# Combined Iterative Channel Estimation and Data Detection for Space-Time Block Codes

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***Abstract*—This paper considers the problem of channel estimation for space-time block code (STBC) systems with convolutional codes for error correction. We propose an iterative processing scheme which combines channel estimation and data detection to improve system performance and increase transmission efficiency. In the proposed scheme, pilot symbols are used for initial estimation of channel gains and data detection. The information bits detected during the data period are fed back to the channel estimator for re-estimating the channels and improving data detection. The proposed scheme allows to improve the BER performance and decrease the pilot length to 50% at the cost of only 2 processing iterations.**

## I. INTRODUCTION

Multiple antenna transmission has been known as an effective scheme to increase channel capacity of wireless communications systems. Space-time block codes such as the Alamouti scheme [1] with two transmit antennas are now included in various wireless system standards. Examples are Wimax (Worldwide Interoperability for Microwave Access) and the third generation of mobile communications such as UTMS (Universal Terrestrial Mobile Systems). For full exploitation of the STBC performance, it is required that the receiver have full knowledge about the MIMO (multiple input multiple output) channels between transmitter and receiver.

The problem of the MIMO channel estimation is thus important and has been a topic for various researches [2]–[4]. In [3], the authors proposed an alternating pilot-symbol assisted (PSA) channel estimator which requires $N$ times larger number of pilot symbols for an STBC with $N$ transmit antennas. In a recent work [5], we proposed a simple channel estimator for the Alamouti's STBC which requires only simple temporal average operation instead of an interpolation filter. We have also shown that at least 16 pilot symbols are required to achieve a similar BER of an ideal system with perfect channel state information at the receiver. Recently, Kulakowski and Ludwin [4] proposed an iterative channel estimation (ICE) scheme for an STBC system under fast varying channels. The idea of the proposed ICE scheme is to use the previously decoded data symbols as a training sequence to re-estimate the channel gain. The simulation results of the paper show that for the pilot length of 10% of the transmission time, about 5 to 10 blocks of training sequences are required to achieve BER performance which is approximately 1-2 dB worse than the perfect CSI case at moderate velocity.

Our approach in this paper is similar to the ICE proposed by [4]. However, the system under consideration is working in a quasi-static channel which means that all the channels are invariant within a data block. Another difference is that our proposed system uses convolutional code for forward error correction (FEC) while the system in [4] is uncoded. The combination of iterative channel estimation and FEC codes helps to estimate channels more correctly in the later iterations and thus allows to achieve better BER performance. The objective of our proposal is to sacrifice several processing iterations to reduce the pilot length so that the transmission efficiency is increased. In our computer simulation experiments the proposed scheme has demonstrated significant improvement in BER performance and transmission efficiency for the cost of only 2 additional processing iterations.

The remainder of the paper is organized as follows. The signal model and detection algorithm of the Alamouti's STBC systems are summarized in Sect.II. Section III presents the conventional channel estimation algorithm for the Alamouti's STBC system. Our proposed combined channel estimation and data detection is presented in Sect.IV. Simulation results are shown in Sect. V, and, finally, conclusions are drawn in Sect. VI.

## II. SIGNAL MODEL

We consider a wireless communications system employing the Alamouti's space-time block code with two transmit antennas and one receive antenna as illustrated in Fig. 1. In our system forward error correction (FEC) codes such as the convolutional code are used in the transmitter as in the contemporary wireless communications systems. At the transmitter, a block of $L_b$ information bits $\boldsymbol{b}_i$ is first encoded using a convolutional code to generate a codeword $\boldsymbol{c}_i$ of length $L_c$. This codeword $\boldsymbol{c}_i$ is then modulated using an $M$-PSK modulation scheme to create $\log_2 M$-bit transmit symbols $s_k$. Next, each pair of symbols $\{s_k, s_{k+1}\}$ are space-time block encoded using the Alamouti STBC according to the following encoding rule

$$\boldsymbol{X}_k = \begin{bmatrix} s_k & s_{k+1} \\ -s_{k+1}^* & s_k^* \end{bmatrix} \qquad (1)$$

where the notation * denotes the complex conjugation.

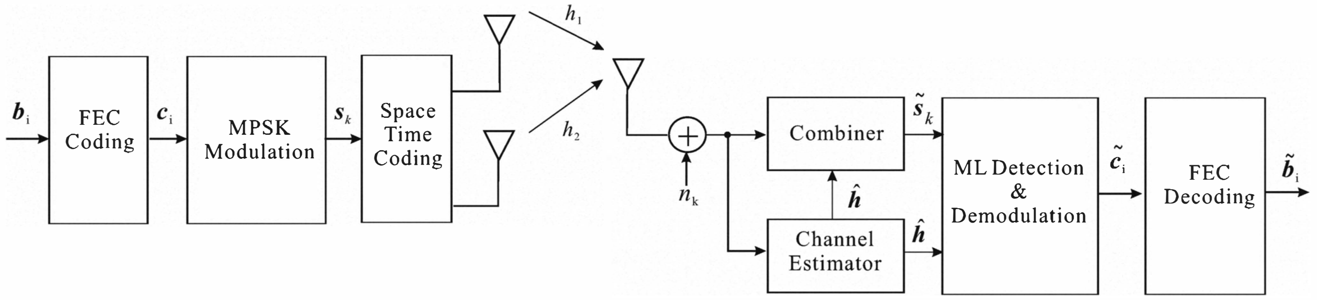

Fig. 1. Configuration of the 2 × 1 Alamouti's STBC system under consideration.

The channel gain between each pair of the receive and transmit antennas $h_n, n = 1, 2$, is assumed quasi-static and independent and identically distributed (iid). It is common to model $h_n$ using a complex Gaussian variable with zero mean and unit variance, i.e., $h_n \in \mathcal{N}_c(0, 1)$, for a Rayleigh fading channel. The additive white Gaussian noise (AWGN) sample $z_k$ at time slot $k$ is assumed iid and modeled using a Gaussian random variable with zero mean and variance $\sigma_z^2$, i.e., $z_k \in \mathcal{N}_c(0, \sigma_z^2)$.

The receive symbols at the receive antenna at two consecutive time slots are expressed as

$$y_k = h_1 s_k + h_2 s_{k+1} + z_k \tag{2}$$

$$y_{k+1} = -h_1 s_{k+1}^* + h_2 s_k^* + z_{k+1}. \tag{3}$$

It is easy to convert the above equations into the vector form as

$$\boldsymbol{y}_k = \boldsymbol{H} \boldsymbol{s}_k + \boldsymbol{z}_k \tag{4}$$

where

$$\boldsymbol{s}_k = \left[s_k, s_{k+1}\right]^T \tag{5}$$

$$\boldsymbol{y}_k = \left[y_k, y_{k+1}^*\right]^T \tag{6}$$

$$\boldsymbol{z}_k = \left[z_k, z_{k+1}^*\right]^T \tag{7}$$

$$\boldsymbol{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}. \tag{8}$$

In order to estimate the transmit vector $\boldsymbol{s}_k$, it is required that the channel gains $h_1$ and $h_2$ be correctly estimated. Given the correct estimates of the channel gains, the transmit vector can be estimated by linearly combining the receive vector $\boldsymbol{y}_k$ with the channel matrix as

$$\tilde{\boldsymbol{s}}_k = \boldsymbol{H}^H \boldsymbol{y}_k = (|h_1|^2 + |h_2|^2)\boldsymbol{s}_k + \boldsymbol{H}^H \boldsymbol{z}_k \tag{9}$$

where $\tilde{\boldsymbol{s}}_k = [\tilde{s}_k, \tilde{s}_{k+1}]^T$.

The estimated symbols $\tilde{\boldsymbol{s}}_k$ is demodulated to receive the estimated codeword $\tilde{c}_i$, and then decoded using the Viterbi decoder to correct the erroneous bits and obtain the information bits $\tilde{\boldsymbol{b}}_i$

## III. CHANNEL ESTIMATION FOR ALAMOUTI' STBC

As summarized in the previous work [5], there are two common methods for channel estimation, namely, decision-directed channel estimation and pilot-symbol-assisted (PSA) channel estimation. The main difference between the two types is that pilot symbols are required in the PSA but not in the decision-directed method. It is also known that the PSA channel estimation method is preferably employed in practical wireless systems. One typical example is the GSM (Global System for Mobile Communications) in which 26 symbols are inserted in each data frame of 148 symbols for channel estimation [6]. Our main interest in this work is the PSA channel estimation with data frames format illustrated in Fig. 2. The pilot symbols, which are known at both the receiver and the transmitter, are used at the receiver to extract the temporal estimates of the channel parameters. These temporal estimates are then filtered to provide estimates of the channel gains. Under quasi-static fading assumption, the channels are not changed during one transmit frame duration. Therefore, the channel gains estimated during the pilot period can be used to estimate the transmitted data symbols during the data transmission period. In the previous work [5], we proposed a simple PSA channel estimation method for the Alamouti's STBC in which the temporal average operation is used in stead of an interpolation filter. The proposed method is briefly explained below.

Let us define the transmit pilot matrix during the two time slots $t$ and $t + 1$ of the training period as

$$\boldsymbol{P}_t = \begin{bmatrix} p_t & p_{t+1} \\ -p_{t+1}^* & p_t^* \end{bmatrix}. \tag{10}$$

Due to the orthogonality of the STBC, we have $\boldsymbol{P}_t \boldsymbol{P}_t^H = \boldsymbol{I}_2$. Note that the resultant identity matrix is due to the fact that the transmit power is normalized to one. Define the channel vector $\boldsymbol{h}_t = [h_1, h_2]^T$ and the noise vector $\boldsymbol{n}_t = [n_t, n_{t+1}]^T$, the receive vector $\boldsymbol{r}_t = [r_t, r_{t+1}]^T$ during the training period is given by

$$\boldsymbol{r}_t = \boldsymbol{h}_t \boldsymbol{P}_t + \boldsymbol{n}_t. \tag{11}$$

Performing right multiplication of both sides of the above equation with $\boldsymbol{P}_t^H$ gives

$$\tilde{\boldsymbol{h}}_t = \boldsymbol{r}_t \boldsymbol{P}_t^H = \boldsymbol{h} + \boldsymbol{n}_t \boldsymbol{P}_t^H. \tag{12}$$

The estimate of the channel vector $\tilde{\boldsymbol{h}} = [\tilde{h}_1, \tilde{h}_2]^T$ is obtained by averaging $\tilde{\boldsymbol{h}}_t$ over the $L_p$ pilot symbols, i.e.

$$\tilde{\boldsymbol{h}} = \frac{1}{2L_p} \operatorname{sum}(\tilde{\boldsymbol{h}}_t, 2) \tag{13}$$

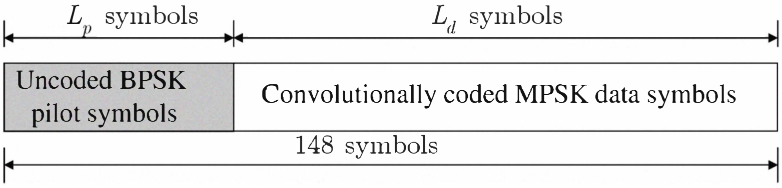

Fig. 2. Data frame structure

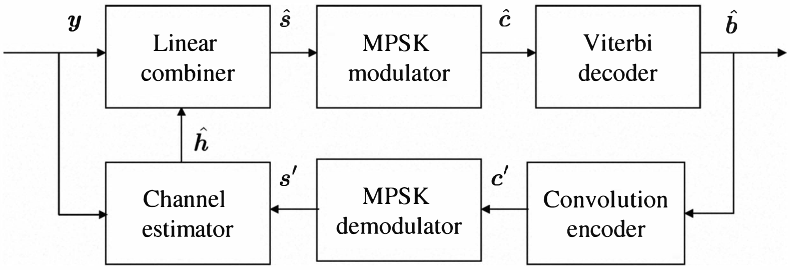

Fig. 3. Iterative receiver for channel estimation and data detection

where the index 2 in the sum function denotes the row-wise summation.

We have shown in the previous work [5] that for an Alamouti's STBC system without FEC codes at least 16 pilot symbols are required to obtain BER performance which is 0.5 dB worse than the case with perfect CSI. Applying to the GSM case, for example, this requirement accounts for $16/148 = 11\%$ reduction in the transmission efficiency. This reduction results in significant waste of bandwidth and reduces the data transmission rate. Clearly, a more efficient channel estimation method with less pilot symbols is essential for high-rate data transmission systems.

## IV. COMBINED ITERATIVE CHANNEL ESTIMATION AND DATA DETECTION

With the aim to reduce the waste of bandwidth due to a long pilot period, we propose to use a combined scheme of iterative channel estimation and data detection. Configuration of the receiver with iterative processing is illustrated in Fig. 3. It is worth noting that although the idea of the iterative processing is similar to that of [4], the target application and system features are different. The objective of using iterative processing of [4] is to estimate the time-varying channels while ours is to improve the estimates of channel gains and shorten the pilot length. The main features of our system include

- Error correction codes are used during the data period but not pilot period. The use of error correction codes after space-time decoding allows to obtain better decoded information bits. In the iterative channel estimation decoded bits in the previous iteration will server as the pilot bits for re-estimating the channels in the next iteration. Clearly, better decoded bits means better estimates of the channel gains. It is worth noting that error correct codes such as the Reed-Solomon, convolutional, turbo or LDPC codes are already standardized in contemporary wireless communications systems such as GSM or UMTS. Therefore, the proposed iterative channel estimation method in fact does not require further bandwidth expansion. The reason for not using the error correction codes during the pilot period is because the objective of our research is to reduce the pilot length and it is apparent that error correction codes are not effective for a short pilot period of only several symbols.
- Simple BPSK modulation is used during the pilot period instead of higher-order modulation schemes. The use of BPSK modulation allows to maintain full bit energy, which helps to improve the correct estimates of channels. This again helps to reduce the number of required processing iteration.

The iterative processing algorithm is explained as follows.

1. During the pilot period the algorithm uses our previously proposed channel estimation method presented in Sect. III to estimate the channel vector $\tilde{\boldsymbol{h}}_t$ using equation (12) and (13).
2. Having obtained the estimated channel vector $\tilde{\boldsymbol{h}}_t$, the channel matrix

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \tilde{h}_1 & \tilde{h}_2 \\ \tilde{h}_2^* & -\tilde{h}_1^* \end{bmatrix} \tag{14}$$

is formed and the transmit vector $\tilde{\boldsymbol{s}}_k$ can be estimated using (9) as

$$\tilde{\boldsymbol{s}_k} = \boldsymbol{H}^H \boldsymbol{y}_k. \tag{15}$$

The estimated vector $\tilde{\boldsymbol{s}_k}$ will be demodulated and then decoded using the Viterbi decoder to obtain the $L_b$ information bits $\tilde{\boldsymbol{b}}_i$.

TABLE I

ESTIMATED CHANNEL GAINS DURING PROCESSING ITERATIONS.

| | **Eb/No=0dB (4 pilot symbols)** | **Eb/No=10dB (4 pilot symbols)** | **Eb/No=0dB (16 pilot symbols)** | **Eb/No=10dB (16 pilot symbols)** |
|---|---|---|---|---|
| Channel gains $\boldsymbol{h}$ | -0.8636 - 0.7118i<br>0.3243 - 0.0464i | 0.3713 – 0.6968i<br>0.5804 + 0.6088i | -0.6046 - 1.2156i<br>0.1998 - 0.0403i | 0.1703 + 0.7116i<br>1.2120 + 0.2516i |
| Pilot symbol estimation | -0.5890 - 0.5552i<br>0.0827 - 0.3718i | 0.3144 – 0.8763i<br>0.6037 + 0.7049i | -0.6390 - 1.3133i<br>0.1651 - 0.1737i | 0.1827 + 0.6940i<br>1.2219 + 0.2631i |
| After 1st iteration estimation | -0.4703 - 0.4250i<br>0.0791 - 0.1696i | **0.3596 – 0.6951i**<br>**0.5835 + 0.6289i** | -0.5741 - 1.0071i<br>0.1141 - 0.0546i | **0.1594 + 0.7011i**<br>**1.2138 + 0.2319i** |
| After 2nd iteration estimation | -0.4326 - 0.3280i<br>0.1520 - 0.0749i | **0.3596 – 0.6951i**<br>**0.5835 + 0.6289i** | -0.4976 - 0.6814i<br>-0.0551 - 0.0331i | **0.1594 + 0.7011i**<br>**1.2138 + 0.2319i** |

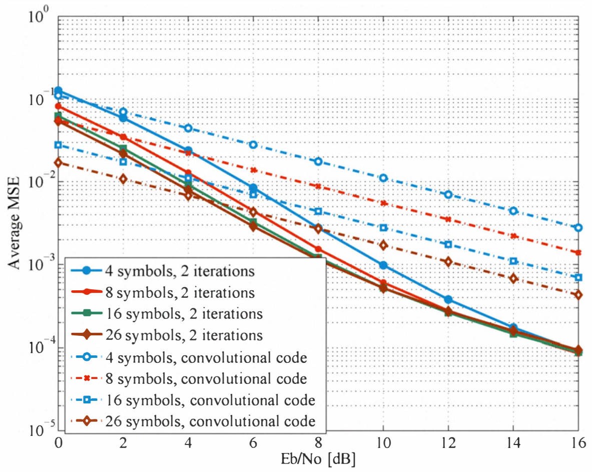

Fig. 4. MSE performance of a 2 × 1 Alamouti's STBC system with the proposed iterative processing for 2 iterations and different pilot symbol lengths.

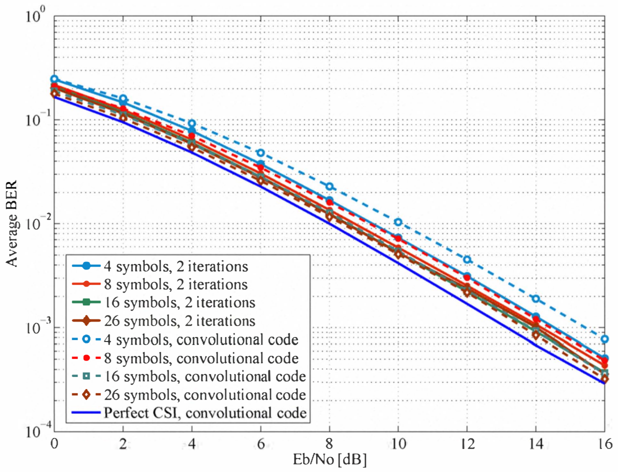

Fig. 5. BER performance of a 2 × 1 Alamouti's STBC system with the proposed iterative processing for 2 iterations and different pilot symbol lengths.

3. Start iterative process of joined channel estimation and data detection. The decoded bit $\tilde{\boldsymbol{b}}_i$ obtained in step 2 are now served as the pilot bits. They will pass through a process similar in the transmitter from being encoded using convolutional codes, modulated using MPSK, and STBC encoded to obtain $\boldsymbol{s}'$ as illustrated in Fig. 3. Applying $\boldsymbol{s}'$ to the received vector $\boldsymbol{y}$ during the data period to re-estimate the channel vector $\hat{\boldsymbol{h}}$ as in step 1. Again $\hat{\boldsymbol{h}}$ will be used to re-detect the transmit information bits $\tilde{\boldsymbol{b}}_i$.
4. Repeat this iterative process until the required BER has been met.

## V. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed algorithm, we have carried out a Monte-Carlo simulation for a $2 \times 1$ Alamouti's STBC system. The channel gains between the receive and transmit antennas $h_1$ and $h_2$ are modeled using two iid complex Gaussian variables with mean zero and unit variance. The data frame format is the same as shown in Fig. 2 with $L_p$ pilot symbols and $L_d$ data symbols making a 148 symbol frame. The $L_p$ pilot symbols are modulated using BPSK while the $L_d$ data symbols are modulated using 8-PSK. The error correction code used for simulation is the rate $1/2$ convolutional code with constrain length of 7 and code generator (171,133) available in Matlab.

Table I shows an example of the channel gains simulated by Matlab and estimated using the pilot symbol estimation and iterative processing after one and two iterations. The first important observation is that iterative processing at high $E_b/N_0$ helps to obtain better estimated channels. After one iteration the estimated channel gains become closer to those of the simulated channels. This is true for both the case of short (4 symbols) and long (16 symbols) pilot period. However, it is interesting to note that iterative processing is only effective for the first iteration. Running further iterations do not help to increase the estimation accuracy. The reason for the convergence after one iteration can be explained by using the fact that the hard decision from the Viterbi decoder was used for iterative processing. Since the Viterbi decoder performs well at high $E_b/N_0$, it is clear that the estimation process will converge after some iterations. The actual number of iterations after which the estimation converges depends on the $E_b/N_0$ value. In our experiment at the selected $E_b/N_0 = 10$dB, the estimation process converges after one iteration for both the case of 4 and 16 pilot symbols. Experiments to find out the number of iterations and corresponding values of $E_b/N_0$ for estimation convergence have not been performed yet. Another observation is that at low $E_b/N_0$ iterative processing would even cause the estimates worse. Examples can be seen at $E_b/N_0 = 0$dB for both the case of 4 and 16 pilot symbols. The reason for this tendency is because the Viterbi decoder does not perform well at low $E_b/N_0$, and this causes error propagation in the further iteration.

In order to justify the above observations, we have run Monte-Carlo simulations to obtain BER and MSE performance. Figures 4 and 5 compare the BER and MSE performance obtained for a system without using convolutional code and iterative processing. It can be clearly seen that the proposed scheme improves the performance significantly. More importantly, for the case of the convolutional code at least 16 pilot symbols are required to achieve BER close to the perfect case while our scheme needs only 8 pilot symbols and 2 iterations[1]. This allows to shorten the pilot length to 50% and results in increased transmission efficiency from 89% to 95%. Further iterations have also been tried but no further improvement was realized and thus the results were no shown in the figures. This improvement in both BER performance and transmission efficiency proves the effectiveness of our proposed scheme.

## VI. CONCLUSION

In this paper we have proposed a combined iterative channel estimation and data detection scheme for the Alamouti's STBC system. The proposed scheme allows to improve the BER performance and decrease the pilot length by 50%. Although the results were shown only for the case of a 2 × 1 MISO (multiple input multiple output) system, the method can be easily extended to the case of MIMO systems.

## ACKNOWLEDGMENT

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[1] Please note 2 data detection iterations contain only one channel estimation iteration.