1,2]$, as shown in Fig.1, the optimum CR angle is found to be 30.94° which gives $\delta_{\min} = 7.33$.

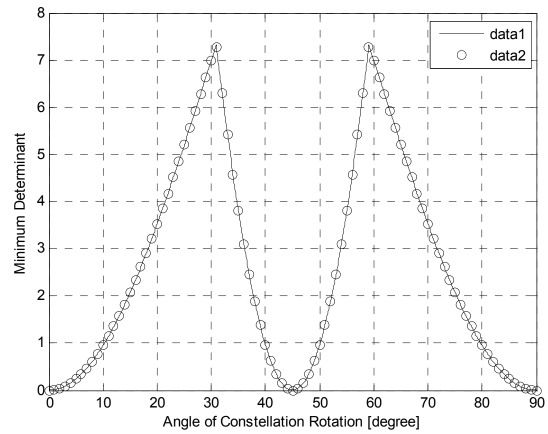

Fig.1: Optimization of 4-QAM rotation angle for the proposed TTS-STBC; data1 and data2 represents respectively values of $\delta_{\min,1}$ and $\delta_{\min,3}$.

IV. Decoding of The Proposed TTS-STBC

To illustrate decoding complexity of the proposed STBC codes using single-symbol ML, we will derive the decision metric used for the ML detection for a single antenna receiver as follows.

We assume that the channel is quasi-static, i.e., the channel coefficients are constant over a period of 3 transmission slots, and they can be changed independently from one codeword transmission to the next one. Moreover, we also assume that channel state information (CSI) is perfectly known at the receiver but not at the transmitter. We then obtain received signal in vector/matrix form as $\boldsymbol{r} = \boldsymbol{Sh} + \boldsymbol{n}$, where $\boldsymbol{r} = [r_1\ r_2\ r_3]^{\mathrm{T}}$ is the received signal vector. $h = \left[h_1\ h_2\right]^{\mathrm{T}}$ is the channel coefficient vector, where h_k, k = 1,2 represents the channel gain from the k-th transmit antenna to the receive antenna and is independent identically distributed (*iid*) with $\mathcal{CN}(0, 1)$. $\boldsymbol{n} = [n_1\ n_2\ n_3]^{\mathrm{T}}$ is complex random Gaussian noise vector with *iid* $\mathcal{CN}(0, N_0)$ entries. The ML decision metric is $M\left(\boldsymbol{S}\right) = \left|\boldsymbol{r} - \boldsymbol{Sh}\right|^2$. The ML decoding is to find the optimal $\boldsymbol{S}$ among all possibilities which minimizes the metric $M(\boldsymbol{S})$. After some manipulations with $\boldsymbol{S}$ substituted with (1) and removing irrelevant items, the metric $M(\mathbf{S})$ can be expanded as a sum of three terms

$$M\left(\boldsymbol{S}\right) = f_1(x_1) + f_2(x_2) + f_3(x_3) \quad (11)$$

where

$$f_1(x_1) = \left(|h_1|^2 + |h_2|^2\right)x_{1I}^2 + |h_1 + h_2|^2 x_{1Q}^2 - \\ -2\,\mathrm{Re}\left\{\left(h_1 r_1^* + h_2^* r_2\right)x_{1I} + j\left(h_1 + h_2\right) r_3^* x_{1Q}\right\} \tag{12}$$

$$f_2(x_2) = \left(|h_1|^2 + |h_2|^2\right)|x_2|^2 + 2\,\mathrm{Re}\left\{\left(h_1^* r_2 - h_2 r_1^*\right)x_2\right\} \tag{13}$$

$$f_3(x_3) = |h_1 + h_2|^2 x_{3I}^2 + \left(|h_1|^2 + |h_2|^2\right)x_{3Q}^2 - \\ -2\mathrm{Re}\left\{\left(h_1 + h_2\right) r_3^* x_{3I} + j\left(h_1 r_1^* + h_2^* r_2\right)x_{3Q}\right\} \tag{14}$$

Since each term $f_k(x_k)$, $k = 1, 2, 3$ is independent of x_m for $k \neq m$, minimizing the ML metric $M(\boldsymbol{S})$ is equivalent to minimizing each term $f_k(x_k)$, $x_k \in \mathcal{A}e^{j\theta}$, independently. This implies that the proposed code can perform single-symbol ML decoding without sacrificing the performance.

V. Simulation Results

In the Fig.2, we provide simulation results of the proposed STBC for two transmit antennas and one receive antenna on a quasi-static flat fading channel. The transmitted symbols were 4-QAM modulated. The performance results for the Alamouti code [3], the H-STBC [4] as well as the Q-STBC [5][6] are also provided for comparison.

From Fig.2, we can observe that the proposed code outperforms H-STBC as a result of the full-diversity, especially at high SNR, e.g., a performance gain of 4 dB at BER of 10^{-3}. In comparison to Q-STBC [5], the proposed code has not only lower decoding complexity due to single-symbol ML decoding, but also a performance gain of 1.5 dB at BER of 10^{-3}. This is explained as follows. Although both H-STBC and Q-STBC have the same transmit diversity order (order 2), the proposed TTS-STBC achieves maximum coding gain while Q-STBC cannot. The reason is due to the fact that Q-STBC focuses on diversity gain and thus its coding gain is not optimized [6 – Section III.A.2]. Therefore, the proposed TTS-STBC outperforms Q-STBC.

In comparison to the Alamouti code, the performance gap between the proposed TTS-STBC and the Alamouti code is due to the fact that the Alamouti code is an orthogonal design with a higher coding gain whereas the proposed TTS-STBC is non-orthogonal. However, the Alamouti code can not apply for 3-slot transmission.

Next, we compare our proposed TTS-STBC and Q-STBC based on the following six criteria:

- Rate: it is clear that both the codes achieve the same rate one, i.e. full-rate transmission.

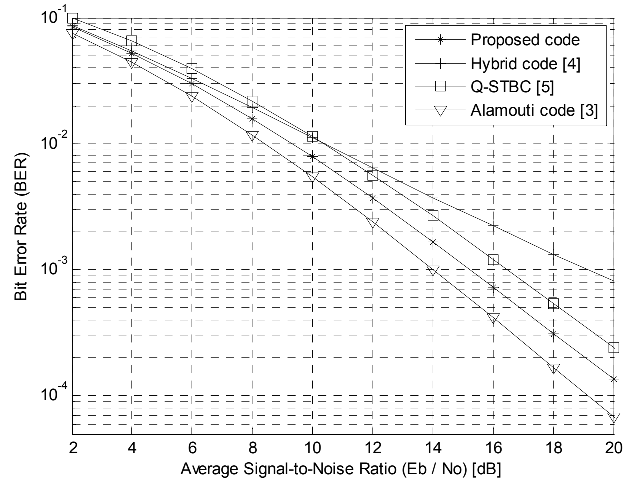

Fig. 2: Performance comparison for 3-slot transmissions

- Full-diversity: with two transmit antennas and N_R receive antennas both the codes achieve the same diversity order of $2N_R$, i.e., full-diversity transmission.
- Backward compatibility: the proposed TTS-STBC does not have this property. However, it is worth noting that this property is not always necessary in many orthogonal/non-orthogonal STBC designs [8]. If necessary, it is possible to find a technical solution for this problem with slightly increased complexity.
- Receiver complexity: our proposed TTS-STBC requires less computational complexity than that of Q-STBC due to single-symbol ML decoding. Decoding complexity of our proposed code is of $O(3M)$ while that of Q-STBC is $O(M) + O(M^2)$, with M denoting the modulation order. Detailed complexity comparisons are illustrated in Table I.
- Transmitter complexity: from the code structure in (4) and Q-STBC in (2) we can observe that while our code requires only one phase rotator, Q-STBC needs up to four phase rotators ($-2\pi/5$, $-\pi/5$, $\pi/5$, $2\pi/5$). Therefore, our code requires lower computational complexity for the transmitter.
- Power fluctuation: it can be seen that data symbols are transmitted from the two transmit antennas with fixed power $E\left(|u_1|^2\right) = E\left(|u_2|^2\right) = E\left(|u_3|^2\right)$. As a result, our proposed code allows for minimizing the signal power fluctuation at the transmitter.

In the Table I, we present a comparison of decoding time for our proposed TTS-STBC and Q-STBC [5][6]. The system used for simulation is as follows. The simulation system employs two transmit antennas and one receive antenna operating under quasi-static flat fading channel. The computer used for simulation has a dual-core Pentium CPU E3500 with clock rate of 2.60 GHz and 3.2 Gb RAM.

Decoding time was recorded for transmitted 10,000 symbols taken from 4, 8, 16, 32, 64, 128 and 256-QAM constellations.

Table I: Comparison of decoding time (sec) of LYC-STBC [5][6] and our proposed TTS-STBC for 10,000 symbols

M-QAM Modulation.	**4**	**8**	**16**	**32**	**64**	**128**	**256**
Q-STBC [5][6]	0.64	1.32	3.48	11.85	43.83	171.47	673.13
Proposed TTS-STBC	0.33	0.46	0.65	0.96	1.48	2.46	4.07

It can be seen clearly from Table I that our proposed TTS-STBC code is more efficient than Q-STBC in term of computational complexity, particularly for the case of high-order modulation.

VI. CONCLUSIONS

In this paper, we have proposed a new STBC for three time slots and two transmit antennas. The proposed TTS-STBC achieves simultaneously four desirable properties such as full-rate, full-diversity, maximum coding gain and single-symbol ML decoding.

The proposed TTS-STBC has significant advantage over the previous H-STBC [4] and Q-STBC [5][6] because these codes cannot achieve simultaneously all these four desirable properties. The proposed TTS-STBC can be a prospective candidate for the LTE-Advanced system.

REFERENCES

[1] 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 8)," 3GPP TS 36.213 V8.3.0 (2008-05)

[2] Alcatel Shanghai Bell, Alcatel-Lucent, "STBC-II scheme for uplink transmit diversity in LTE-Advanced," R1-082500, 3GPP TSG RAN WG 1 Meeting #53 bis, Jun-Jul. 2008.

[3] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," IEEE J. Select. Areas Comm., vol. 16, pp. 1451-1458, Oct. 1998.

[4] Alcatel Shanghai Bell, Alcatel-Lucent, "STBC-II scheme with non-paired symbols for LTE-Advanced uplink transmit diversity," R1-090058, 3GPP TSG RAN WG 1 Meeting #55 bis, Jan. 2009

[5] Z.D. Lei, C. Yuen and F. Chin, "Three-Time-Slot Quasi-Orthogonal Space-Time Block Codes", In Proc. IEEE ICC, Cape Town, South Africa, May 2010, pp. 1-5.

[6] Z.D. Lei, C. Yuen and F. Chin, "Quasi-Orthogonal Space-Time Block Codes for Two Transmit Antennas and Three Time Slots", IEEE Trans. on Wireless Commun., Accepted for publication, pp 1-9, 2011.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," IEEE Transactions on Information Theory, Vol.44, No.2, pp. 744-765, March 1998.

[8] M. Z. A. Khan and B. S. Rajan, "Single-Symbol Maximum-Likelihood Decodable Linear STBCs," IEEE Transactions on Information Theory, vol. 52, no. 5, pp. 2062-2091, 2006.