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Stress state analysis of laminated shells with piezoelectric layers based on the refined theory

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Abstract. This paper presents stress state analysis of composite cylindrical shells bonded piezoelectric layers in the top and bottom surfaces. The laminated smart shell is subjected to electric potential and mechanical loading. The mathematical model of electroelasticity behaviour is based on the refined theory and virtual work principle. The governing differential equations are reduced to ordinary differential equations by means of trigonometric function expansion for displacement and electric potential. Operative method is used to solution of boundary problem. Stress-strain state of smart cylinder for electromechanical loads with clamped support is provided. Numerical results are presented showing the effect of boundary layer at clamped edges and comparison between the obtained results to the classical theory of the Kirchhoff-Love type is accurate.

1. Introduction

Nowadays, piezomaterials are widely found in various fields of mechanical engineering, automation, computational technology, etc., due to its direct and inverse effects. In the aerospace industry, piezoelectric materials are used as sensors and actuators on adaptive aircraft systems to improve the quality of aerodynamics and effective management of their deformations [1]. In addition, composite materials are commonly used to reduce the masses and increase the strength on aircraft. The combination of piezoelectric and composite materials allows improving the properties of next generation modern aircraft as controlled systems [2-3].

The main design diagrams of aircraft structural elements are thin plates and shells. The calculations of the stress-strain state of plates and shells are based on the assumptions of the classical deformation theory (CDT) of the Kirchhoff-Love [4] and Timoshenko-Gol'denveizer [5-6]. CDT is used to represent a three-dimensional equations of elasticity theory in two-dimensional form. Based on CDT, H S Tzou [7], Reddy [8] extended to piezoelectric plates and shells and presented electromechanical model by joining elasticity equations with Maxwell' equations. Mindlin [9], Reissner [10] provided first order shear deformation theory (FSDT) with hypothesis is the transverse normals do not remain perpendicular to the mid-surface after deformation. Reddy [8, 11] developed third order shear deformation theory (TSDT) and high order shear deformation theory by using the nonlinear polynomial function to describe the shear stress distribution for not only isotropic but also anisotropic and piezoelectric magnetic laminated composite materials.

In recent years, many studies have been conducted in the field of the electromechanical state of smart laminated shells. Based on FSDT Rajeev Kumar [12] and A. Benjeddou [13] introduced finite element formulation to 9 nodes to static and dynamic analysis of laminated composite shells subjected to electrical, mechanical and thermal loadings. Ehsan Arshid and Khorshidvand [14] used Hamilton's variational principle to derive the governing motion equations for a circular plate made up of a porous material integrated by piezoelectric actuator patches. Mitchell and Reddy [15] have published solutions for axisymmetric composite cylinder with embedded piezoelectric layers under axial load. Diego Amadeu, Paulo Mendonca [16] and Kant T [17] developed a formulation HSDT for solution of piezoelectric laminated plates and sandwich plates. By using differential quadrature method R. Akbari Alashti [18], Santosh Kapuria [19] provided three-dimensional elastic and thermoelastic analysis of a piezoelectric cylindrical shell with functionally graded layers under the effect of axisymmetric and asymmetric thermo-electro-mechanical loads. Aghalovyan [20] presented asymptotic method to solution of the electroelasticity problem for piezoceramic shell.

However, in these works, the boundary condition only for fully simply-supported are studied and the stress-strain state in the clamped edges of piezoelectric laminated shells has not mentioned much. In the paper, the refined theory is presented to model the electromechanical state of smart cylindrical shells. The displacement and potential field, in this case, satisfied the energy compatibility conditions proposed by Lurie, Vasiliev [21]. The generalized Lagrange principle is utilized to derive the governing equations of composite shell bonded piezoelectric layers. The operative method is used to analyze the stress concentration state of piezolaminated cylindrical shells due to electric potential and mechanical loading applied at the top and the bottom with clamped edges. The comparison of results for cylindrical shell with classical theory is given.

2. Materials and methods

2.1. Shell model

Consider a composite cylindrical shell in the orthogonal curvilinear coordinate systems with the geometrical parameters according to figure 1, is bonded piezoelectric layers on the top and bottom surfaces. For a piezoelectric laminated shell subject to a prescribed surface tractions $q_{i3} = q_{i3}^{\pm}(\xi,\theta)$, $(i=1,2,3)$ and electric potentials $\varphi_i = \varphi^{\pm}(\xi,\theta)$ at the upper and lower surfaces $z = \pm h$ of shell, respectively.

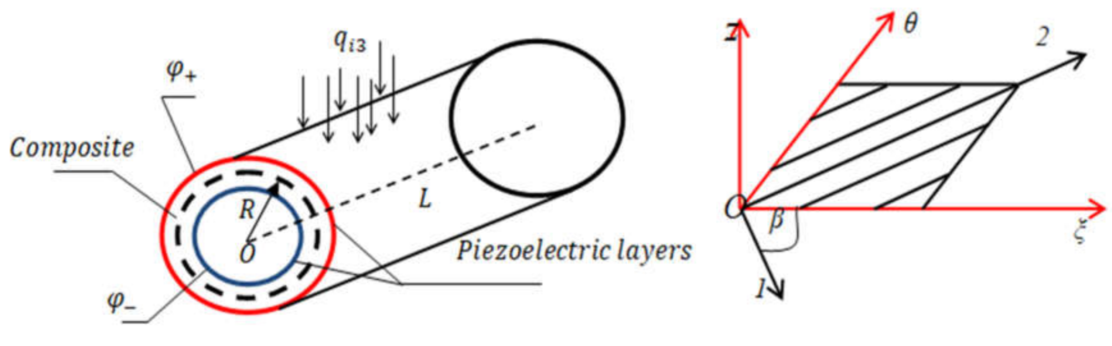

Figure 1. Composite cylindrical shell bonded piezoelectric layers.

2.2. Displacement field

The displacement field and electric potentials of the shell in the orthogonal curvilinear coordinate systems $O\xi\theta z$ is analyzed [22-24] as follows

$$\begin{aligned}
u(\xi,\theta,z) &= u_0(\xi,\theta)+u_1(\xi,\theta)z+u_2(\xi,\theta)\frac{z^2}{2!}+u_3(\xi,\theta)\frac{z^3}{3!},\\
v(\xi,\theta,z) &= v_0(\xi,\theta)+v_1(\xi,\theta)z+v_2(\xi,\theta)\frac{z^2}{2!}+v_3(\xi,\theta)\frac{z^3}{3!},\\
w(\xi,\theta,z) &= w_0(\xi,\theta)+w_1(\xi,\theta)z+w_2(\xi,\theta)\frac{z^2}{2!},\\
\varphi(\xi,\theta,z) &= \varphi_0(\xi,\theta)+\varphi_1(\xi,\theta)z+\varphi_2(\xi,\theta)z^2.
\end{aligned} \tag{1}$$

2.3. The strain and stress fields

The strain components in above equations are related to the displacement components, which are determined by equations:

$$\begin{aligned}
&\varepsilon_\xi=\frac{1}{R}\frac{\partial u}{\partial \xi},\ \varepsilon_\theta=\frac{1}{R+z}(\frac{\partial v}{\partial \theta}+w),\ \gamma_{\xi\theta}=\frac{1}{R}\frac{\partial v}{\partial \xi}+\frac{1}{R+z}\frac{\partial u}{\partial \theta},\\
&\varepsilon_z=\frac{\partial w}{\partial z},\ \gamma_{\xi z}=\frac{1}{R}\frac{\partial w}{\partial \xi}+\frac{\partial u}{\partial z},\ \gamma_{\theta z}=\frac{1}{R+z}\frac{\partial w}{\partial \theta}+\frac{\partial v}{\partial z}-\frac{v}{R+z}.
\end{aligned} \tag{2}$$

2.4. Piezoelectric layers

Linear constitutive equations of piezoelectric layers [7-8] in the general coordinate system $O\xi\theta z$ is expressed as

$$\begin{aligned}
\{\sigma\} &= [C]\{\varepsilon\}-[e]^T\{E\},\\
\{D\} &= [e]\{\varepsilon\}+[\mu]^T\{E\}.
\end{aligned} \tag{3}$$

where $\sigma=\{\sigma_\xi,\sigma_\theta,\sigma_z,\sigma_{\xi\theta},\sigma_{\theta z},\sigma_{z\xi}\}$ is the stress vector, $D=\{D_\xi,D_\theta,D_z\}$ is the electric displacement vector, $E=\{E_\xi,E_\theta,E_z\}$ is the electric field vector. $C=C_{ij}\ (i=\overline{1,6}, j=\overline{1,6})$ is the elastic stiffness matrix, $\varepsilon=\{\varepsilon_\xi,\varepsilon_\theta,\varepsilon_z,\gamma_{\xi\theta},\gamma_{\theta z},\gamma_{z\xi}\}$ is the strain vector, $e=e_{ij}\ (i=\overline{1,3}, j=\overline{1,6})$ is the piezoelectric matrix, $\mu=\mu_{ij}\ (i=\overline{1,3}, j=\overline{1,3})$ is the dielectric matrix of material.

According to the properties of piezoelectric materials, under the influence of mechanical and electrical loads, charges $Q_{\pm h}$ appear on the surface $z=\pm h$ of shell.

At top and bottom surface $z=\pm h$ of shell, electric potentials are $\varphi_{-h}=\varphi_-$ and $\varphi_h=\varphi_+$. Combining the forth equation of Eq. (5), we have

$$\varphi_1=\frac{\varphi^+-\varphi^-}{2h},\quad \varphi_2=-\frac{\varphi_0}{h^2}+\frac{\varphi^++\varphi^-}{2h^2}. \tag{4}$$

Maxwell's equations for the electric field, neglecting magnetic effects, can be reduced to electrostatic equations

$$E_\xi=-\frac{\partial\varphi}{A_1\partial\xi},\ E_\theta=-\frac{\partial\varphi}{A_2\partial\theta},\ E_z=-\frac{\partial\varphi}{\partial z}. \tag{5}$$

where $A_1=R,\ A_2=1+z/R$ - Lame’s parameters of cylindrical shell.

2.5. Composite layers

The neutral plane is the mid-surface of the shell. The main direction of fiber reinforcement of each layer coincides with the direction of the local coordinate system *O123*, correspondingly. The angle between the direction of fiber reinforcement and the vertical axis *Oz* of the general coordinate system is β. The structure includes *n* layers, the total thickness *2h* (figure 1).

Hooke's law for the layer *k* in the local coordinate system *O123* has the following form

$$\left\{\sigma_{123}^{(k)}\right\}=\left[C^{(k)}\right]\left\{\varepsilon_{123}^{(k)}\right\}. \tag{6}$$

Eq. (6) is the relationship between the stress field and the strain field of layer *k* in the local coordinate system *O123*. So we need to perform the coordinate transfer to general coordinate system $O\xi\theta z$ with matrix transformation $T_2^{(k)}$ of layer *k*.

Then, the relationship between the stress field and the strain field of layer *k* in the general coordinate system now becomes

$$\left\{\sigma_{\xi\theta z}^{(k)}\right\}=\left[T_2^{(k)}\right]^T\left[C^{(k)}\right]\left[T_2^{(k)}\right]\left\{\varepsilon\right\}. \tag{7}$$

where $\sigma_{\xi\theta z}^{(k)}=\left\{\sigma_\xi,\sigma_\theta,\sigma_z,\sigma_{\xi\theta},\sigma_{\theta z},\sigma_{z\xi}\right\}$ - the stress vector of layer *k* in general coordinate system $O\xi\theta z$.

Matrix transformation of layer *k* is determined by:

$$\left[T_2^{(k)}\right]=\begin{bmatrix}\cos^2\beta^{(k)} & \sin^2\beta^{(k)} & 0 & 0 & 0 & \sin\beta^{(k)}\cos\beta^{(k)}\\ \sin^2\beta^{(k)} & \cos^2\beta^{(k)} & 0 & 0 & 0 & -\sin\beta^{(k)}\cos\beta^{(k)}\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & \cos\beta^{(k)} & -\sin\beta^{(k)} & 0\\ 0 & 0 & 0 & \sin\beta^{(k)} & \cos\beta^{(k)} & 0\\ -\sin 2\beta^{(k)} & \sin 2\beta^{(k)} & 0 & 0 & 0 & \cos^2\beta^{(k)}-\sin^2\beta^{(k)}\end{bmatrix} \tag{8}$$

2.6. Governing equations and boundary conditions

The Lagrange principle is used to establish the equilibrium equation, the total potential energy is the minimum value, it means:

$$\delta U-\delta A=0. \tag{9}$$

where U is the electroelastic potential energy, A is the work done by external forces $q_{i3}=q_{i3}^{\pm}(\xi,\theta)$, $(i=1,2,3)$ and electric charges $Q_{\pm h}$ at the upper and lower surfaces. The variation of the electroelastic potential energy is defined in the following formula:

$$\delta U-\delta A=\sum\int(\sigma\delta\varepsilon+D\delta E)dV-\sum\int(q_{i3}(\delta u_i+\delta v_i+\delta w_i)dS-\sum\int Q_{\pm h}\delta\varphi dS. \tag{10}$$

Equilibrium equations are governed by integrating separately the expression (10) according to potential and displacement components, then taking independently the possible potential and displacement equal zero. From there, we get the following equilibrium equations system as follows

$$
\begin{aligned}
&\frac{\partial N_{11}^{(0)}}{\partial \xi}+\frac{\partial N_{21}^{(0)}}{\partial \theta}=0,\\
&\frac{\partial N_{11}^{(i)}}{\partial \xi}+\frac{\partial N_{21}^{(i)}}{\partial \theta}-RN_{13}^{(i-1)}=0,(i=\overline{1,3}),\\
&\frac{\partial N_{12}^{(0)}}{\partial \xi}+\frac{\partial N_{22}^{(0)}}{\partial \theta}+N_{23}^{(0)}=0,\\
&\frac{\partial N_{12}^{(i)}}{\partial \xi}+\frac{\partial N_{22}^{(i)}}{\partial \theta}+N_{23}^{(i)}-RN_{23}^{(i-1)}-iN_{23}^{(i)}=0,(i=\overline{1,3}),\\
&\frac{\partial N_{13}^{(0)}}{\partial \xi}+\frac{\partial N_{23}^{(0)}}{\partial \theta}-N_{22}^{(0)}+Rp_z^{(0)}=0,\\
&\frac{\partial N_{13}^{(j)}}{\partial \xi}+\frac{\partial N_{23}^{(j)}}{\partial \theta}-N_{22}^{(j)}-RN_{33}^{(j-1)}+Rp_z^{(j)}=0,(j=\overline{1,2}),\\
&\frac{\partial (h^2 ND_1^{(0)}-ND_1^{(2)})}{\partial \xi}+\frac{\partial (h^2 ND_2^{(0)}-ND_2^{(2)})}{\partial \theta}-2ND_3^{(1)}=0.
\end{aligned}
\tag{11}
$$

where $N_{ij}^{(s)}(i=\overline{1,3}, j=\overline{1,3}, s=\overline{0,3})$ - mechanical forces and moments, $ND_i^{(s)}(i=\overline{1,3}, s=\overline{0,2})$ - electrical forces and moments and the following designations are adopted:

$$
\begin{aligned}
&\left(N_{11}^{(i)},N_{12}^{(i)},N_{13}^{(i)}\right)=\sum_{k=1}^{n+2}\int_{-h}^{h}\left(\sigma_\xi,\sigma_{\xi\theta},\sigma_{\xi z}\right)\frac{z^i}{i!}dz,\ (i=\overline{0,3}),\\
&\left(N_{22}^{(i)},N_{21}^{(i)},N_{23}^{(i)}\right)=\sum_{k=1}^{n+2}\int_{-h}^{h}\left(\sigma_\theta,\sigma_{\xi\theta},\sigma_{\theta z}\right)\frac{z^i}{i!}dz,\ (i=\overline{0,3}),\\
&N_{33}^{(j)}=\sum_{k=1}^{n+2}\int_{-h}^{h}\sigma_z\frac{z^j}{j!}dz,\ (j=\overline{0,2}),\\
&ND_1^{(i)}=\sum_{k=1}^{n+2}\int_{-h}^{h}D_1(1+\frac{z}{R})\frac{z^i}{i!}dz,\\
&ND_2^{(i)}=\sum_{k=1}^{n+2}\int_{-h}^{h}D_2\frac{z^i}{i!}dz,\ (i=\overline{0,2}),\\
&ND_3^{(i)}=\sum_{k=1}^{n+2}\int_{-h}^{h}D_3(1+\frac{z}{R})\frac{z^i}{i!}dz,(i=\overline{0,2}),\\
&p_z^{(i)}=q_{33}^{+}(\xi,\theta)(1+\frac{h}{R})(\frac{h^i}{i!})-q_{33}^{-}(\xi,\theta)(1-\frac{h}{R})(\frac{(-h)^i}{i!}),\ (i=\overline{0,3}).
\end{aligned}
\tag{12}
$$

The boundary conditions at the edges are:

- for clamped supported: $u_i=v_i=w_i=0,\ \varphi_j=0,\ i=\overline{0,3},\ j=\overline{0,2}.$
- for simply supported: $\sigma_\xi^i=v_i=w_i=0,\ \varphi_j=0,\ i=\overline{0,3},\ j=\overline{0,2}.$
- for free from supported: $\sigma_\xi^i=\sigma_{\xi\theta}^i=\sigma_{\xi z}^i=0,\ \varphi_j=0,\ i=\overline{0,3},\ j=\overline{0,2}.$

2.7. Analytical solution

In order to satisfy the cyclic boundary condition abided by the coordinate, the displacement, potential field and the loads according to the single trigonometric series are expanded as follows:

$$
\begin{aligned}
&q(\xi,\theta)=\sum_{m=1}^{\infty}Q_m(\xi)\cos(m\theta)+Q_0(\xi),\\
&u_i(\xi,\theta)=\sum_{m=1}^{\infty}U_i(\xi)\cos(m\theta)+U_{i0}(\xi),\ i=\overline{0,3},\\
&v_i(\xi,\theta)=\sum_{m=1}^{\infty}V_k(\xi)\sin(m\theta)+V_{i0}(\xi),\ i=\overline{0,3},\\
&w_j(\xi,\theta)=\sum_{m=1}^{\infty}W_j(\xi)\cos m\theta+W_{j0}(\xi),\ j=\overline{0,2},\\
&\varphi(\xi,\theta)=\sum_{m=1}^{\infty}\varphi_m(\xi)\cos(m\theta)+\varphi_0(\xi).
\end{aligned}
\tag{13}
$$

Substituting Eq. (13) into Eq. (11) then perform some mathematical transformations, we obtain differential equations to determine $u_i(\xi,\theta),\ v_i(\xi,\theta),\ w_i(\xi,\theta),\ \varphi_j(\xi,\theta),\ i=\overline{0,3},\ j=\overline{0,2}$ functions as follows

$$
\begin{aligned}
&\sum(Ki^{\varphi_0}_{d2\xi}\frac{\partial^2}{\partial\xi^2}+Ki^{\varphi_0}_{d1\xi}\frac{\partial}{\partial\xi}-m^2Ki^{\varphi_0}_{d2\theta})\varphi_0+\sum_{j=0}^{3}(Ki^{uj}_{d2\xi}\frac{\partial^2}{\partial\xi^2}+Ki^{uj}-m^2Ki^{uj}_{d2\theta})u_j+\\
&+\sum_{j=1}^{3}mKi^{vj}_{d2\xi\theta}\frac{\partial}{\partial\xi}v_j+\sum_{j=0}^{2}Ki^{wj}_{d1\xi}\frac{\partial}{\partial\xi}w_j+(K_i^{q^+_{13}}q^+_{13}+K_i^{q^-_{13}}q^-_{13})=0,\qquad (i=\overline{1,4})\\
&\sum(-mKi^{\varphi_0}_{d2\xi\theta}\frac{\partial}{\partial\xi}+Ki^{\varphi_0})\varphi_0+\sum_{j=0}^{3}-mKi^{uj}_{d2\xi\theta}\frac{\partial}{\partial\xi}u_j+\sum_{j=0}^{3}(Ki^{vj}_{d2\xi}\frac{\partial^2}{\partial\xi^2}-mKi^{vj}_{d2\theta}+Ki^{vj})v_j+\\
&+\sum_{j=0}^{2}-mKi^{wj}_{d1\theta}w_j+(K_i^{q^+_{i3}}q^+_{23}+K_i^{q^-_{i3}}q^-_{23})=0,\qquad (i=\overline{5,8})\\
&(Ki^{\varphi}_{d2\xi}\frac{\partial^2}{\partial\xi^2}+Ki^{\varphi})\varphi_0+\sum_{j=0}^{3}Ki^{uj}_{d1\xi}\frac{\partial}{\partial\xi}u_j+\sum_{j=0}^{2}(Ki^{wj}_{d2\xi}\frac{\partial^2}{\partial\xi^2}+Ki^{wj}-mKi^{wj}_{d2\theta})w_j+\\
&+\sum_{j=0}^{3}mKi^{vj}_{d1\theta}v_j+(K_i^{q^+_{33}}q^+_{33}+K_i^{q^-_{33}}q^-_{33})+(K_i^{\varphi^+}\varphi_++K_i^{\varphi^-}\varphi_-)=0,\qquad (i=\overline{9,11})\\
&\sum(K^{\varphi_0}_{d2\xi}\frac{\partial^2}{\partial\xi^2}-m^2K^{\varphi_0}_{d2\theta}+K^{\varphi_0})\varphi_0+\sum_{j=0}^{3}K^{uj}_{d1\xi}\frac{\partial}{\partial\xi}u_j+\sum_{j=1}^{3}mK^{vj}_{d1\xi\theta}v_j+\\
&+\sum_{j=0}^{2}K^{wj}_{d1\xi}\frac{\partial}{\partial\xi}w_j+(K^+\varphi_++K^-\varphi_-)=0.
\end{aligned}
\tag{14}
$$

The coefficients K are the constants, which depend on geometric parameters, elastic and electrical constants of the shell material. Stresses σ_ξ, $\sigma_{\xi\theta}$ and $\sigma_{\xi z}$ are founded by integrating the equilibrium equation the 3D elasticity theory:

$$\sigma_{\xi z}=-\frac{1}{1+rz}\int_{-h}^{z}\left(\frac{1+rz}{R}\frac{\partial\sigma_{\xi}}{\partial\xi}+\frac{1}{R}\frac{\partial\sigma_{\xi\theta}}{\partial\theta}\right)dz+\frac{(1+rz)_{z=-h}}{1+rz}q_{13}^{-},$$
$$\sigma_{\theta z}=-\frac{1}{(1+rz)^{2}}\int_{-h}^{z}\left(\frac{1+rz}{R}\frac{\partial\sigma_{\theta}}{\partial\theta}+\frac{(1+rz)^{2}}{R}\frac{\partial\sigma_{\xi\theta}}{\partial\xi}\right)dz+\frac{(1+rz)^{2}_{z=-h}}{(1+rz)^{2}}q_{23}^{-}, \quad (15)$$
$$\sigma_{z}=-\frac{1}{1+rz}\int_{-h}^{z}\left(\frac{1+rz}{R}\frac{\partial\sigma_{\xi z}}{\partial\xi}+\frac{1}{R}\frac{\partial\sigma_{\theta z}}{\partial\theta}-\frac{1}{R}\sigma_{\xi}\right)dz+\frac{(1+rz)_{z=-h}}{1+rz}q_{33}^{-}.$$

3. Numerical results and discussion

Let us consider a fully clamped support piezoelectric laminated circular cylinder shell. The piezoelectric layers are assumed to be made of polyvinylidene fluoride (PVDF). The laminated shell includes *3* layers and are made by Graphite-Epoxy (AS/3501) with the angles of fiber reinforcement β= [-90^0/0/90^0]. Elasticity and piezoelectricity properties of materials [19, 25] is given in table 1 and table 2.

Table 1. Elasticity properties of materials.

Property (GPa)	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
PVDF	3.0	1.5	1.5	3.0	1.5	3.0	0.75	0.75	0.75
Graphite-Epoxy	12.0	2.5	2.5	9.6	-3.3	9.6	4.5	5.4	5.4

Table 2. Piezoelectricity properties of materials.

Property*	e_{15}	e_{24}	e_{31}	e_{32}	e_{33}	μ_{11}	μ_{22}	μ_{33}
PVDF	0	0	-30.0	23.0	3.0	3.14	3.08	3.08
Graphite-Epoxy	0	0	0	0	0	0	0	0

* *The unit of matrix e is Cm^{-2} and for the matrix μ is Fm^{-1}.*

Geometrical parameters of shell are used: radius *R = 1.0 m*, length *L = 4R*, relative length $h/R=1/100$. Two cases of circular laminated cylinder shell are considered:

- The shell is under the action of mechanical load on its outer surface of circular shell.
- The shell is under the action of electric potential on its outer surface of circular shell.

Here, the abbreviation 'Clas' corresponds to the results of calculation according to classical theory and 'HSDT' corresponds to results of calculation according to refined theory.

In the first case, mechanical loads are presented sinusoidal function $q_{33}^{+}(\xi,\theta)=Q_0\cos(m\xi)$, $(q_{13}^{\pm}=q_{23}^{\pm}=q_{33}^{-}=0)$ with $Q_0=const$. Stress state of piezoelectric laminated shell is showed in figures 2-3. See figure 2 we can notice that, at the rigidly fixed position for clamped support show additional stress of "boundary layer". The normal stress σ_z, negligible in the classical theory, according to presented theory, account for about 23% of the maximum normal stress σ_ξ .

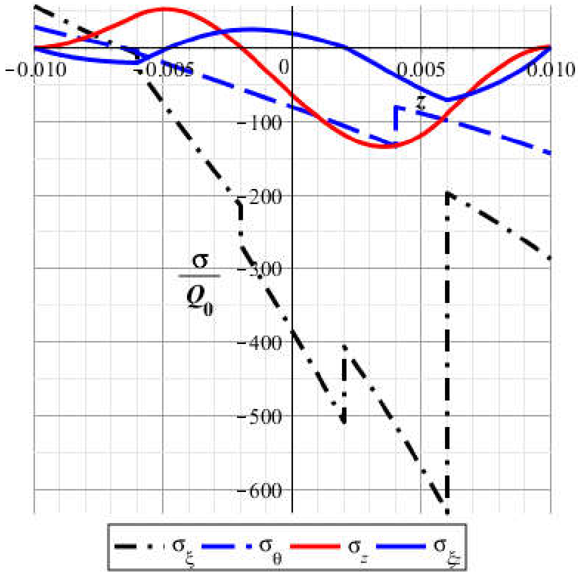

Figure 2. Stress distribution at boundary position by the thickness in case 1.

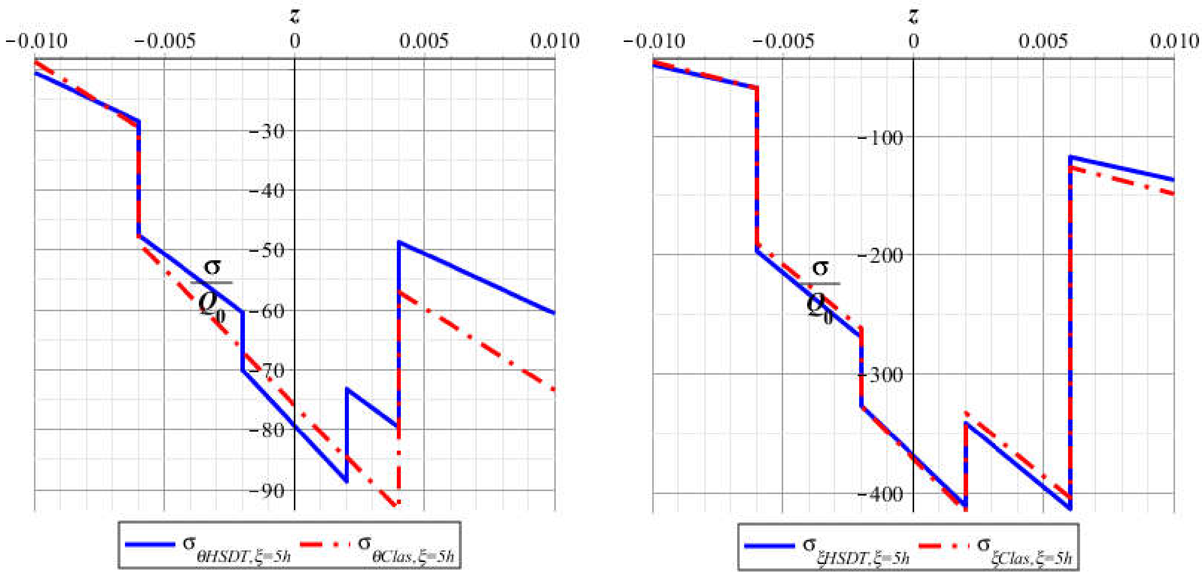

Figure 3. Comparison of stress the distribution σ_ξ and σ_θ at position $\xi = 5h$ by the thickness with classical theory in case 1.

In the second case, electric potentials are considered sinusoidal function $\varphi^+(\xi,\theta) = V_0 \cos(m\xi)$, $(\varphi^-(\xi,\theta) = 0)$ with $V_0 = const$. Similarly as in the first case, we can see the boundary effect at clamped edges of shell. The normal stress σ_z, negligible in the classical theory, according to the revised theory, account for about 33% (figure 4) of the maximum normal stress σ_ξ.

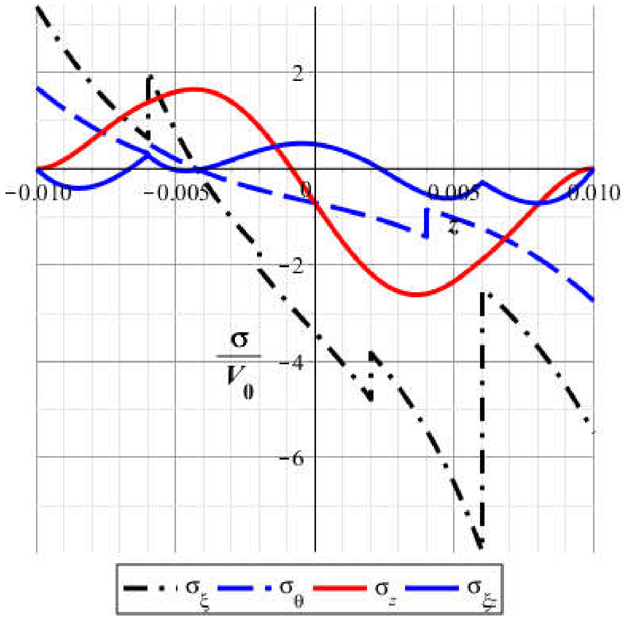

Figure 4. Stress distribution at boundary position by the thickness in case 2.

Comparison with classical theory of Kirchhoff-Love at distance away from edge zone, the distribution of the main stresses σ_ξ and σ_θ of the shell (figure 3 and figure 5) in this work for both cases meet a very good agreement and practically coincides, which confirms the reliability of the results obtained.

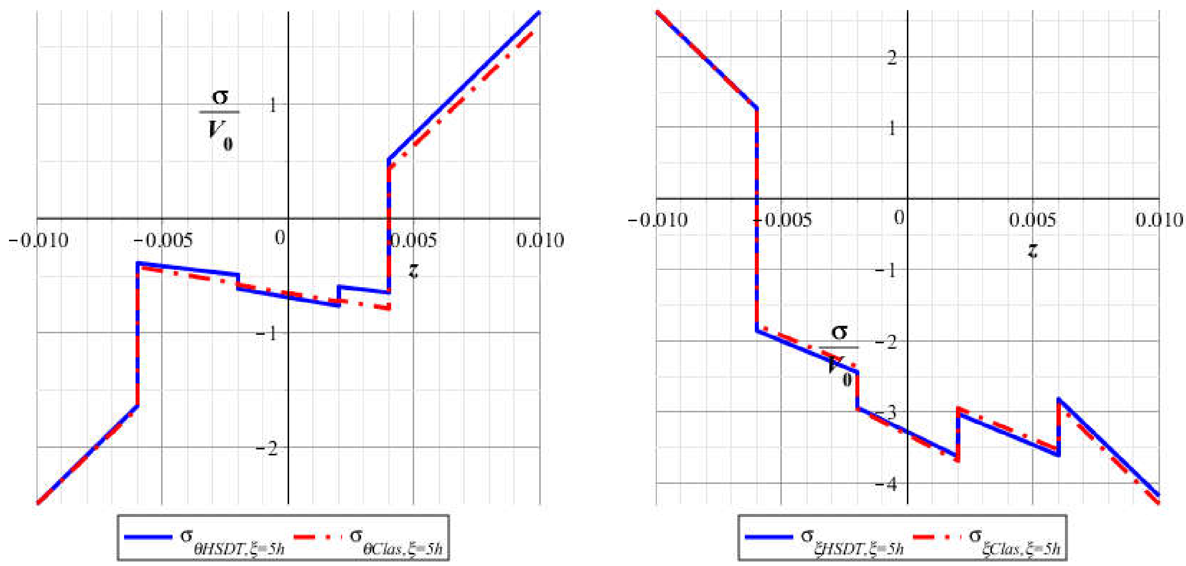

Figure 5. Comparison of stress the distribution σ_ξ and σ_θ at position $\xi = 5h$ by the thickness with classical theory in case 2.

Table 3 shows results of maximum stresses in two cases for calculating by refined theory in this paper and CDT of Kirchhoff-Love.

Table 3. Comparison of maximum stresses to CDT.

Maximum stresses in case 1			
Stress	σ_ξ	σ_θ	σ_z
CDT	645.8	146.5	0
Present	641.5	143.6	145.3
Maximum stresses in case 2			
Stress	σ_ξ	σ_θ	σ_z
CDT	8.3	2.0	0
Present	7.8	2.7	2.6

4. Conclusions

Based on the proposed theory and computed results presented in this work, we have some highlight conclusions as follows:

- The equations for state analyzing the laminated cylindrical shell bonded piezoelectric layers based on refined theory have been derived. The formulae given here can be used to obtain the governing equations for other piezoelectric composite structures and structures mounted piezoelectric layers such as beams and plates.
- Stress state of circular cylindrical laminated shell with piezoelectric layers on the top and bottom surfaces for fully clamped support is studied.
- The results obtained in this work showed there are transverse normal and tangential stresses near clamped edges of laminated shell, which are neglected in classical theory. This additional stresses must be calculated for calculating the strength of elements in various joint designs.

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