Application of Interpolation for DBIM Reconstruction of Ultrasound Tomography

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Abstract:
Ultrasonic tomography technique has a great potential value in many applications such as medicine, underwater acoustics, non-destructive testing, etc. However, one of the most disadvantages of this technique is time consumption in reconstruction process. The distorted Born iterative method (DBIM) has been utilized to reconstruct the ultrasound tomography. It can take several iterations depend on the quantity of measured signals. This paper has proposed a solution of fast reconstruction by combining interpolation technique and DBIM. The tomography of scatters' sound contrast is first reconstructed with a raw meshed integration area. We can easily obtain the convergent even after one or two iterations using DBIM. After that, the interpolation is applied to the signal to obtain a dense image. A method like nearest neighbor is chosen because it doesn't generate any new data values. Finally, the dense image is brought back to DBIM to continue the reconstruction process. The simulation show that we can save up to 50% of reconstructed time.

1. Introduction

Ultrasound imaging and tomography play important roles in clinical detection. Most of ultrasound scanners are based on a pulse echo method that use the time of light energy reflected by object's boundaries [1]. By extending the number of angles around the subject, the inverse technique offered better reconstructed image in case of strong scattering. Works in ultrasound tomography has usually been trying to calculate the size of tissue (scatter area) and the speed of sound crossing the shell. In present, there are a few commercial tomography devices introduced to market. The reason is that the inverse scatter has to face with limitations of high computation and efficiency. Initial approaches utilized the projection theory that widely used in X-ray and nuclear tomography [2][3]. However, these ray-based methods were not suitable with diffraction properties of ultrasound propagation. Then, the Gauss-Newton method combined has been proposed to solve the inverse problem However, the limitation of this approach noise sensitivity [4]. After that, the Born Iterative Method (BIM) based on first-order Born approximation has been introduced as one of efficient diffraction tomography approaches [5]. This method is robust to noise, but its computation is quite high. Based on this trend, the same authors has developed Distorted Born Iterative Method (DBIM) in order to improve BIM by using Green's function updated each iteration [6]. DBIM has been proved to outperform DIM by its fast convergence. Unfortunately, when the scattering is strong, using this method can obtain a poor initial value that makes the time consuming become so high. Recently, the multi-frequency technique is applied in order to speed up the convergence of DBIM [7]. However, this technique requires large number of measurements because it needs a set of data for each corresponding frequency.

The interpolation technique is proposed in this paper for obtaining a quick and correct initial value. At first, the problem is solved using raw meshing (i.e. low resolution) to quickly obtain an average object function. This result is then used as the initial value for reconstruction of the object with dense mesh (i.e. standard resolution).

This paper is organized as following: Section 2 presents the theoretical foundations of first-order Born tomography and our proposed scheme. Consequently, the simulation illustrated the performance of this approach is present in Section 3. Lastly, section 4 is remarks and conclusions.

2. Material and Method

2.1 Distorted Born Iterative Method

We set up a measurement configuration of transmitters and receivers in order to obtain the scattered data (see Fig. 1). At an instance, only one transmitter and one receiver are active to obtain a corresponding measured data value. This data was processed using DBIM to reconstruct the sound contrast of scatters. In this way, we can detect if there is any tissue in this medium.

We assume that there is an infinite space containing homogeneous medium such as water whose background wave number is k_0. There is also an object with constant density and a wave number *k(r)* put inside this medium (see Fig. 2). Note that k(r) depends on the space. The wave equation of the system can be shown as:

$$p(\vec{r}) = p^{inc}(\vec{r}) + p^{sc}(\vec{r}) \quad (1)$$

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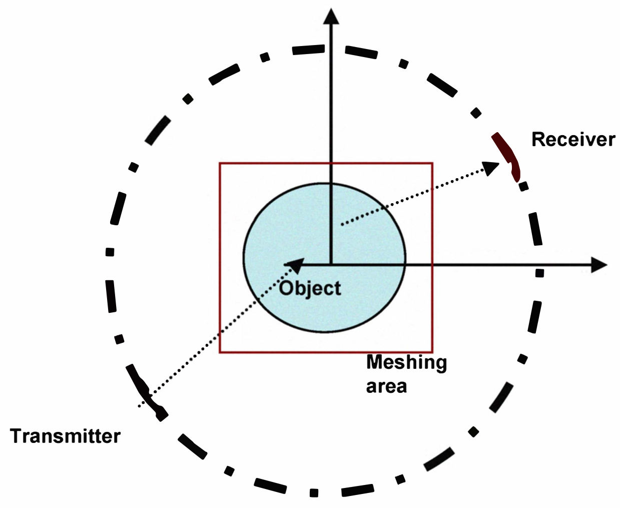

Figure 1. Geometrical and acoustical configuration.

Where $p(\vec{r})$, $p^{inc}(\vec{r})$, and $p^{sc}(\vec{r})$ are the total pressure, incident pressure and scattered pressure fields, respectively.

This equation can be expressed in detail:

$$p(\vec{r}) = p^{inc}(\vec{r}) + \iint O(\vec{r}')p(\vec{r}')G_0(|\vec{r}-\vec{r}'|)d\vec{r}' \quad (2)$$

Where $G_0(.)$ is the homogenous Green function, and $O(\vec{r}) = k(r)^2 - k_0^2$ is the object function need to be reconstructed from scattered data.

One of the effective solutions to solve the equation (1) by discreting is Method of Moment (MoM). The pressure in the grid points (see Fig. 1) can be computed in vector form with size $N^2 \times 1$:

$$\overline{p} = \left(\overline{I} - \overline{C}.D(\overline{O})\right)\overline{p}^{inc} \quad (3)$$

And the exterior points give scatter signal:

$$p^{sc} = \overline{B}.D(\overline{O})\overline{p} \quad (4)$$

Where $\overline{B}$ is the matrix with Green's coefficient $G_0(r,r')$ from each pixel to the receiver, $\overline{C}$ is the matrix with Green's coefficient $G_0(r,r')$ among all pixels, $\overline{I}$ is identity matrix, and D(.) is an operator that transform a vector into a diagonal matrix. The detail of calculation of $\overline{B}$ and $\overline{C}$ can be found in [10].

There are two unknown variables $\overline{p}$ and $\overline{O}$ in equations (3) and (4). In this case, the first Born approximation has been applied and the forward equation (3) and (4) can be rewritten:

$$\begin{aligned}\Delta p^{sc} &= \overline{B}.D(\overline{p})\overline{\Delta O} \\ &= \overline{M}.\Delta O\end{aligned} \quad (5)$$

Where $\overline{M}. = \overline{B}.D(\overline{p})$.

For each transmitter and receiver, we will have a matrix $\overline{M}$ and a scalar value Δp^{sc}. Realize that unknown vector $\overline{O}$ has N×N variables which are equal to the number of pixels in ROI. The object function can be computed by iterations:

$$\overline{O}^{(n)} = \overline{O}^{(n-1)} + \Delta\overline{O}^{(n-1)} \quad (6)$$

Where $\overline{O}^{(n)}$ and $\overline{O}^{(n-1)}$ are object functions at present and previous steps, respectively; $\Delta\overline{O}$ can be found by solving Tikhonov regularization problem:

$$\overline{\Delta O} = \arg\min_{\Delta O} \left\| \Delta\overline{p}^{sc} - \overline{M}_s \overline{\Delta O} \right\|_2^2 + \gamma \left\| \overline{\Delta O} \right\|_2^2 \quad (7)$$

Where $\Delta\overline{p}^{sc}$ is the $(N_tN_r \times 1)$ vector contains the difference between measured and predicted scattered ultrasound signals; $\overline{M}_s$ is system matrix $(N_tN_r \times N^2)$ formed by N_tN_r different matrixes $\overline{M}$; and γ is the regularization parameter that needs to be carefully selected [8] [9].

2.2 DBIM based on Interpolation

The proposed method consists of three parts. The first is reconstruction process with a raw meshed integration area with the size of $N_1 \times N_1$. We can easily obtain the convergent even after one or two iterations using DBIM. The result obtained at this part is the average back ground value of the object. In the second part, the interpolation is applied to the signal to obtain a dense image. Finally, the dense image is brought back to DBIM to continue the reconstruction process with the desired size of $N_2 \times N_2$.

Nearest neighbor is the simplest and fastest implementation of interpolation. There are various kinds of complex interpolation algorithms such are bilinear, bicubic, spline, etc. However, we only focus to nearest neighbor technique due to its time consumption. Figure 2 illustrate the principle of this method [11]. We have a reference image and using this image as an initiation to construct a new scaled image. Size of the constructed image will be determined by the scaling ratio. It can be smaller, larger, or equal in size. In this work, we want to enlarge an image, thus, we need some empty spaces in the original base picture. In Fig. 2, the image with dimension (4×4 pixels) is to be enlarged to (8×8 pixels). The black pixels represent empty spaces where this technique is needed for the complete picture. For the nearest neighbor technique, the empty spaces will be filled with the nearest neighboring pixel in the left side.

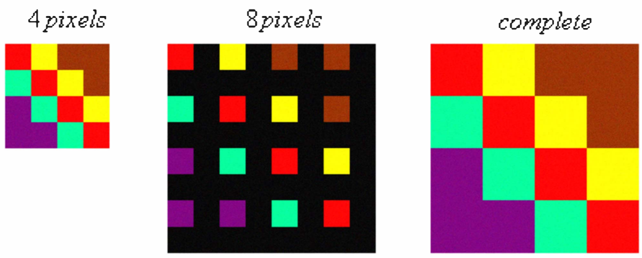

Figure 2. Ilustration of nearest neighbor image scaling [11]

The following algorithm summarizes the reconstructed process:

Algorithm 1. Modified DBIM

1: Choose an initial $\overline{O}_{N_1}{}^{(0)}$ and $\overline{p}^{(0)} = \overline{p}_{N_1}{}^{inc}$

2: While (n < N_{max1}) or (RRE_{N1} < ε_{N_1}) do

{

3: Calculate two matrices $\overline{B}_{N_1}$ and $\overline{C}_{N_1}$; $\overline{p}$ and p^{sc} correspond to $\overline{O}_{N_1}{}^{(n)}$ using equation (3) and (4).

4: Calculate the vector $\Delta\overline{p}^{sc}{}_{average}$

5: Calculate RRE correspond to $\overline{\Delta O}_{N_1}{}^{(n)}$ using (10)

6: Calculate a new $\overline{O}_{N_1}{}^{(n+1)}$ by using (5)

7: n=n+1; }

8: Interpolate $\overline{O}_{N_1}{}^{(n)}$ to obtain $\overline{O}_{N_2}{}^{(0)}$

9: Init $\overline{p}^{(0)} = \overline{p}_{N_2}{}^{inc}$, n=0

10: While (n < N_{max2}) or (RRE_{N2} < ε_{N_2}) do

{

11: Calculate two matrices $\overline{B}_{N_2}$ and $\overline{C}_{N_2}$; $\overline{p}$ and p^{sc} correspond to $\overline{O}_{N_2}{}^{(n)}$ using equation (3) and (4).

12: Calculate the vector $\Delta\overline{p}^{sc}{}_{average}$

13: Calculate RRE correspond to $\overline{\Delta O}_{N_2}{}^{(n)}$ using (10)

14: Calculate a new $\overline{O}_{N_2}{}^{(n+1)}$ by using (5)

15: n=n+1;

}

Where N_{max1} and N_{max2} are the maximum numbers of iterations, ε_{N_1} and ε_{N_2} are stopping errors determined by noise floor [12], RRE_{N1} and RRE_{N2} are relative residual errors [10].

3. Simulation and Results

The modified DBIM reconstruction has been tested by numerical simulation. The table below summarizes the scenario's simulation.

Table 1. Simulation parameters

Parameters	Values
Frequency of ultrasound signal	F=1 Mhz
Number of pixels	$N_1 \times N_1, N_1=6$ $N_2 \times N_2, N_2=12$
Number of transmitters	$N_t=24$
Number of receivers	$N_r=12$
The radius of the cylinder	$R=5\lambda$
Speed of sound contrast	1%

We assume that there is a cylindrical tissue in the center of the medium. The ideal object function that can be expressed as:

$$O(\vec{r}) = \begin{cases} \omega^2\left(\frac{1}{c_1^2} - \frac{1}{c_0^2}\right) & if \ |\vec{r}| \le R \\ 0 \ \ if \ |\vec{r}| > R \end{cases} \tag{8}$$

Where ω is radial frequency, $k_0 = \frac{\omega}{c_0}$ is the wave number in reference medium, c_1 is the speed of the sound in the object (cylinder here) and R is radius of the object.

The incident field for a Bessel beam of zero order in 2-D is given by:

$$p^{inc}(\vec{r}) = J_0\left(k_0\left|\vec{r} - \vec{r_k}\right|\right) \tag{9}$$

Where J_0 is the 0^{th} – order Bessel function and $\left|\vec{r} - \vec{r_k}\right|$ is the distance between the transmitter and the k^{th} point in the ROI.

The simulated results are showed in Fig. 3. Fig. 3e is the ideal object function at the desired scale. It can be seen that the object is in the center of the medium. Fig. 3a is the 6×6 pixels scale DBIM reconstruction only after single iteration. Under low resolution, the average of desired object function has converged quickly. Fig. 3b is 12×12 pixel scale DBIM reconstruction after another iteration utilizing interpolation results from (a). We can that it take only two DBIM iteration to complete the reconstruction.

It is necessary to compare our proposed scheme with conventional DBIM. Figs. 3c and 3d show the 12×12 pixel tomography after the first and fourth iterations using conventional DBIM (i.e. without interpolation). It is obvious that the conventional method takes four DBIM iterations but the reconstructed quality is still not as good as our method.

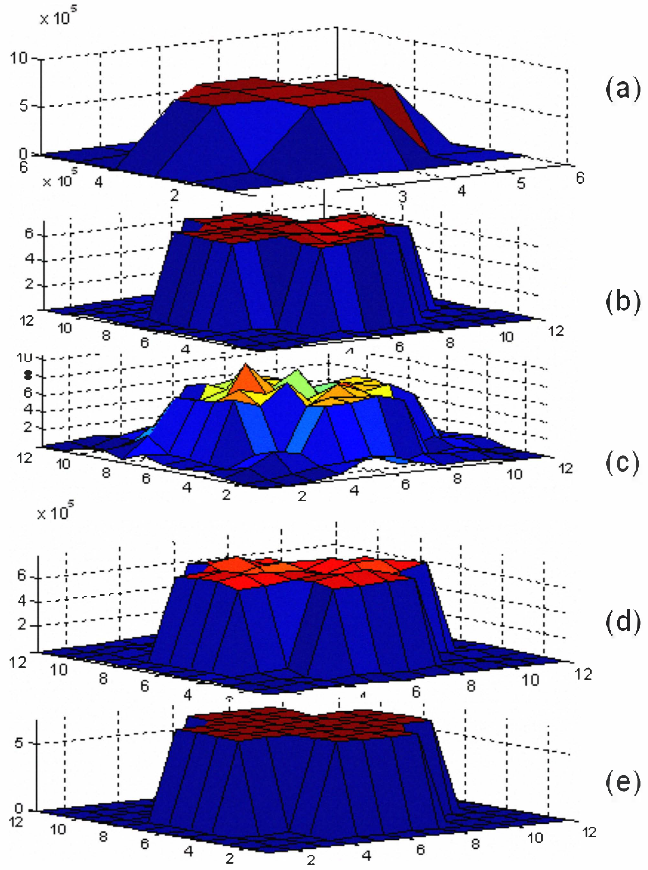

Figure 3. DBIM reconstruction results: (a) 6×6 pixel tomography after one iteration; (b) 12×12 pixel tomography after only one iteration, using interpolation from (a); (c) 12×12 pixel tomography after first iteration, without interpolation; (d) 12×12 pixel tomography after four iterations, without interpolation; (e) 12×12 pixel tomography in perfect reconstruction.

4. CONCLUSIONS

This paper has successful in applying interpolation technique in order to speed up the reconstructed process of sound contrast using DBIM. A simple simulation scenario of sound contrast reconstruction has been performed to prove the effectiveness of this method. The simulated results showed that not only the time consuming is save up to twice, but also the reconstructed quality is ensured. The next step of this work is verified this proposed scheme with the experimented data in order to apply in real-time medical imaging.

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