MSE Based Antenna Selection for MIMO-SDM Systems

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***Abstract*—Antenna selection is a promising technique which provides performance improvement for multiple input multiple output (MIMO) systems at low cost of additionally employed antennas and signal processing. Various antenna selection algorithms based on different criteria have been proposed in the literature. This paper presents a new antenna selection algorithm for MIMO spatial division multiplexing (MIMO-SDM) systems which uses the mean square error (MSE) as a criterion for selecting the best subset of transmit/receive antennas to improve the system error performance. The proposed algorithm helps to achieve lower the bit error rate (BER) for MIMO-SDM systems with linear detectors compared with the previous channel matrix norm based and signal to noise ratio (SNR) based selection algorithms. Compared with the SNR based algorithm, the proposed algorithm provides double diversity gain in receive antenna selection systems and the same diversity order but about 1dB SNR gain in transmit antenna selection systems.**

I. Introduction

Multiple input multiple output (MIMO) transmission is a promising technique which helps to achieve the system capacity linearly increased with the number of employed antennas [1]. Spatial division multiplexing (SDM) is one type of MIMO systems which aims at increasing the spectral efficiency of the channel [2]. At the transmitter, the transmit data symbols are demultiplexed into parallel streams and transmitted simultaneously over transmit antennas. This encoding scheme is simple at the transmitter but burdens complexity on the receiver for resolving the co-channel interference (CCI) problem among the transmit parallel streams. Various signal detection schemes have been proposed for MIMO-SDM [2]–[5]. The maximum likelihood detectors such as that proposed in [3] are optimum in minimizing bit error rate (BER) but often too complex to be implemented. Other detectors combine linear detection with either successive interference cancelation [2], lattice reduction [4], or maximum likelihood detection [5] to reduce detection complexity while achieving suboptimal error performance.

Another trend of improving error performance for MIMO-SDM systems is to combine linear detection with antenna selection which attracted significant attention recently [4],[7]–[9]. The idea of antenna selection was first proposed for receive diversity selection combining systems [10] and then extended to MIMO systems [7]. It is based on the assumption that fading is statistically independent between antenna elements. The concept of antenna selection for MIMO-SDM is to choose from the set P of all possible subsets of antennas an optimal subset for transmission. The size of the selected antenna subset depends on the number radio frequency (RF) chains used in the transmitter or the receiver. Antenna selection algorithms can be classified into the capacity based and error performance based algorithms. It was shown in [7] that the error performance based algorithm outperforms the capacity based algorithm. Also shown in [7] is that the signal to noise (SNR) based algorithm is the best transmit antenna selection approach in terms of minimizing bit error rate (BER). This algorithm, however, does not take into account the effect of co-channel interference (CCI) among transmit streams and thus does not guarantee the best error performance. Motivated by this fact, in this paper we propose a mean square error (MSE) based antenna selection algorithm which combines the effect of CCI for antenna selection. Simulation results show that our proposed algorithm provides better BER performance compared to the previous channel gain based and SNR based algorithm.

The remainder of the paper is organized as follows. The signal model for MIMO-SDM with antenna selection is presented in Sect.II. Previous antenna selection algorithms based on channel norm and SNR are summarized in Sect. III. Sect. IV presents our proposed algorithm. Simulation results and comparative analysis are given in Sect. V. Finally, Sec. VI concludes the paper.

II. Signal Model

We consider a MIMO-SDM system with N_t transmit antennas and M_r receive antennas as illustrated in Fig. 1. There are $N < N_t$ and $M < M_r$ RF chains at the transmitter and receiver, respectively. The MIMO channel between the transmitter and receiver is described by an $M_r \times N_t$ complex matrix $\boldsymbol{H}$ whose elements are modeled using independent and identically distributed (iid) Gaussian random variables with zero mean and unit variance, i.e. $h_{mn} \sim \mathcal{N}_c(0,1)$. It is also assumed that the channel is non-frequency selective and varies slowly. Although both transmit and receive antenna selection are illustrated in the system, we only consider the case in which either transmit or receive antennas are selected.

The principle of the system is described as follows. At the transmit side, the transmit data symbols s_n are demultiplexed

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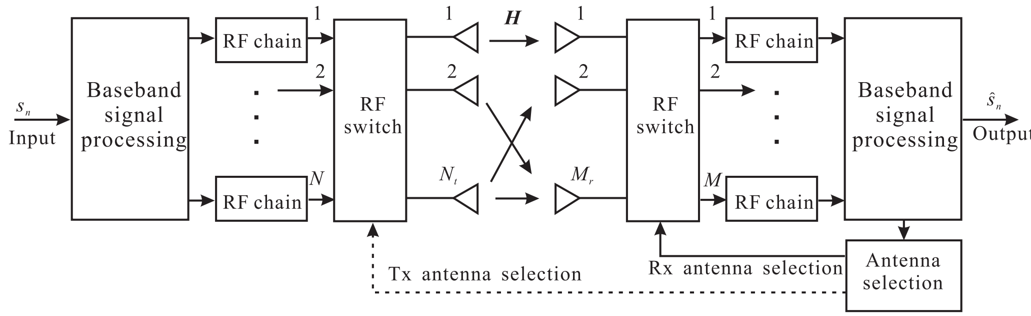

Fig. 1. A MIMO-SDM system with antenna selection

into N parallel streams. These streams are up-converted into the carrier frequency using N RF chains and then transmitted through N selected transmit antennas. The selection of the N out of N_t antennas for transmission is done using the feedback information from the receive side. At the receive side, M out of M_r antennas are selected to receive the transmitted signals and feed them to the M RF chains. The transmit and receive antenna selection algorithm is main topic of this paper and will be presented the following section.

Let $\boldsymbol{s}$ denote the transmit vector and $\boldsymbol{H}_p$ the selected channel matrix corresponding to the selected N transmit and M received antennas. The received signal vector is

$$\boldsymbol{y} = \sqrt{\frac{E_s}{N}}\boldsymbol{H}_p\boldsymbol{s} + \boldsymbol{z} \tag{1}$$

where $\boldsymbol{z}$ is the additive white Gaussian noise (AWGN) vector at the input of the receive RF chains. Each sample z_m of $\boldsymbol{z}$ is assumed iid and modeled using a Gaussian random variable with zero mean and variance N_0, i.e., $z_m \in \mathcal{N}_c(0, N_0)$. The square root term means the total transmit power is normalized to the number of transmit RF chains. The received signal vector can be then detected using various detection algorithms such as the linear zero-forcing (ZF) or the minimum mean square error (MMSE) detector, the combined linear detection and successive interference cancelation (SIC) [2], the combined MMSE and maximum likelihood (ML) detection [5], or the combined MMSE and lattice reduction aided (LRA) detection [4]. In this paper, in order to demonstrate the performance of antenna selection algorithms we will consider only the case of the linear ZF and MMSE detectors.

For the linear detectors the linear combining weight matrix $\boldsymbol{W}$ is given by

$$\boldsymbol{W}^{\text{ZF}} = \boldsymbol{H}_p\left(\boldsymbol{H}_p^H\boldsymbol{H}_p\right)^{-1} \tag{2}$$

for ZF detectors and

$$\boldsymbol{W}^{\text{MMSE}} = \left(\boldsymbol{H}_p\boldsymbol{\Lambda}\boldsymbol{H}_p^H + N_0\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_p\boldsymbol{\Lambda}, \tag{3}$$

for MMSE. Here $\boldsymbol{I}_k$ denotes a $k \times k$ identity matrix and $\boldsymbol{\Lambda} = \frac{E_s}{N}\boldsymbol{I}_N$ is the power matrix. The estimates of $\boldsymbol{s}$ are given by $\hat{\boldsymbol{s}} = Q\{\boldsymbol{W}^H\boldsymbol{y}\}$, where $Q\{\bullet\}$ denotes the quantization operation.

III. PREVIOUS ANTENNA SELECTION ALGORITHMS

In this section we summarize previous antenna selection algorithms and introduce our algorithm based on MSE. Our described algorithms here are mainly for the receive side and so the number of transmit antenna is fixed to N. The presentation for the transmit side is similar with the number of receive antennas fixed to M.

A. Norm-based selection algorithm

The most basic algorithm for antenna selection is to choose the optimal subset p from the set P of all $\binom{M_r}{M}$ possible subsets of receive antennas. The norm-based algorithm involves choosing the antenna subset which gives the maximum equivalent channel gain [6]

$$h_{eq} = \frac{1}{N}\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}|h_{mn}|^2} \tag{4}$$

Choosing this equivalent channel requires a large number of computation and an alternative approach with the same performance is to calculate the norm of channel vectors as

$$\hat{h}_{eq}(m) = \sqrt{\sum_{m=1}^{M}|\boldsymbol{h}_m|^2} \tag{5}$$

where $\hat{h}_{eq}(m)$ is the channel gain corresponding to the mth antenna. The algorithm then selects M receive antennas corresponding to the M largest values.

B. SNR-based selection algorithm

The SNR-based antenna selection algorithm was proposed in [7] together with other singular value and capacity based algorithms. It was clear shown in [7] that the SNR-based algorithm provides the best BER performance among the three algorithms and in this paper we are only interested in this algorithm. The algorithm makes selection based on the post processing SNR expressed as

$$\text{SNR}_n = \frac{E_s\left\|\boldsymbol{w}_n^H\boldsymbol{h}_n\right\|^2}{NN_0\left\|\boldsymbol{w}_n\right\|^2}. \tag{6}$$

For the MMSE detector the SNR is given by [7]

$$\mathrm{SNR}_n^{(\mathrm{MMSE})} = \frac{E_s}{NN_0\left[\boldsymbol{H}_p\boldsymbol{H}_p^H + N_0/E_s\boldsymbol{I}_M\right]_{nn}^{-1}} \tag{7}$$

and

$$\mathrm{SNR}_n^{(\mathrm{ZF})} = \frac{E_s}{NN_0\left[\boldsymbol{H}_p^H\boldsymbol{H}_p\right]_{nn}^{-1}}. \tag{8}$$

for the ZF detector.

Using the singular value decomposition, the minimum SNR was shown to be lower bounded by [7]

$$\mathrm{SNR}_{min}^{(\mathrm{ZF})} \geq \lambda_{min}^2(\boldsymbol{H_p})\frac{E_s}{NN_0} \tag{9}$$

where $\lambda_{min}^2(\boldsymbol{H}_p)$ expresses the minimum singular value of the candidate matrix $\boldsymbol{H}_p$. The expression of (9) leads to the conclusion that the performance of linear receivers improves as the minimum singular value increases. Therefore, the algorithm is for every subset of the receive antennas $p \in P$ compute the weight matrix $\boldsymbol{W}$ and the corresponding SNR_{min} for the desired linear detector, select the subset with the largest SNR_{min}.

IV. Proposed MSE-based Antenna Selection Algorithm

A. Motivation

Having studied the above presented two selection algorithms, we have the following observations. First, the norm based algorithm makes selection based on only the channel gains. The effect of noise at each receive RF branch and CCI among transmit streams are not used for selection. Intuitively, the algorithm cannot provide the best performance. The expected improvement is only SNR gain due to a better selected subset of channels. No diversity gain is expected for the norm based algorithm. Second, for the SNR based algorithm, although the effect of noise is used for selection, the effect of CCI is still neglected. Thus the algorithm is also not able to give the best performance. The expected gain in this case depends on the implementation location. For the case of transmit selection, the successful choice of the subset of transmit antennas with the highest SNRs allows reduction in CCI among transmit streams and helps to improve the diversity gain. When implemented at the receiver, the successfully selected subset of receive antennas allows better signal combination, which provides some SNR gain. These two observations pave the way for new selection algorithms which take into account both the effect of noise and CCI. Motivated from this fact, we propose a new selection algorithm based on MSE. Data streams with minimum MSE will be better detected, i.e, with less bit error. Thus the algorithm will provide improved diversity gain in both transmit and receive selection mode. Detailed description of the algorithm is presented in the following.

B. Algorithm

The mean square error of the linear detector is given by

$$\begin{aligned}\mathrm{MSE} &= \mathrm{E}\{\|\Delta s\|^2\} = \mathrm{E}\left\{\|\boldsymbol{s} - \boldsymbol{W}^H\boldsymbol{y}\|^2\right\} \\ &= \frac{1}{N}\mathrm{trace}\left(\boldsymbol{R}_{\Delta s}\right)\end{aligned} \tag{10}$$

where $\mathrm{E}\{\cdot\}$ denotes the ensemble average, and $\boldsymbol{R}_{\Delta s}$ is the error covariance matrix. After some mathematical manipulations we have

$$\boldsymbol{R}_{\Delta s} = \boldsymbol{\Lambda} - \boldsymbol{R}_{ys}^H\boldsymbol{W} - \boldsymbol{W}^H\boldsymbol{R}_{ys} + \boldsymbol{W}^H\boldsymbol{R}_{yy}\boldsymbol{W} \tag{11}$$

where $\boldsymbol{R}_{ys} = \mathrm{E}\{\boldsymbol{y}\boldsymbol{s}^H\} = \boldsymbol{H}\boldsymbol{\Lambda}$ is the cross-correlation matrix between the receive and transmit vector and $\boldsymbol{R}_{yy} = \mathrm{E}\{\boldsymbol{y}\boldsymbol{y}^H\} = \boldsymbol{H}\boldsymbol{\Lambda}\boldsymbol{H}^H + N_0\boldsymbol{I}_M$ is the covariance matrix of the receive signal.

Replacing (2) and (3) into (11) and (10), we have MSE for the nth transmit stream using the ZF as

$$\mathrm{MSE}_n^{(\mathrm{ZF})} = N_0\left(\boldsymbol{w}_n^H\boldsymbol{w}_n\right) \tag{12}$$

with $\boldsymbol{w}_n = \boldsymbol{W}^{\mathrm{ZF}}(:,n)$, and using MMSE as

$$\mathrm{MSE}_n^{(\mathrm{MMSE})} = \frac{E_s}{N}\left(1 - \boldsymbol{h}_n^H\boldsymbol{w}_n\right) \tag{13}$$

with $\boldsymbol{w}_n = \boldsymbol{W}^{\mathrm{MMSE}}(:,n)$. Here we have denoted the nth column of $\boldsymbol{W}$ by $\boldsymbol{w}_n = \boldsymbol{W}(:,n)$. Assume that the noise spectral density N_0 are the same for all antennas, it is clear that the MSE of both the detectors only depends on the channel and the received signal covariance matrix, which makes it possible to use MSE as a criterion for antenna selection. Our proposed algorithm is as follows: *for every subset of the receive antennas $p \in P$ compute the maximum value of MSE (MSE_{max}), choose the subset with the minimum MSE_{max}.*

The algorithm can be summarized in the form of the pseudo code as follows

1) BEGIN: Input M_r, M, N, $\boldsymbol{H}$
2) Compute $P = \binom{M_r}{M}$
3) For $p = 1$ to P
4) For $n = 1$ to N
5) If 'ZF detector'
6) $\boldsymbol{W}_p = \boldsymbol{H}_p\left(\boldsymbol{H}_p^H\boldsymbol{H}_p\right)^{-1}$
7) $\boldsymbol{w}_n = \boldsymbol{W}(:,n)$
8) $\mathrm{MSE}_n = \boldsymbol{w}_n^H\boldsymbol{w}_n$
9) Elseif 'MMSE detector'
10) $\boldsymbol{R}_{yy} = \mathrm{E}\{\boldsymbol{y}\boldsymbol{y}^H\}$
11) $\boldsymbol{W}_p = \boldsymbol{R}_{yy}^{-1}\boldsymbol{H}_p$
12) $\boldsymbol{w}_n = \boldsymbol{W}(:,n)$
13) $\mathrm{MSE}_n = 1 - \boldsymbol{H}_p^H(:,n)\boldsymbol{w}_n$
14) End
15) End
16) $\mathrm{MSE}_p^{max} = \max\left(\mathrm{MSE}_n\right)$
17) End
18) $p = \arg\min_{p \in P}\left(\mathrm{MSE}_p^{max}\right)$
19) END: Return $\boldsymbol{H}_p$

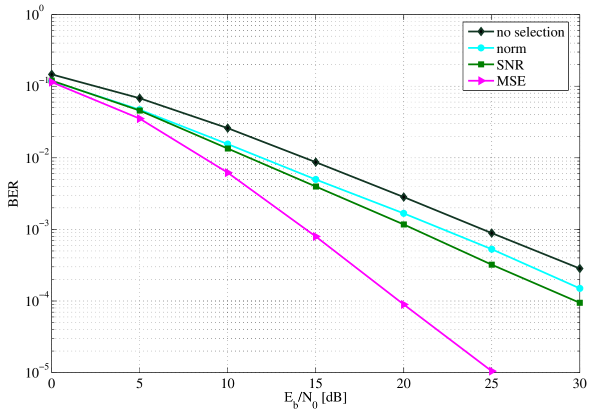

Fig. 2. BER of 2×3 MIMO-SDM system with Rx antenna selection

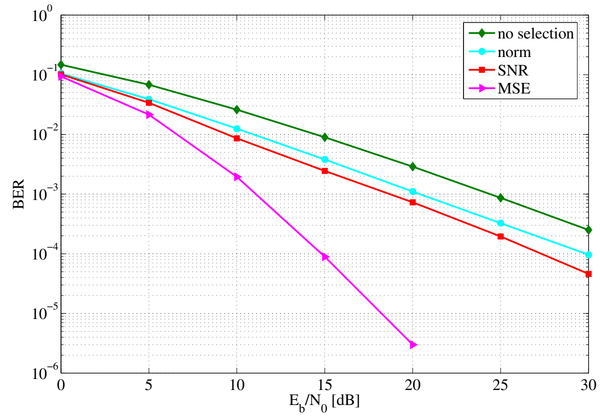

Fig. 3. BER of 2×4 MIMO-SDM system with Rx antenna selection.

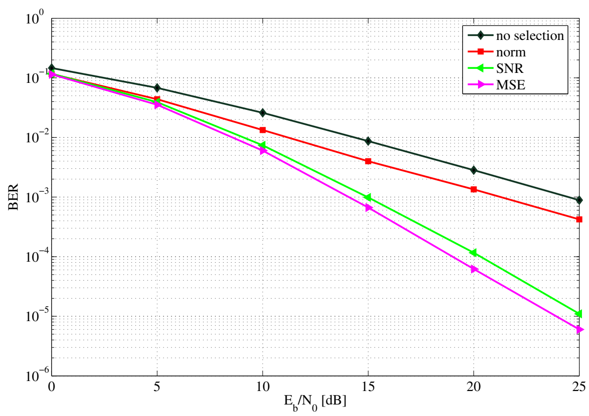

Fig. 4. BER of 3×2 MIMO-SDM system with Tx antenna selection.

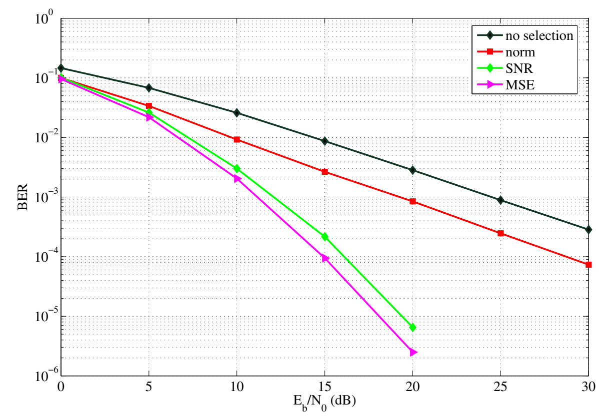

Fig. 5. BER of 4×2 MIMO-SDM system with Tx antenna selection

V. PERFORMANCE EVALUATION

In this section, we compare performance of the above presented three selection criteria using Monte Carlo simulations. In our simulations, BPSK is used for modulation and the coefficients of channel are generated using i.i.d. complex Gaussian random variables with zero mean and variance 0.5 for each dimension. Noise at each receive antenna element is also generated using a Gaussian random variable with zero mean, but variance depending on the target E_b/N_0.

The following scenarios are set up for simulation:

(i) A receive antenna selection MIMO-SDM system with 2 transmit antennas, and 3 to choose 2 receive antennas.
(ii) A receive antenna selection MIMO-SDM system with 2 transmit antennas, and 4 to select 2 receive antennas.
(iii) A transmit antenna selection MIMO-SDM system with 2 receive antennas, and 3 to select 2 transmit antennas.
(iv) A transmit antenna selection MIMO-SDM system with 2 receive antennas, and 4 to select 2 transmit antennas.

For presentation simplicity, only results for MMSE detection are illustrated.

In the case of receive antenna selection systems, it is clearly shown in Fig. 2 and Fig. 3 that our proposed MSE based algorithm provides significantly improved diversity gain compared with the previous norm based and SNR based algorithm. In particular, the 2×3 and 2×4 MIMO system provide about double diversity gain compared with the other two algorithms. It is also clear from the figures is that the previous two algorithms provide only SNR gain but no diversity gain. As a result, compared with the SNR based antenna selection, our algorithm provides up 10dB and 12dB SNR gain at BER $= 10^{-4}$ in a 2×3 and 2×4 MIMO system, respectively.

For systems with transmit antenna selection in Figs. 4 and 5, improvement in diversity gain is recorded for both the SNR based and the proposed MSE based selection algorithm. However, our proposed MSE based algorithm has merit over the SNR based algorithm in providing some improvement in SNR gain. For both 3×2 and 4×2 MIMO system, the SNR gain is about 1dB compared with the SNR based selection algorithm.

We also would like to note that similar observations have also been recorded for MIMO-SDM systems with ZF detection, but not presented here for presentation simplicity.

VI. CONCLUSION

In this paper, we have proposed an MSE based antenna selection algorithm for MIMO-SDM systems. The proposed algorithm was demonstrated to be superior to the previously proposed algorithms of SNR based and norm based selection in terms of minimizing BER. The algorithm can be applied to both transmit and receive antenna selection systems.

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