Optimization of signal points in Bit-Intreleaved Coded Modulation system with Iterative Decoding (BICM-ID)

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***Abstract*—A new method for adjusting positions of signal points of a traditional signal constellation used in the BICM-ID (Bit-Interleaved Coded Modulation with Iterative Decoding) system is proposed. Optimal rotation angle versus SNRs (Signal-to-Noise Power ratios) in the sense of providing the system with attainable lowest BER (Bit Error Rate) are reported.**

***Index Terms*— Bit Interleaved Coded Modulation with Iterative Decoding, signal constellation**

I. Introduction

ZEHAVI [1] has shown that, in the trellis coded system, the using of an interleaver increases the diversity degree of the system which results in some coding gain on the phading channel. This technique is called Bit Interleaved Coded Modulation (BICM). However, the performance of a BICM scheme decreases on the Gaussian channel because the interleaving doesn't optimize Euclidean distances between the symbol sequences. According to X. Li and J. Ritcey [3] [4], the effective Euclidean distances in the BICM system can be improved when we use Iterative Decoding (ID). This system is called Bit Interleaved Coded Modulation with Iterative Dercoding (BICM-ID). According to Chi–Hsiao Yih's results [5], the performance of BICM-ID system depends on the positions of the signal points in the constellation. The author has carried out a survay of error floor (i.e., at high SNRs). Results for other ranges of SNR have not been reported.

In this paper we propose the method to adjust the signals' position through carrying out a study of system's performance's changing when we rotate a part of the signal constellation out of its symmetric position. The result of simulation with 8-PSK signals shown that the performance of system is improved significantly in the whole SNR range, especially at high SNRs. The result can be used as a basis to design an adaptive model with attainable lowest BER (Bit Error Rate). The next section describes a mathematical model of BICM-ID system in to find the optimal rotation angle and reports imperical formulas to calculate the optimum angles depending on SNR.

II. The BICM-ID system

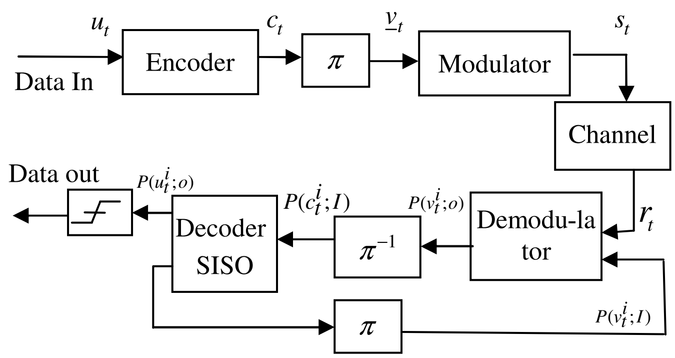

Figure 1. BICM-ID system

The trnsmitter in a traditional BICM-ID system consists of three serialy concatenated blocks: a binary convolutional encoder, a bit interleaver, and a M-ary modulator (Fig. 1, due to [2]). The rate of encoder is *k/n*, meaning that for k information bits $u=[u^1,u^2...u^k]$, we have *n* bits $c=[c^1,c^2...c^n]$ as the encoder output. The interleaver disoderes the encoded bit block of length *N*, $\underline{c}=[c_1^1,c_1^2...c_1^n,c_2^1,c_2^2...c_2^n,\ldots,c_{N/n}^1,c_{N/n}^2...c_{N/n}^n]$, and divides them into N/m sub-blocks each consisting of $m=\log_2 M$ bits, say, $\underline{v}_t=(v_t^1,v_t^2,\ldots,v_t^m)$, and being mapped into a symbol s_t in the constellation S with M signal points according to μ : $s_t=\mu(\underline{v}_t)$, $s_t\in S$, $t=1,2,\ldots,N/m$. For example, $S=(e^{jl2\pi/M}, l=0,1,...,M-1)$ for an MPSK set [4].

The received signal at the receive site is $r_t=\rho_t\sqrt{E_s}\,s_t+n_t$, where ρ_t is phading coefficient, E_s is energy of symbol, n_t is AWGN noise of spectral density N_0. On the fading channel, ρ_t has Rayleigh distribution with $E(\rho_t^2)=1$. When there is a perfect CSI (Channel State Information) at the receiver site, we say that ρ_t is perfectly estimated. Hereafter we consider the AWGN channel so that we set $\rho_t=1$.

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Receiver uses the iteration with exchanging extrinsic information between the decoder and the demodulator. The SISO (Soft Input Soft Output) decoder feeds back its output to the demodulator as the A Posteriori Probability so that the demodulator can calculate the bits's $LLR(v_t^i;O)$ (Log Likelihood Ratio), for $t=1,2,\ldots,N/m,\ i=1,2,\ldots,m$, by the Maximum A Postriori Probability algorithm (MAP).

$$LLR(v_t^i;O)=\log\frac{P(v_t^i=1/r_t)}{P(v_t^i=0/r_t)}=\log\frac{\sum\limits_{s_t\in S_1^i}P(r_t/s_t)P(s_t)}{\sum\limits_{s_t\in S_0^i}P(r_t/s_t)P(s_t)}$$

$$=\log\frac{\sum\limits_{s_t\in S_1^i}\exp\left[\frac{-\|r_t-\rho_t s_t\|^2}{N_0}\right]\prod\limits_{j=1}^{m}P(\underline{v}_t^i)}{\sum\limits_{s_t\in S_0^i}\exp\left[\frac{-\|r_t-\rho_t s_t\|^2}{N_0}\right]\prod\limits_{j=1}^{m}P(\underline{v}_t^i)} \qquad (1)$$

In (1), S_b^i is the subset of S consisting of signal points whose binary label under the mapper μ has bit b at the position i. Extrinsic information calculated by demodulator as

$$L_e(\underline{v}_t^i)=\log\frac{P(\underline{v}_t^i=1/r_t)}{P(\underline{v}_t^i=0/r_t)}-\log\frac{P(\underline{v}_t^i=1)}{P(\underline{v}_t^i=0)}$$

$$=\log\frac{\sum\limits_{s_t\in S_1^i}\exp\left[\frac{-\|r_t-\rho_t s_t\|^2}{N_0}\right]\prod\limits_{j=1,j\neq i}^{m}P(\underline{v}_t^i)}{\sum\limits_{s_t\in S_0^i}\exp\left[\frac{-\|r_t-\rho_t s_t\|^2}{N_0}\right]\prod\limits_{j=1,j\neq i}^{m}P(\underline{v}_t^i)} \qquad (2)$$

In (1) and (2), the probabilities $P(\underline{v}_t^i)$ are calculated as

$$P(\underline{v}_t^i=1)=\frac{\exp(L_e(\underline{v}_t^i))}{1+\exp(L_e(\underline{v}_t^i))} \text{ and } P(\underline{v}_t^i=0)=\frac{1}{1+\exp(L_e(\underline{v}_t^i))}.$$

Extrinsic information (2), after being deinterleaved, will be taken to the SISO decoder. When we calculate the metric of each bit, we only use the a priori probability of the others in the same symbol label. The metric is updated in each iteration and finally hard-dicided after a certain number of iterations. The SISO decoder works in accordance with LOG-MAP algorithm. In expression (2), the sum of the probabilities is approximately equal to the maximum operand, and with using the seaching table, the complexity of the system can be decreased remarkably. We have the so called max*-LOG-MAP.

III. THE BASIC MAPPING RULES AND METHODS OF ADJUSTING THE SIGNAL POINT POSITON

There are many ways to map the bit sub-block into the symbols of a constellation. Among these, 3 basic mappings are usually used: Gray, Set partition and Semi Set partition. For example, with the 8-PSK constellation the mappings are illustrated in Fig. 2. The line between the pair of signals presents the Euclidean distance between that pair of signals whose labels differ only in one bit. We define the i-th bit distance to be the Euclidean distance between a pair of signals whose binary labels differ in *i*-th bit. That is

$$d_i=d_E(\mu(v^1,\ldots,0,\ldots,v^m),\mu(v^1,\ldots,1,\ldots,v^m)) .$$

In case of the traditional symetric 8-PSK signals (Fig. 3a.), the values of bit distances are reported in Table 1 for each bit with different mappings.

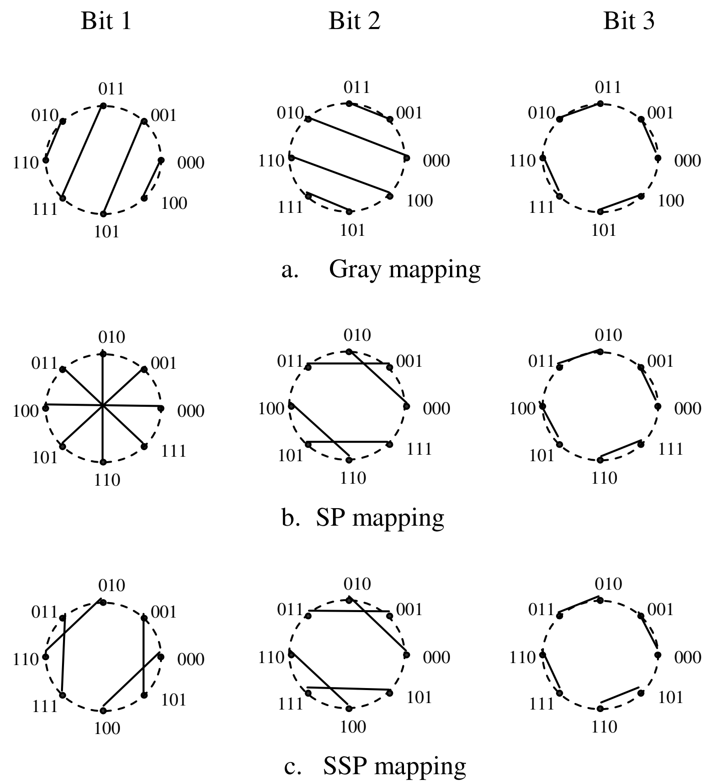

Figure 2. Gray, SP and SSP mappings

Each bit's distance related to reliability when it is decided to be 0 or 1. Obviously, the bit with a larger bit distance has higher protection level and, since, a smaller probability of error [6]. We realize that SP and SSP are the mappings which have uniform protection levels, that is the error probability of a bit does not depend on the specific signal point and depends only on the bit's position in the label. This property holds when each bit's position has only one value d_i. In contrast, the Gray mapping does not have uniform protection level.

TABLE 1. Bit distances of basic mappings of 8PSK signals

Mapper	bit 1: d_1	bit 2: d_2	bit 3: d_3
Gray	$2\sin(\pi/8)$ and $2\sin(3\pi/8)$	$2\sin(\pi/8)$; $2\sin(3\pi/8)$	$2\sin(\pi/8)$
SP	2	$\sqrt{2}$	$2\sin(\pi/8)$
SSP	$\sqrt{2}$	$\sqrt{2}$	$2\sin(\pi/8)$

Since each pair of nearest signals in Gray mapping have labels which differ only 1 bit, so in case of no iteration, the Gray mapping has the lowest BER among three basic mappings. In the system with iterations, supposed that the a priori information is reliable (with perfect feedback, say, at high SNR), then the SP mapping gives the lowest BER because of the largest 1-th bit distance. The result shown in Fig. 6 proves that. At low SNRs

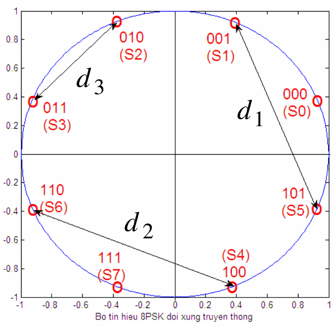

Figure 3a: Original 8PSK signals

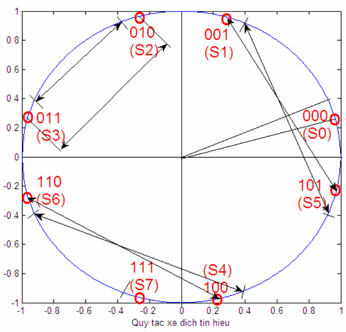

Figure 3b: Adjusted 8PSK signals

SP mapping is worse than SSP mapping because of high Symbol Error Ratio (SER) and unreliable feedback information. But when the SNR increases, the SER decreases. Thus SP is better due to the larger average distance of bits. In BICM-ID system, good combination between the first iteration's result reliability and feedback information's improvement will bring better performance [4].

Let us consider the SSP mapping of the traditional symmetric 8PSK constellation in Fig. 3. The mapping rule is represented by a vector which contains the integers from 0 to 7. The number itself represents a 8PSK signal point while its position in the vector relates to the binary label of that signal point. For example, if the mapping rule is given as $Rule = [0\ 1\ 2\ 3\ \ 6\ 7\ 4\ 5]$, then the binary label of the integer i is given to the signal

$$S_i = e^{(j(2Rule(i+1)+1)\frac{\pi}{8})};\ i=0,...,7$$

Bit istances d_1, d_2, d_3, respectively, of bits 1,2,3 are shown in Fig. 3a. (their values are given in Tab. 1). Now we adjust the constellation by rotating the i-th signal point an angle α or $-\alpha$ depending on if i is even or odd. That is, the i-th signal point now becomes $S_i e^{j[1-2(i \bmod 2)]\alpha}$.

Fig. 3b presents the positions of signal points after they have been rotated with an angle $\alpha < 0$ (we call it the negative rotation). It can be seen that the distances of 1^{st} bit and 2^{nd} bit have not changed, while the 3^{th} bit distance have increased, i.e., $d_1^* = d_1$; $d_2^* = d_2$; $d_3^* > d_3$. On the contrary, if $\alpha > 0$ (positive rotation) then the distance of 3^{th} bit will decrease when distances of 1^{st} bit and 2^{nd} bits keep unchanged.

To find the good rotation angles we carried out simulation with α varying from $-\pi/16$ to $\pi/16$ at different SNR values. The simulation result is shown in Fig. 4. Each curve correspondends to a specific value of E_b/N_0 changing from 1dB to 4.5dB with a step of 0.5dB. The lowest point of each curve show the best askew angle for the value E_b/N_0 the curve is related to. It can be seen in the figure that it is possible to choose good rotation angles so that there is some linear relationship in E_b/N_0. In comparision to traditional symetric postions of signal point ($\alpha = 0$), the performance of the system is improved if the signals are adjusted to match the SNR value. In the BICM-ID system, thanks to the interleaver, bits in the binary label of an M-PSK symbol are independent on each other and supposed to be transmitted (at high SNRs) on $m = \log_2 M$ separate parallel BPSK channels with different energies, depending on the corresponding bit distances. Thus at high SNRs the symbol error rate (SER) is low, consequently it is possible and desired for us to make signal points in pairs $(s_0, s_5), (s_1, s_2), (s_3, s_6), (s_7, s_4)$ closer to each other, in order to increase the third bit distance. The more we adjust the negative rotation to increase the distance of bit 3, the lower the error floor will be. Contrarily, at small SNRs the SER is high, then we need to use the positive rotation to make the points in signal pairs $(s_0, s_1), (s_2, s_3), (s_6, s_7), (s_4, s_5)$ further from each other to have smaller SER (like Gray mapping).

By analysing simulation results of the BICM-ID system in consideration using the 8PSK constellation with SSP mapping, we propose the following experimental formulae which can be used to calculate suitable askew angles for each SNR zone:

$$\begin{cases} \alpha = (-0.4E_b/N_0 + 1.1)\dfrac{\pi}{16} & \text{when} \quad E_b/N_0 \le 2 \\ \alpha = (-0.2E_b/N_0 + 0.4)\dfrac{\pi}{16} & \text{when} \quad 2.5 \le E_b/N_0 \le 3.5 \\ \alpha = (-0.2E_b/N_0 + 0.3)\dfrac{\pi}{16} & \text{when} \quad 4 \le E_b/N_0 \le 6 \\ \alpha = -\dfrac{\pi}{16} & \text{when} \quad E_b/N_0 > 6 \end{cases}$$

TABLE 2. Best rotation angles versus SNR values

E_b/N_0 (dB)	0	0.5	1	1.5	2	2.5	3
α	$\frac{1.1\pi}{16}$	$\frac{0.9\pi}{16}$	$\frac{0.7\pi}{16}$	$\frac{0.5\pi}{16}$	$\frac{0.3\pi}{16}$	$\frac{-0.1\pi}{16}$	$\frac{-0.2\pi}{16}$
E_b/N_0 (dB)	3.5	4	4.5	5	5.5	6	≥ 6.5
α	$\frac{-0.3\pi}{16}$	$\frac{-0.5\pi}{16}$	$\frac{-0.6\pi}{16}$	$\frac{-0.7\pi}{16}$	$\frac{-0.8\pi}{16}$	$\frac{-0.9\pi}{16}$	$\frac{-\pi}{16}$

We provide a quick-look-up table for appropriate rotation angles versus SNRs (see Tab. 2) so that the system can behave accordingly in an adaptive manner when CSI is known. Using these values for the adaptive rotation when the SNR is changing form 0dB to 4.5dB, we can show graphically in Fig. 5 the relationship between the best rotation angles for each SNR and the BER curve versus SNR.

We compare the system using the traditional symmetric signal constellation with the system using the adjusted signals by simulation. The convolutional code is the RSC code with generators [1,5/7]. The interleaver is random with the length of 1000 bits. The constellation is 8PSK with SSP mapping. The result shows that, at $BER = 10^{-5}$, there is a gain of about 1dB compared to the traditional symmetric constellation (Fig. 6).

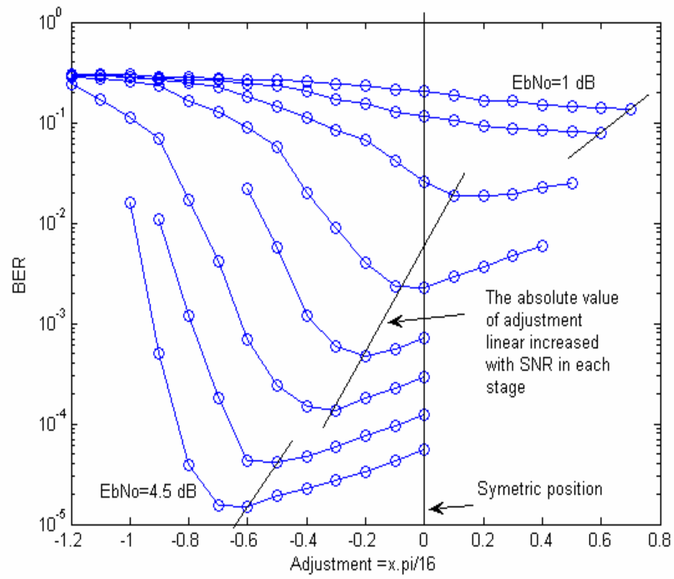

Figure 4. The changes of BER versus rotation angles

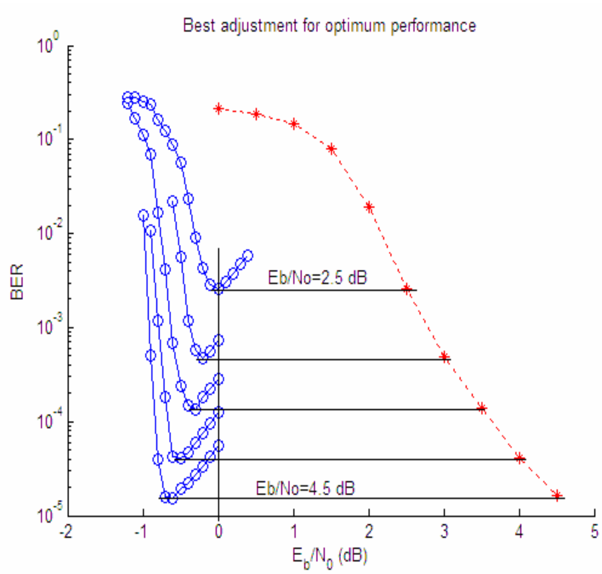

Figure 5. Best adjustments for optimum performance

IV. CONCLUTION

In this paper, we have studied the relation between the position of 8PSK signal points with Semi-Set Partition mapping and the performance of the related BICM-ID system. We have shown that there exist an adjusting rotation angle for each SNR value that gives rise to the best BER of the system. We have also reported mathematic formulae together with a lookup table for the appropriate rotation angles versus SNR values. These values can used in an adaptive scheme to obtain the best performance in all SNR ranges. The proposed method can be directly extended to other signal constellations, like MPSK and MQAM, with different mapping rules.

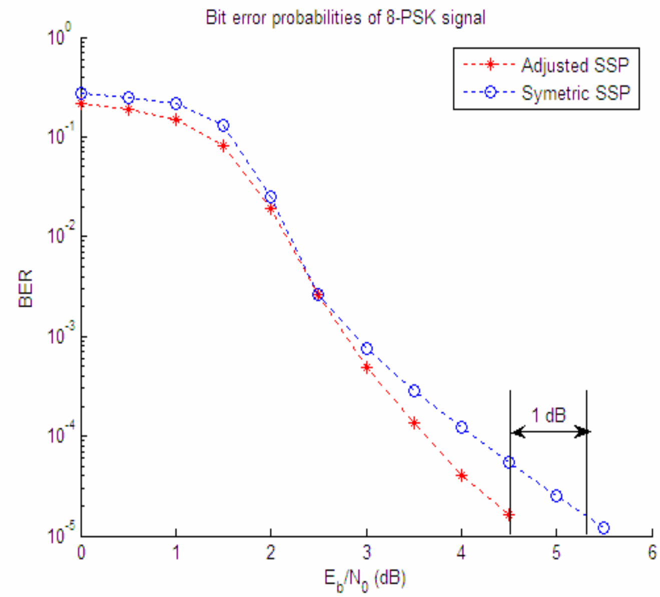

Fig. 6. Comparison of performance of the traditional 8PSK and adaptively adjusted 8PSK constelations.

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