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New Solution Determining Optimal Amplitude Distribution for Sparse Cylindrical Sonar Arrays

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*Abstract***:** This paper proposes a new solution to determine the optimal amplitude distribution reducing side-lobe level (SLL) to less than the required value and generate the narrowest halfpower beamwidth (HPBW) in sparse cylindrical sonar arrays. The proposed solution was implemented based on the explicit expression of a beam pattern, separation of the amplitude distribution into the row and the column, and an analysis of the beam patterns from simulations. The merits of the proposed solution were evaluated by simulation in two cases with isotropic elements and directional elements.

*Keywords***:** Phased Array Antenna, Cylindrical Arrays, Sparse Arrays, Sonar Arrays, Amplitude **Distribution**

1. Introduction

ylindrical arrays have been used in many applications, such as radar, sonar, navigation, and communication, due to the capability of 360° coverage using an omnidirectional beam or multiple beams, or a narrow beam that can be steered over 360° [1, 2]. In fish-finding sonar, the cylindrical sonar array is usually mounted below the vessel hull to generate an umbrella-shaped beam pattern in transmit mode and single beams simultaneously steering over 360° in receiver mode [\[3\].](#page-6-0) **C**

As a cylindrical array, the sparse cylindrical sonar array (SCSA) (also called cylindrical arrays with triangular grid [\[4\]\)](#page-6-1) is advantageous in mitigating the number of elements while maintaining the aperture and image quality [\[3\].](#page-6-0) With this advantage, SCSA has been used widely in sonar applications to reduce the complexity of both hardware and operation [1, 3].

With a complicated geometry, it is difficult for designers to reduce the side-lobe level (SLL) and halfpower beamwidth (HPBW) in the beam pattern of SCSA when the main beam is steered to any desired direction in the azimuth plane. Some studies used an Annealing Algorithm [3, 5] to control the SLL and HPBW in the array, which was implemented by adding and removing the non-active elements in the array. With this solution, the computation complexity in determining the beam pattern was increased due to the increased number of calculations. In addition, describing the amplitude distribution mathematically is challenging when controlling SLL by this solution. Another solution for investigating the beam pattern in the azimuth plane used only two consecutive circles [\[6\].](#page-6-2) These solutions in [3, 5, 6] generated high SLLs > -16 dB and difficult to control.

This paper proposes a determination solution of the amplitude distribution generating a low SLL by exploiting the mathematical expression of the beam pattern and analyzing simulation results in two cases with isotropic elements and directional elements. Using the proposed solution, the optimal amplitude distribution was determined to control the SLL to be lower than a required value and obtain the narrowest HPBW corresponding to the SLL. The effectiveness of the proposed solution was evaluated by comparing the beam patterns derived from different amplitude distributions and releasing the optimal distribution satisfying the particular requirements.

2. Geometry Model of SCSA

Fig. 1 (a) presents a geometry model of a full SCSA. In this figure, *R*, *N*, and *P* denote the radius of each circle, number of elements in a circle, and number of circles in the full SCSA, respectively. The angle between two consecutive elements and the distance between two adjacent circles are denoted by $\Delta\theta = 2\pi/N$ and *h*, respectively. Therefore, the total number of elements in the full SCSA is *N*×*P*. When the beam is steered to the desired direction in the azimuth plane (usually in receiver mode), the angle of the active sector in each circle might be chosen as 60°, 90°, or 120° [1, 2], and the number of active elements in each circle is Q ($Q \leq N$) (Fig. 1(b)).

Figure 1. Sparse cylindrical sonar array (SCSA) (a) Full SCSA, (b) SCSA with active elements

With the selection of element at the point A_1 in Fig. 1(b) as the 1st element of the SCSA, the nth element in the pth (1) ≤ *p* ≤ *P*) circle and the q^{th} (1 ≤ q ≤ *Q*) column will satisfy the condition of $n = Q(p-1)+q$. The coordinates were determined as follows [1, 4]

$$
x_n = R\cos\left(\left(q - 1 + \frac{1}{2}\left\lceil\frac{p}{2} - \left\lfloor\frac{p}{2}\right\rfloor\right\rceil\right)\Delta\theta\right) \tag{1}
$$

$$
y_n = R \sin \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta \right) \tag{2}
$$

$$
z_n = (p-1)h \tag{3}
$$

where $\lceil t \rceil$ and $\lfloor t \rfloor$ denote the round functions toward integers of arbitrary real number $t: \lceil t \rceil = \min\{n \in \mathbb{Z}, n \ge t\}$ and $\lfloor t \rfloor$ = $max \{ n \in Z, n \le t \}.$

Based on the explicit expressions of the coordinates for each element in the array, the phase distribution and beam pattern can be determined when steering the main beam to any desired direction [\[4\].](#page-6-1) By exploiting the mathematical

expression of the beam pattern and using simulation tools, the optimal amplitude distribution was determined to control the SLL and HPBW when elements in the array were isotropic and directional (Section 3).

3. Determination of Optimal Amplitude Distribution Controlling SLL and HPBW

When the phase reference ($\psi_1 = 0$) is chosen at position $A_1 = (R, 0, 0)$, to steer the main beam to point M_0 d determined by the unit direction vector $\vec{u}_0 = (\cos\theta_0 \cos\varphi_0, \sin\theta_0 \cos\varphi_0, \sin\varphi_0)$, the phase of the *n*th element at position $A_n = (x_n, y_n, z_n)$ must be excited as follows [\[4\]:](#page-6-1)

*A*¹ 0 0 0 0 0 0 0 1 (,) cos 1 1 cos cos 2 2 2 1 sin 1 sin cos (1) sin 2 2 2 *n p p kR q p p kR q k p h* = − − + − − − − + − − − (4)

where $k = \frac{2\pi}{4}$ $=\frac{2\pi}{\lambda}$ is the wavenumber, and λ is the

wavelength.

The far-field continuous-wave (CW) beam pattern of the array with $P\times Q$ isotropic elements in direction (θ, φ), [1, 4]

which is called the array factor (AF), is determined to be
\n[1, 4]
\n
$$
\int \exp\left(j k R \left(\cos \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta \right) - 1 \right) (\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0) \right)
$$
\n
$$
AF(\theta, \varphi) = \sum_{n=1}^{p \cdot \varphi} a_n \left(\sec \varphi \left(j k R \sin \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta \right) (\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0) \right)
$$
\n
$$
\times \exp \left(j k (p - 1) h (\sin \varphi - \sin \varphi_0) \right)
$$
\n(5)

Based on expression (5), the amplitude distribution *aⁿ* is separated into the product of distributions on the row and column as follow[s \[1\]:](#page-6-3)

$$
a_n = a_p \times a_q \tag{6}
$$

 λ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$

J

where a_p and a_q are the amplitude distributions on the column and the row, respectively.

To investigate the beam pattern of SCSA with directional elements, the element patterns of the nth element in the azimuth and the evaluation were suitably chosen as the forms, $1+\cos(\theta_{en})$ and $1+\cos(\phi_{en})$, respectively [2, 7]. In these formulae, θ_{en} is the angle between the normal vector of the radiation surface of the nth element and considered direction in the azimuth plane, and φ_{en} is the angle between this normal vector and considered direction in the evaluation plane*.* When elements are arranged as shown in Fig. 1, *φen* is also the

desired evaluation angle *φ*, and *θen* is determined as follows:

$$
\cos(\theta_{en}) = \cos\left(\left(q - 1 + \frac{1}{2}\left[\frac{p}{2} - \left\lfloor \frac{p}{2} \right\rfloor\right]\right) \Delta \theta - \theta\right) \tag{7}
$$

As a result, the beam pattern of SCSA when considering the beam pattern of each element can be expressed as d as
 $\left[\begin{array}{c} a_{1} \times a_{1} \times \left(1+\cos\left(\left(a-1+\frac{1}{a}\right)\frac{p}{a}\right)\right) \times \left(1+\cos\left(\varphi\right)\right) \\ a_{2} \times a_{3} \times \left(1+\cos\left(\left(a-1+\frac{1}{a}\right)\frac{p}{a}\right)\right) \times \left(1+\cos\left(\varphi\right)\right) \\ \text{res}\right] \end{array}\right]$

expressed as
\n
$$
F(\theta,\varphi) = \sum_{n=1}^{P\cdot\mathbb{Q}} \begin{vmatrix} a_p \times a_q \times \left(1 + \cos \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta - \theta \right) \right) \times (1 + \cos(\varphi)) & \text{resy} \\ \times \exp \left(jkR \left(\cos \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta \right) - 1 \right) (\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0) & \text{azin} \\ \times \exp \left(jkR \sin \left(\left(q - 1 + \frac{1}{2} \left[\frac{p}{2} - \left[\frac{p}{2} \right] \right] \right) \Delta \theta \right) (\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0) & \text{cosh} \end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix}
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\n
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\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix}
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\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix}
$$
\n
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\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \\
\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0\n\end{vmatrix
$$

In the evaluation plane, the SCSA can be considered a uniform linear array antenna (ULAA) with uniformly spaced elements by distance *h*. Therefore, the amplitude distribution on column *a^p* was selected according to the Dolph-Chebyshev distribution [2, 8].

In the azimuth plane, to control the SLL and HPBW in the beam pattern of SCSA, the amplitude distributions on row *a^q* were chosen to be a uniform window, Dolph-Chebyshev window, Gaussian window, Hamming window, Taylor window, and Kaiser window. A raised cosinesquared weighting (A cosine squared on a pedestal distribution) was also chosen to control the SLL and HPBW in the beam pattern of SCSA [9, 10]. This raised cosine-squared distribution (aperture distribution) was determined to be [10]

$$
a_q(q) = \Delta + (1 - \Delta)\cos^2\left(\frac{q - \frac{Q + 1}{2}}{Q - 1}\right)
$$
 (9)

where Δ denotes the pedestal level, which determines the SLL and HPBW in the beam pattern [\[10\]](#page-6-4). When $\Delta = 1$, the aperture distribution becomes a uniform distribution. With $\Delta = 0.08$, the aperture distribution becomes a Hamming distribution [\[9\].](#page-6-5)

With the selected distributions, the beam pattern of SCSA was analyzed in the azimuth plane by the simulation tools to determine the optimal amplitude distribution satisfying the particular requirements of a lower SLL than a required value and the narrowest HPBW. Based on this solution, the optimal amplitude will be determined as section 4 in two cases with isotropic elements and directional elements.

4. Simulation Results

To determine the optimal amplitude distribution satisfying the requirements of the SLL and the narrowest HPBW, this study considered an example of SCSA with 54 elements (*N* $= 54$) on a circle and 16 circles ($P = 16$). Choosing the active sector as 120°, the number of active elements on a circle was 19 $(Q = 19)$, and the total number of active elements in the array was $16\times19 = 304$ elements.

The simulation was carried out at frequency $f = 30$ kHz, with the velocity of sound in seawater being $c = 1500$ m/s. The distances between two consecutive circles and between two consecutive elements on a circle were chosen as $h = 0.5 \times \lambda = 2.5$ (cm) and $d = 0.8 \times \lambda = 4$ (cm), respectively. As a result, the radius of a circle in the SCSA was $R = 34.46$ (cm). The desired steering angles in the azimuth plane and elevation plane were assumed to be $\theta_0 = 60^\circ$ and $\varphi_0 = 0^\circ$, respectively.

As a ULAA in the vertical direction, a Dolph-Chebyshev with the side-lobe attenuation (SLA) -25 dB was chosen for a_p to control the SLL and HPBW. To determine the optimal amplitude distribution providing a lower SLL than a required value and the narrowest HPBW in the horizontal direction, the amplitude distributions were selected; the beam patterns were simulated according to the selected amplitude distributions, and the simulation results were compared. The simulations were implemented for two cases with isotropic elements and directional elements of the form $1+\cos(\theta_{en})$.

To generate a beam pattern with a lower SLL than -23 dB when isotropic elements are used, the amplitude distributions *a^q* in the azimuth plane were chosen, which included: Dolph-Chebyshev window *SLA* = -45 dB, Gaussian window with the standard deviation $\sigma = 2.09$, Hamming window, Taylor window with nearly constantlevel side-lobes adjacent to the mainlobe $n = 5$ and a maximum side-lobe level ASLL = -45.5 dB [\[8\],](#page-6-6) Kaiser window with $\beta = 5$, and raised cosine-squared weighting with $\Delta = 0.16$. In addition, a uniform window was also chosen to compare the parameters of the beam patterns. Based on MATLAB software and reference (9), these distributions can be generated, and the beam patterns with isotropic elements and directional elements were also generated to compare the results.

Fig. 2 (a) and Fig. 2 (b) show the simulation results of the beam pattern of SCSA in the azimuth plane in the two cases. In these figures, the beam patterns obtained from uniform distribution are denoted by curves with black squares. The beam patterns derived from the Dolph-Chebyshev distribution, Gaussian distribution, Hamming distribution, Taylor distribution, Kaiser distribution, and raised cosine-squared distribution ($\Delta = 0.16$) are denoted by solid blue curves, dashed red curves, dashdot brown curves, dotted cyan curves, curves with green triangles, and curves with magenta circles, respectively. The parameters of beam patterns, including SLL and HPBW, are depicted more clearly in Table 1.

From Fig. 2(a) and table 1, a uniform distribution provides the narrowest HPBW but generates an uncontrollably high SLL (-10.20 dB), which is higher than in ULAA (approximately -13 dB [\[10\]\)](#page-6-4). In the investigated amplitude distributions generating an $SLL < -23$ dB in the

case with isotropic elements, the Hamming distribution provided the lowest SLL (-25.06 dB) but generated a wide HPBW (5.44°) and an uncontrollable SLL. Compared to the amplitude distributions providing an $SLL < -23$ dB, the raised cosine-squared distribution ($\Delta = 0.16$) can provide both an SLL \le -23 dB (-23.99 dB) and the narrowest HPBW (5.04°). The others, including the Dolph-Chebyshev distribution (SLA =-45 dB), Taylor distribution (ASLL = -45.5 dB), Kaiser distribution (β =5), and Gaussian distribution ($\sigma = 2.09$) generated both a higher SLL and a wider HPBW than the above aperture distribution.

Considering the element patterns in the azimuth plane of the form $1+\cos(\theta_{en})$, the SLLs decreased significantly compared to when neglecting the beam pattern of each element, as shown in Fig. 2(b) and Table 1. In the amplitude distributions mitigating the SLL, except for the Kaiser distribution (β = 5) generating an SLL of -24.89 dB, the others provided $SLLs < -25$ dB. In contrast to the $SLLs$, HPBWs increased slightly compared to the case with isotropic elements. By comparing the HPBWs, when element patterns in azimuth plane were chosen in the form $1+\cos(\theta_{en})$, the raised cosine-squared distribution (Δ = 0.16) also provided the narrowest HPBW (5.24°) according to $SLL < -25$ dB (-26.57 dB). Therefore, the aperture distribution ($\Delta = 0.16$) is the optimal amplitude distribution for SCSA according to the SLL and HPBW in the two cases with isotropic elements and directional elements.

Figure 2. Beam patterns of SCSA with the amplitude distributions (a) Isotropic elements, (b) Directional elements

	Isotropic elements		Directional elements				
Distribution for a_q	SLL (dB)	HPBW(°)	SLL (dB)	HPBW(°)			
Uniform	-10.20	3.76	-11.57	3.92			
Dolph-Chebyshev $(SLA = -45 dB)$	-23.16	5.3	-25.13	5.5			
Gaussian (σ = 2.09)	-23.16	5.1	-25.54	5.30			
Hamming	-25.06	5.44	-26.98	5.62			
Taylor $(ASLL = -45.5 dB)$	-23.08	5.27	-25.07	5.46			
Kaiser (β = 5)	-23.09	5.49	-24.89	5.66			
Raised cosine-squared distribution ($\Delta = 0.16$)	-23.99	5.04	-26.57	5.24			

Table 1. Parameters of the SCSA Beam Patterns with the Amplitude Distributions

To investigate the change in SLL and HPBW according to parameter Δ of the aperture distribution, this study considered the beam pattern of SCSA with directional elements of the form $1+\cos(\theta_{en})$ when Δ changes from 0 to 0.22 with step 0.02. Table 2 lists the parameters of beam pattern, including SLL and HPBW.

A larger parameter Δ indicated a lower HPBW (Table 2). On the other hand, SLL did not increase monotonously according to Δ . When Δ changed from 0 to 0.1, the SLL of the beam pattern decreased gradually and achieved the

minimum value (-27.20 dB) at $\Delta = 0.1$. With pedestal levels larger than 0.1, the SLLs rose along with the increases in pedestal levels (Δ) . Based on expressions (8) and (9), Fig. 3 presents the beam patterns of SCSA in four cases with $\Delta = 0.2$, $\Delta = 0.4$, $\Delta = 0.6$, and $\Delta = 0.8$ when steering the main beams to $(60^{\circ}, 0^{\circ})$. In this figure, solid red curve, dashed blue curve, dashdot green curve, and dotted black curve express the beam patterns in cases Δ = 0.2, Δ = 0.4, Δ = 0.6, and Δ = 0.8, respectively.

Table 2. Parameters of the SCSA Beam Patterns with the Values of Pedestal Levels (Δ)

Pedestal level (Δ)	Directional elements			Directional elements	
	SLL (dB)	HPBW(°)	Pedestal level (Δ)	SLL (dB)	HPBW(°)
0	-24.26	6.16	0.12	-27.19	5.42
0.02	-25.11	6.01	0.14	-26.96	5.32
0.04	-25.88	5.87	0.16	-26.57	5.24
0.06	-26.52	5.81	0.18	-26.06	5.16
0.08	-26.98	5.62	0.20	-25.46	5.09
0.10	-27.20	5.52	0.22	-24.82	5.02

Figure 3. Beam patterns of the SCSA with the parameters Δ

According to Table 2 and Fig.3, parameter Δ was determined to achieve beam patterns with SLLs decreasing to -27.20 dB and the narrowest HPBW. To achieve SLLs < -27.20 dB, a Gaussian distribution with a large σ can be chosen as the optimal amplitude distribution because, except for a raised cosine-squared distribution, the Gaussian distribution provides the narrowest HPBW according to the SLL less than requirement value, as listed in Table 1.

For example, with $\sigma = 2.5$, a beam pattern with SLL -33.24 dB and HPBW 5.93° for the above configured SCSA based on (8) were obtained. With the Gaussian distribution, a larger σ indicates a lower SLL. In contrast to the SLL, HPBW decreased with increasing σ. Therefore, parameter σ should be larger than 2.5 to reduce SLL to less than -33.24 dB.

With the explicit expression of the beam pattern and the analysis of the simulation results, the parameters of the optimal distributions (the aperture distribution in cases of SLL decreasing to -27.20 dB and Gaussian distribution in the other cases) were also determined when changing the required SLL value and requiring the narrowest HPBW. Therefore, it is more flexible and simpler for designers to choose the optimal amplitude distribution for SCSA using the proposed solution compared to using the annealing algorithm.

To evaluate the merits of the proposed solution more clearly, the beam pattern in the azimuth plane of SCSA raised by the amplitude distribution in [\[11\]](#page-6-7) was compared with that derived by the optimal amplitude distribution from the proposed solution. The comparison was implemented based on expression (8) with the same conditions of SCSA configured as above, and the element patterns of each element in the form $1+\cos(\theta_{en})$. For simplicity, the amplitude distribution on the row in this reference was determined by considering the cylindrical array as a simpler circular array, whose individual elements represent the columns of the cylindrical array [\[11\].](#page-6-7) To reduce SLL, the amplitude distribution in [\[11\]](#page-6-7) was chosen as a cosine on a pedestal distribution with a pedestal value equal to 0.4 [\[9\].](#page-6-5) With this distribution, the beam pattern in the azimuth plane had an SLL of -15.94 dB and HPBW of 4.43°. Based on the proposed solution, the optimal amplitude distribution on the row for the above SCSA was the raised cosine-squared distribution (Δ = 0.57), which provided a beam pattern in the azimuth plane with $SLL = -16.07$ dB and HPBW = 4.30°. Fig 4 presents these beam patterns in the azimuth plane. The amplitude distribution from the solution generated both a lower SLL and narrower HPBW than the amplitude distribution in [\[11\].](#page-6-7)

Figure 4. Beam patterns of the SCSA obtained from the conventional solution and proposed solution

The proposed solution can be used for SCSA in sonar systems owing to its flexibility and simplicity. On the other hand, experiments have not been carried out using the proposed solution because of the lack of measurement equipment. This work will be implemented in the future.

5. Conclusion

This paper proposed a new solution for determining the optimal amplitude distribution in SCSA that reduces the

SLL to \lt -23 dB in cases with isotropic elements and reduces the SLL to \lt -25 dB in cases with directional elements, and generated the narrowest HPBW in each case. Based on the proposed solution, SLL and HPBW in the beam pattern of SCSA have been controlled to ensure the particular requirements for SLL and HPBW in two cases with isotropic elements and directional elements. In addition, by analyzing the simulation results, the distribution parameters were defined when the requirements of SLL and HPBW were changed.

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