Identification Vibration Characteristics of Structures by Operational Modal Analysis (OMA) Technique

Trung Duc Tran, Anh Tuan Le, and Dinh Huong Vu

Abstract The paper presents how to identify natural frequencies and mode shapes of structures by Operational Modal Analysis (OMA) technique, in which the Frequency Domain Decomposition (FDD) method is used. This method is an experimental method only base on the data of measuring the dynamic response of the structures under the excitation due to ambient forces and operational loads to determine the vibration characteristics. Measure vibration (acceleration) and determine spectral density matrix, using the singular values decomposition method of spectral density matrix to determine the natural frequencies and mode shapes of structures. The calculation results show that the natural frequencies, the mode shapes form determined by the OMA technique is consistent with the calculation results according to the theory and show the reliability of the method.

Keywords Natural frequency · Mode shape · Identify · EMA · OMA · FDD

1 Introduction

The use of experimental tests to obtain information about the dynamic response of buildings is an important content in the inspection of the structure and monitoring of the building's health. The activity of the building structure is expressed as a combination of modes, each of which is characterized by a set of parameters (natural frequency, damping ratio, mode shape) and depends characteristics of geometry, materials and boundary conditions [3, 4, 7].

Experimental Modal analysis (EMA) determines these parameters from measurements of applied force and structural response [7]. EMA have been applied in various fields, such as automotive engineering, aerospace engineering, industrial machinery and construction engineering. The determination of dynamic parameters by EMA technology becomes more difficult in the case of building structures because of their large size and low frequency range.

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Applying measurable and controllable stimuli is often a difficult work that requires expensive and complex equipment. For this reason, researchers have recently focused more on the advantages provided by Operational Model Analysis (OMA) techniques [3, 4]. The OMA allows the testing of estimating structural dynamics parameters only from vibration response measurements. The idea behind OMA is to take advantage of the natural excitation that is available from surrounding forces (wind, vehicle, shock, etc.) to replace artificial stimulation. Since the OMA only requires the measurements of the structure's dynamic response under operating conditions, when subjected to ambient stimulation, it is also called different names, such as identifying surrounded vibration pattern or analyze only the output model (Output-only). OMA techniques include methods such as frequency domain decomposition method (FDD) [4, 5], stochastic subspace identification (SSI) method [8].

The paper presents the theoretical basis to determine the dynamic parameter of the structure according to the theory, vibration measurement test of structure and determine the natural frequencies, the mode shapes of the steel beam structure by OMA technique uses frequency domain decomposition (FDD) method.

2 Methods

2.1 Analytical Method to Determine the Dynamic Parameters of Cantilever Beams

Consider a beam structure have any distribution mass m(x), with distributed load q (x,t) [1] (Fig. 1).

Differential equation for free vibration without considering the effect of resistance is written in the form.

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2X}{dx^2}\right] = \omega^2 m(x)X \tag{1}$$

In which, E is the elastic modulus of the beam material, J (x) is the moment of inertia of the beam cross-section, X is the bending form of beam structure (mode shape) only depends on x, ω is natural frequency, m(x) is the mass per unit length, x is the distance from the fixed end.

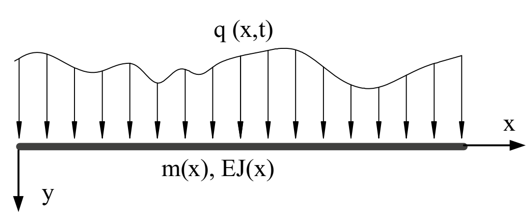

Fig. 1 Analytical diagram

If beams have constant stiffness and mass evenly distributed, we have:

$$\frac{d^4X}{dx^4} - \omega^2 \frac{m}{EJ} X = 0 \tag{2}$$

With the above equation and the boundary conditions corresponding to the cantilever beam, we can write the formula to calculate the specific vibration frequency as follows:

$$\omega_i = \alpha_i^2 \sqrt{\frac{EJ}{ml^4}} \tag{3}$$

In which, E is the elastic modulus of the beam material, J is the moment of inertia of the beam cross-section, m is the mass per unit length, l is the length of the cantilever beam. α_i is the coefficient, get the values $\alpha_i = 1,875;\ 4,694;\ 7,885\ ; \ldots; \pi(2i+1)/2$.

Corresponding to the natural frequency ω_i, we have the ith mode shape.

2.2 *Frequency Domain Decomposition (FDD) Method*

Frequency domain decomposition is proposed by Brincker et al. [5]. This method decomposes the spectral density matrix at each frequency into singularity values and singularity vectors by the singular value decomposition (SVD). Frequency domain decomposition is an extension of the basic frequency domain technique or commonly known as the Pick Peaking technique, in which natural frequencies is identified by finding peaks in the spectral density matrix.

The relationship between unknown input x(t) and measured response output y(t) can be expressed as follows:

$$[G_{yy}(\omega)] = [H(\omega)]^*[G_{xx}(\omega)][H(\omega)]^T \tag{4}$$

where:

$[G_{xx}(\omega)]$ is the Power Spectral Density (PSD) matrix of the input;
$[G_{yy}(\omega)]$ is the PSD matrix of the responses;
$[H(\omega)]^*$ is the complex conjugate matrix of Frequency Response Function (FRF);
$[H(\omega)]^T$ is the transpose matrix of FRF.
The FRF can be written in partial fraction

$$[H(\omega)] = \sum_{1}^{N} \frac{[\mathrm{R}_k]}{j\omega - \lambda_k} + \frac{[\mathrm{R}_k]^*}{j\omega - \lambda_k^*} \tag{5}$$

$$\lambda_k = -\sigma_k + j\omega_{dk} \tag{6}$$

where:n is the number of modes, λ_k is the pole of the kth mode shape, σ_k is minus the real part of the pole and ω_{dk} is the damped natural frequencies of the k^{th} mode shape.

[R_k] is the residue expressed as follows.

$$[\mathrm{R}_k] = \phi_k.\gamma_k^T \tag{7}$$

where: ϕ_k is the mode shape vector, γ_k the modal participation vector.

Suppose the input is white noise, its power spectral density is constant or. $[G_{xx}(\omega)] = \mathrm{C}$, (C is constant). Formula (4) is rewritten as follows:

$$[G_{yy}(\omega)] = \sum_1^N \sum_1^N \left[\frac{[\mathrm{R}_k]}{j\omega - \lambda_k} + \frac{[\mathrm{R}_k]^*}{j\omega - \lambda_k^*} \right] .C. \left[\frac{[\mathrm{R}_k]}{j\omega - \lambda_k} + \frac{[\mathrm{R}_k]^*}{j\omega - \lambda_k^*} \right]^T \tag{8}$$

Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows:

$$[G_{yy}(\omega)] = \sum_1^N \frac{[A_k]}{j\omega - \lambda_k} + \frac{[A_k^*]}{j\omega - \lambda_k^*} + \frac{[B_k]}{-j\omega - \lambda_k} + \frac{[B_k^*]}{-j\omega - \lambda_k^*} \tag{9}$$

where: $[A_k]$ is the kth residue matrix of the output PSD.

At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Thus, in the case of a lightly damped structure, the response spectral density can always be written:

$$[G_{yy}(\omega)] = \sum_{k \in Sub(\omega)} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{d_k^* \phi_k^* \phi_k^{*T}}{j\omega - \lambda_k^*} \tag{10}$$

where: $k \in \mathrm{Sub}(\omega)$ is the set of modes be denoted at a specific frequency, ϕ_k is the mode shape vector and λ_k is the pole of the kth mode shape.

The Frequency domain decomposition technique is based on the singular value decomposition of the Hermitian response spectral density matrix.

$$[G_{yy}(\omega)] = [\mathrm{U}][S][\mathrm{U}]^H \tag{11}$$

where: [S] is a diagonal matrix holding the scalar singular values, [U] is a unitary matrix holding the singular vectors and $[U]^H$ is a Hermitian matrix.

From vibration measurement data of the structure (acceleration), we calculate the spectral density matrix $[G_{yy}(\omega)]$ and decompose the singular value according to formula (11) to determine the natural frequencies of the structure.

Table 1 The physical parameters of the test structure

No.	Parameter	Value	Unit
1	Length	710	mm
2	Density weight	7850	kg/m^3
3	Modulus of elasticity	2.03×10^5	Mpa
4	Width	60	mm
5	Height	8	mm

3 Test on Real Structures

3.1 Test Objectives

The test to obtain dynamic responses (acceleration) of steel beam structures at nodes over time. The result of vibration measurement is used to identify the natural frequencies, mode shapes of the structure.

3.2 Test Model

Test structure is a steel beam. The physical parameters of the structure are shown in Table 1.

3.3 Test Equipment

The equipment used in the test is listed in Table 2.

Table 2 The physical parameters of the test structure

No.	Equipment name	Code	Company	Measuring range	Quantity
1	Vibration measurement equipment	NI cDAQ-9137	National Instrument	Multi -channel	01
2	Accelerometer	PCB 352C68	PCB Group	± 50 g (100 mV/g)	01
3	Accelerometer	PCB 353B33	PCB Group	± 50 g (100 mV/g)	01

3.4 Test Layout

The test layout for determining the natural frequencies of the steel beam is arranged as shown in Fig. 2. In which, using two accelerometer sensors to measure the vibration of the beam, the position of the sensors is shown in Fig. 3, the NI cDAQ-9137 Connected with accelerometer sensors and display. Accelerometer measurements are collected and displayed through the NI Signal Express software pre-installed.

Proceed with the installation and install parameters for measuring equipment, Create vibration for the structure by any stimulus is large enough for the structure to work in the elastic stage. The measured data are recorded as the value of the acceleration overtime at the location where the acceleration is mounted.

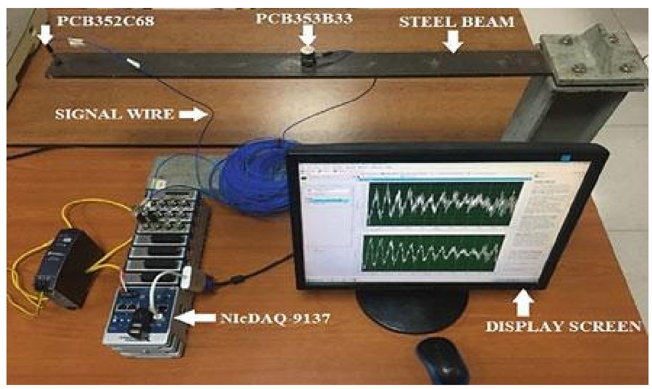

Fig. 2 Experiment setup of the real structure

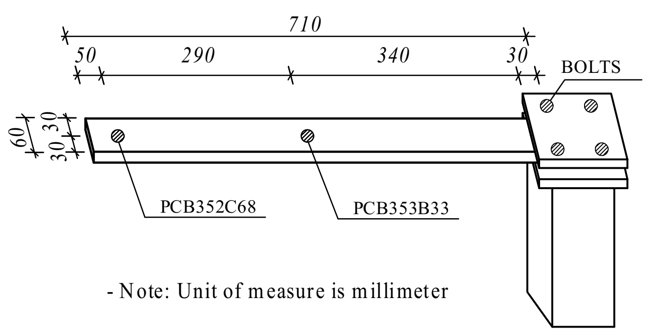

Fig. 3 The position of the sensors

4 Results

4.1 Vibration Results of the Structure

After measuring the vibration of the structure, acceleration at the nodes on the steel girder structure is obtained over time. The data of one measurement is shown in Figs. 4 and 5.

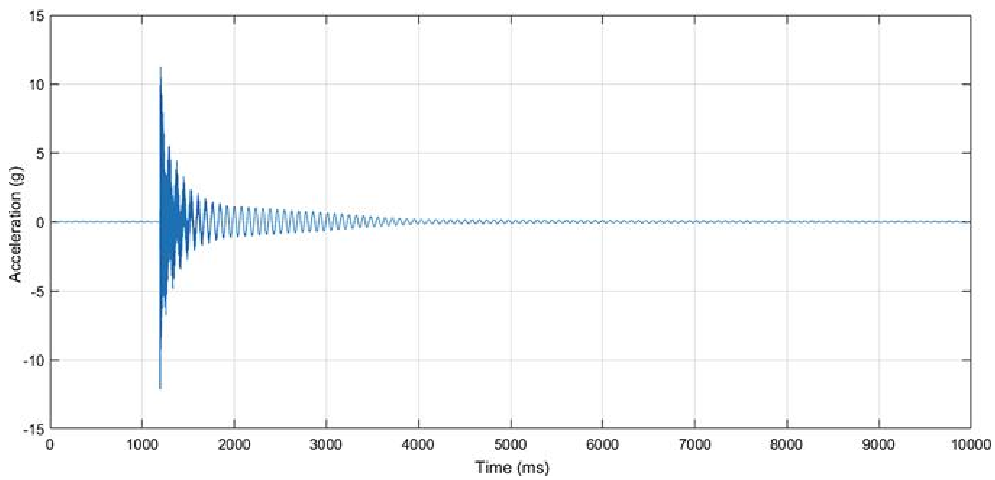

Fig. 4 Results of acceleration at the middle of the beam

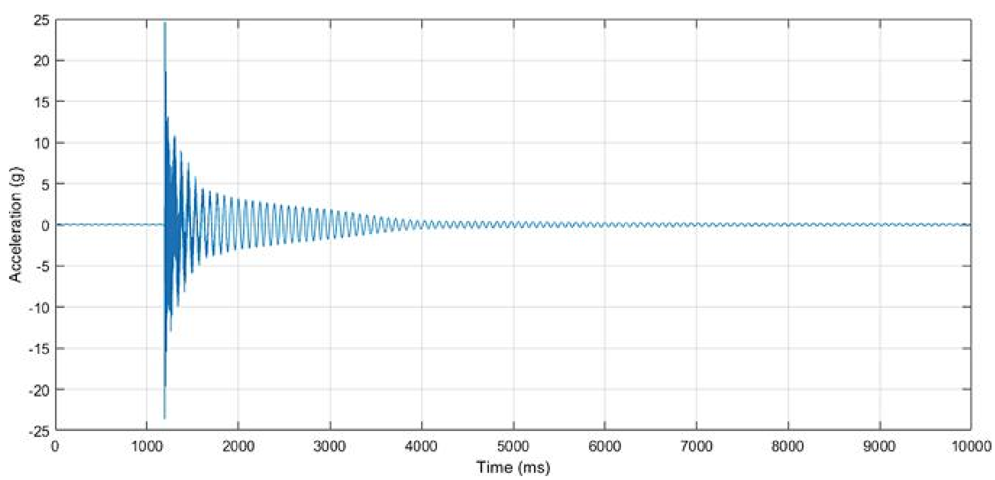

Fig. 5 Results of acceleration at the free position of the beam

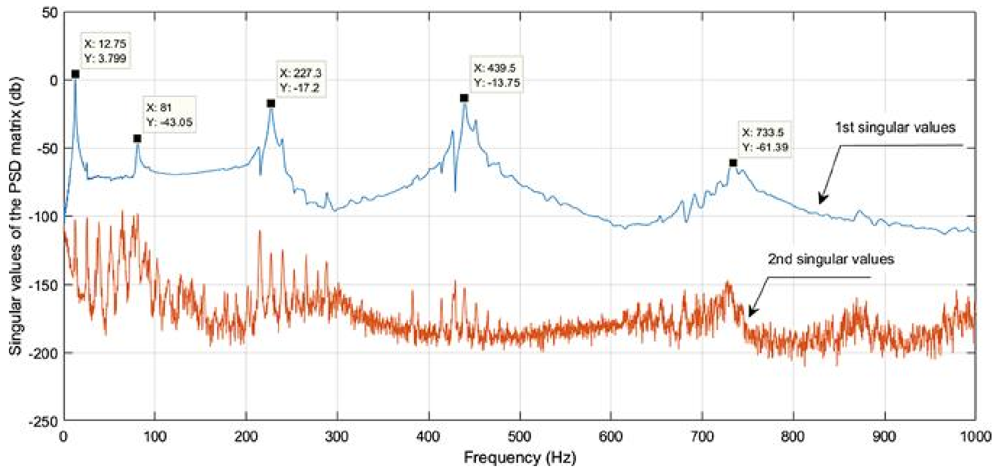

Fig. 6 Power spectral density (PSD)

Table 3 Comparison of natural frequencies between methods

No.	Mode	FDD (Hz)	EMA (Hz)	Error (%)	Theory (Hz)	Error (%)
1	1	12.75	12.8	0.4	12.9	1.2
2	2	81.0	79.8	1.5	80.9	0.1
3	3	227.3	228.6	0.6	226.6	0.3
4	4	439.5	446.1	1.5	444	1.01
5	5	733.5	735.6	0.3	734	0.07

4.2 The Identification Results of Natural Frequencies

With the acceleration data obtained from the experiment, calculate and estimate the power spectral density according to Welch's estimation method and resolve the singularity values by SVD algorithm according to formula (8). We determine the natural frequencies of the structure corresponding to the positions of the maximum power spectral density function. Results of identifying the five natural frequencies are shown in Fig. 6.

Comparing the natural frequencies obtained by the FDD method and the results of the calculation of the natural frequencies by the experimental modal analysis (EMA) method [2] and according to theory [1] are shown in the Table 3.

4.3 Identify Mode Shapes

Most OMA methods provide their results in the form of complex eigenvalues and complex eigenvectors. Since the estimates of specific vibrational-form are in the form of complex vectors, a distinction is needed between the real modes, characterized by the real oscillator vector real values and the complex modes. From SVD singularity resolution, we can determine the complex eigenvectors corresponding to the corresponding frequencies, at the specific vibration frequency values there are specific vibrations of the structure, real part of the vector particularly is the amplitude of the structure vibration at the locations put the accelerometer head.

To accurately determine the specific vibration pattern of the structure, it is necessary to use many vibration probes located at different positions. Because only two accelerometers are used, it is necessary to carry out many measurements, the fixed sensor is used as a reference and move the other sensor at different positions. Through the measurements, determine the amplitude of the vibration at the positions and standardize and determine mode shapes of the structure.

Take three measurements, the position of the accelerometer in the measurements is shown as the Fig. 7.

From the measured data, calculated according to FDD, we get the value of amplitude of variation corresponding to the types of vibration in Table 4.

Carry out the combination of amplitudes of the same vibration form separately and draw on the proportions, we get the mode shapes as follows (Fig. 8).

From the results of identifying the natural frequency and mode shapes form by OMA technique, it shows that the natural frequency is very close to the results calculated by the forced excitation method and analytical method, the mode shapes as calculated according to theory. Thus, shows the consistency between theory and experiment and confirms the reliability of the method.

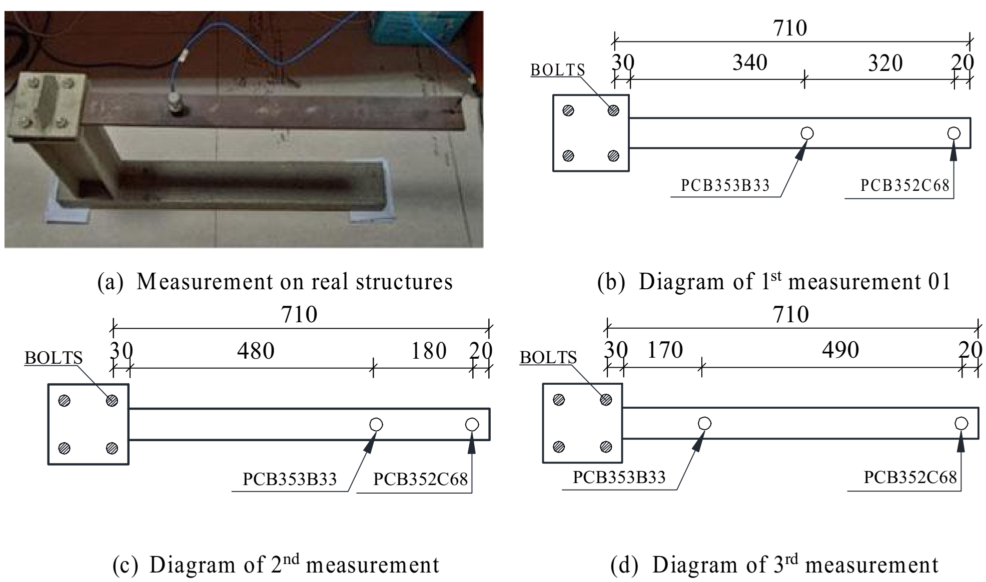

Fig. 7 Experimental diagram to determine the specific vibration pattern

Table 4 The amplitude value

Measured times	Mode	Natural frequency (Hz)	Range of vibration		Amplitude normalization	
			Sensor position 01	Sensor position 02	Sensor position 01	Sensor position 02
1	1	12.75	− 0.9327	− 0.3606	1	0.387
	2	81	− 0.7887	0.6132	1	− 0.777
	3	227.3	− 0.9935	0.1141	1	− 0.115
2	1	12.75	− 0.8204	− 0.5659	1	0.69
	2	81	− 0.9918	0.1253	1	− 0.126
	3	227.3	− 0.8143	0.5802	1	− 0.713
3	1	12.75	− 0.9908	− 0.1352	1	0.136
	2	81	− 0.8361	0.5466	1	− 0.654
	3	227.3	− 0.7323	− 0.681	1	0.93

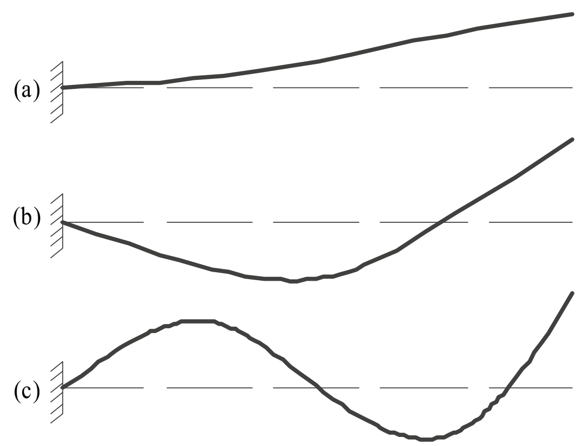

Fig. 8 Mode shapes of the beam **a** 1st mode shape, **b** 2nd mode shape, c 3rd mode shape

5 Conclusion

The paper presents the content of the Operational model analysis (OMA) method, conducting tests on real structures, and identifies the natural frequencies and mode shapes of the steel beam structure.

The results of identifying are consistent with the natural frequency obtained by the forced excitation method and theoretically calculated, with small errors and mode shapes consistent with the calculation theory. This shows the reliability of the experimental and the identification method.

Operational model analysis technique can be developed for the identification of the damping ratio of structures, and for application in monitoring, diagnosing the health of structures and applications in the optimization of shock absorbers. Passive fluctuations, reduce construction damage when it is affected by earthquakes.

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