Global sensitivity analysis for bridge crane system by surrogate modeling

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Abstract. As key equipment for lifting and transporting duties, cranes are used in various industrial fields in modern productions. Thus, the dynamic problem of such crane system is commonly encountered in the design process. This paper presents a Monte Carlo-based global sensitivity analysis of the dynamic model of a bridge crane system using the surrogate model technique. To this regard, physical modeling and differential equation motion of a coupled crane system is first derived using the Lagrange equation. Then, the numerical solution is offered by using the Newmark-β integral method for characterizing dynamic responses of the crane system involving bridge beam, trolley, and payload. In order to compute Sobol sensitivity indices, the input-output correlation is formulated by a neural network-surrogate model formed from the numerical solutions. Finally, for the considered configuration, the importance levels of input variables with the corresponding estimated values of the first and total order sensitivity functions are demonstrated.

Keywords: Overhead crane, dynamic response, global sensitivity, uncertainty, Monte Carlo simulation, neural network-based surrogate.

1 Introduction

Owing to the potential functions, overhead or bridge-type cranes (Fig 1.a) play a significant role in the modern production lines [1]. Vibration of structures of such system subjected to moving loads is commonly encountered in engineering. Cranes are typical of nonlinear mechanical systems in which the motion of the trolley normally results in an undesirable swing of the payload. The dynamic response of this system could be captured by analyzing the girder vibration under a trolley moving without [2, 3] and with [4-6] considering the payload swing.

Generally, the dynamic response of a certain crane system is influenced by its model involving factors (e.g., geometric or material properties, loading conditions). In the conventional dynamic analysis of a whole crane or its component, model parameters are commonly treated as deterministic ones [2-6]. However, in general, the component of this system such as beam, trolley, and payload the have nondeterministic characteristics because of uncertainties in the construction and manufacturing processes as well as due to aging and operational conditions. Probabilistic methods [7, 8] (such as Monte-

Carlo simulations, perturbation, and stochastic finite element method) are widely used for understanding the static and dynamic analysis of structures with random parameters. Thanks to these frameworks, a better description of structural dynamic responses or reliability based-design optimization can be found. For simply structure, system response can be deduced directly from the solving motion equation [9]. However, the more complex system with a higher input number should require a meta-model such as polynomial or machine learning surrogates.

The present work aims at studying the dynamic behavior of the crane system within uncertain parameters. A neural network-based surrogate is first constructed for characterizing dynamics behavior based on the reference data estimated numerically. Then the sensitivity functions of the input variables on the dynamic responses are estimated using Monte Carlo simulations to measure the influence of these inputs on the uncertainty margins of the system response.

2 Mathematical modeling and dynamic response

A model of a bridge crane system is depicted in Fig. 1. The main girder or beam of the crane is introduced by a flexible body with its vertical vibration under consideration. This girder is simply supported and modeled by the unit mass m_b and the length of L. The properties of the beam are defined by Young's modulus E and an inertia moment I. The crane trolley is assumed as a point mass with a mass of m_c traveling at a speed of v, while the payload m_p is assumed to swing angle θ around the trolley center. The suspension cable is simplified without mass, and the rope length of l is kept constant during the trolley moving.

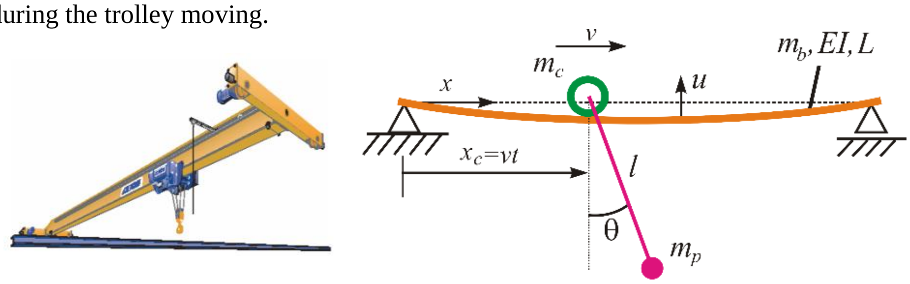

Fig. 1. Crane system [10] (left) and its dynamic model (right) within the swing of payload.

The vertical deflection of the flexible girder can be formulated as follows:

$$u(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t), \;\; \text{with} \;\; \phi_i(x) = \sin\left(\frac{i\pi x}{L}\right), \tag{1}$$

where ϕ_i is the i^{th} modal of the simply supported beam, while $q_i(t)$ and N are the generalized coordinates and coordinate numbers of the elastic displacement of the main girder, respectively.

Using the Lagrange equation, the differential equation motion of the bridge crane was established. The kinetic energy of the coupled system T, includes the girder kinetic energy, the trolley kinetic energy, and the payload kinetic energy, can be expressed by:

$$\begin{aligned} T = &\frac{1}{4} m_b L \sum_{i=1}^{N} \dot{q}_i^2 + \frac{1}{2} m_c \left\{ \dot{x}_c^2 + \left[\dot{x}_c \sum_{i=1}^{N} \phi_i'(x_c) q_i(t) + \sum_{i=1}^{N} \phi_i(x_c) \dot{q}_i(t) \right]^2 \right\} + \\ &\frac{1}{2} m_p \left\{ \left(\dot{x}_c + \dot{\theta} l \cos\theta \right)^2 + \left[\dot{x}_c \sum_{i=1}^{N} \phi_i'(x_c) q_i(t) + \sum_{i=1}^{N} \phi_i(x_c) \dot{q}_i(t) - \dot{\theta} l s\text{in}\,\theta \right]^2 \right\}. \end{aligned} \tag{2}$$

The total potential energy of the coupled crane system V is:

$$V = \frac{EI\pi^4}{4L^3} \sum_{i=1}^{N} i^4 q_i^2(t) - m_c g w(x_c, t) + m_p g \left[w(x_c, t) + l(\cos\theta - 1) \right]. \tag{3}$$

By applying the Lagrange equation, the motion of the system can be derived as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\dot{\boldsymbol{u}} + \boldsymbol{C}\boldsymbol{u} = \boldsymbol{P}, \tag{4}$$

where $\ddot{\boldsymbol{u}}$, $\dot{\boldsymbol{u}}$, and $\boldsymbol{u}$ are the acceleration, velocity, and displacement vector with $\boldsymbol{u} = \left[\theta, q_1, q_2, ..., q_N\right]^T$. $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ denote the mass, damping, and stiffness matrices of the system, and $\boldsymbol{P}$ is a time-dependent loading vector. They are detailed as:

$$\begin{aligned} \boldsymbol{M} &= \begin{bmatrix} m_p l^2 & -2m_p l \sin\theta[\phi_i(x_c)] \\ -2m_p l \sin\theta[\phi_i(x_c)]^{\mathrm{T}} & \frac{m_b L}{2}[\mathbf{I}]_{N\times N} + 2\left(m_p + m_c\right)\mathrm{diag}[\phi_i(x_c)][\boldsymbol{\phi}_i(x_c)] \end{bmatrix}, \\ \boldsymbol{K} &= \begin{bmatrix} \frac{m_p g l \sin\theta}{\theta} & -2m_p l \sin\theta \left\{ \ddot{x}_c[\phi_i'(x_c)] + \dot{x}_c^2[\phi_i''(x_c)] \right\} \\ [\mathbf{0}]_{N\times 1} & \mathrm{diag}[\frac{EI\pi^4 i^3}{2L^3}] + 2\left(m_p + m_c\right) \left\{ \begin{matrix} \ddot{x}_c \mathrm{diag}[\phi_i(x_c)][\boldsymbol{\phi}_i'(x_c)] \\ +\dot{x}_c^2 \mathrm{diag}[\phi_i(x_c)][\boldsymbol{\phi}_i''(x_c)] \end{matrix} \right\} \end{bmatrix}, \\ \boldsymbol{C} &= \begin{bmatrix} 0 & -2m_p l x_c \cos\theta[\phi_i'(x_c)] \\ -2m_p l x_c \sin\theta[\phi_i'(x_c)]^{\mathrm{T}} & 2\left(m_p + m_c\right)\dot{x}_c \mathrm{diag}[\phi_i(x_c)][\boldsymbol{\phi}_i'(x_c)] \end{bmatrix}, \\ \boldsymbol{P} &= \begin{bmatrix} -m_p \ddot{x}_c l \cos\theta & (m_p + m_c)[\phi_i(x_c)]^{\mathrm{T}} \end{bmatrix}. \end{aligned} \tag{5}$$

where $[\boldsymbol{\phi}_i]$ is a matrix having all duplicate rows of $[\phi_i] = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_N \end{bmatrix}$.

Herein, we use the Newmark-β method to solve the matrix equation (4), the acceleration and velocity vectors of the system at (t+Δt) can be discretized as follows [11]:

$$\begin{aligned} \ddot{\boldsymbol{u}}(t+\Delta t) &= a_0\left[\boldsymbol{u}(t+\Delta t) - \boldsymbol{u}(t)\right] - a_2\boldsymbol{u}(t) - a_3\ddot{\boldsymbol{u}}(t), \\ \dot{\boldsymbol{u}}(t+\Delta t) &= \boldsymbol{u}(t) - a_6\boldsymbol{u}(t) - a_7\ddot{\boldsymbol{u}}(t+\Delta t), \end{aligned} \tag{6}$$

in which the velocity vector $\boldsymbol{u}$ at a time (t+Δt) and coefficients are estimated as:

$$\begin{aligned} &\boldsymbol{u}(t+\Delta t) = \tilde{\boldsymbol{K}}(t+\Delta t) \setminus \tilde{\boldsymbol{P}}(t+\Delta t), \\ &\tilde{\boldsymbol{K}}(t+\Delta t) = \boldsymbol{K}(t) + a_0 \boldsymbol{M}(t) + a_1 \boldsymbol{C}(t), \\ &\tilde{\boldsymbol{P}}(t+\Delta t) = \boldsymbol{P}(t+\Delta t) + \boldsymbol{M}(t)\left[a_0 \boldsymbol{u}(t) + a_2 \dot{\boldsymbol{u}}(t) + a_3 \ddot{\boldsymbol{u}}(t)\right] \\ &\qquad + \boldsymbol{C}(t)\left[a_1 \boldsymbol{u}(t) + a_4 \dot{\boldsymbol{u}}(t) + a_5 \ddot{\boldsymbol{u}}(t)\right], \end{aligned} \tag{7}$$

$$\begin{aligned} &a_0 = \frac{1}{\beta \Delta t^2},\ a_1 = \frac{\gamma}{\beta \Delta t^2},\ a_2 = \frac{1}{\beta \Delta t},\ a_3 = \frac{1}{2\beta} - 1, a_4 = \frac{\gamma}{\beta} - 1, \\ &a_5 = \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 1\right),\ a_6 = \Delta t(1-\gamma),\ a_7 = \gamma \Delta t,\ \beta = \frac{1}{4},\ \gamma = \frac{1}{2}. \end{aligned} \tag{8}$$

3 Neural network surrogate and sensitivity analysis

Neural network surrogate is used to construct the relation between the input variables with the output or the system response. The reconstructed neural network is shown in Fig. 2. This architecture consists of four layers, including an input layer (i.e., EI, m_b, m_c, m_p, l), two hidden layers, and an output layer (i.e., u and $\ddot{u}$). The bias or the last neuron in the three left layers is denoted by terms (+1).

In order to generate the reference data for NN training, the above-described procedure for estimating the system dynamics is implemented.

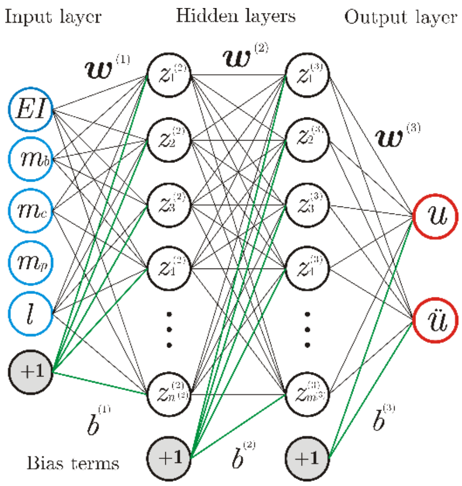

Fig. 2. NN architecture.

The NN variables could typically be a set of raw data, so a normalization task is required to ensure such data relies on the same range of the activation function used (e.g., [0, 1] for a case of the standard sigmoid, $f(v)$=1/[1+exp(-v)]). Herein, the input and output are normalized using their minimum and maximum values as,

$$\bar{\xi} = \frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}} \tag{9}$$

The constructed meta-model serves a great support in terms of the cost of function call for a sensitivity analysis in which the effects (e.g., sensitivity index) of an input factor on the output are measured. In the Sobol's approach, the idea for computation of sensitivity indices is to decompose the function $y(t,\boldsymbol{x})$ into terms of increasing dimensionality as [12]:

$$y(t,x) = f_0(t) + \sum_{i=1}^{d} f_i(t,x_i) + \sum_{1\le i<j\le d} f_{ij}(t,x_i,x_j) + \ldots + f_{1,\ldots p}(t,x_1,\ldots,x_d), \qquad (10)$$

where $\boldsymbol{x} = (EI, m_b, m_c, m_p, l)$ is input variable vector defined on the domain K^d.

We use the Monte-Carlo based numerical procedure to compute the full set of the first and total effect sensitivity indices for the considered model. First, we generate two matrices of data ($\boldsymbol{A}$ and $\boldsymbol{B}$ of size $N\times d$), and a matrix $\boldsymbol{C_i}$ formed by all columns of $\boldsymbol{B}$ except the i^{th} column, which is taken from $\boldsymbol{A}$. Then, the sensitivity indices can be estimated as [13]:

$$\begin{aligned} S_i(t) &= \frac{\frac{1}{N}\left(\sum_{j=1}^{N} y(t,\boldsymbol{x}_A^{(j)})\,y(t,\boldsymbol{x}_{Ci}^{(j)})\right) - \left(\frac{1}{N}\sum_{j=1}^{N} y(t,\boldsymbol{x}_A^{(j)})\right)^2}{\frac{1}{N}\left(\sum_{j=1}^{N}\left(y(t,\boldsymbol{x}_A^{(j)})\right)^2\right) - \left(\frac{1}{N}\sum_{j=1}^{N} y(t,\boldsymbol{x}_A^{(j)})\right)^2}, \\ S_{Ti}(t) &= 1 - \frac{\frac{1}{N}\left(\sum_{j=1}^{N} y(t,\boldsymbol{x}_B^{(j)})\,y(t,\boldsymbol{x}_{Ci}^{(j)})\right) - \left(\frac{1}{N}\sum_{j=1}^{N} y(t,\boldsymbol{x}_A^{(j)})\right)^2}{\frac{1}{N}\left(\sum_{j=1}^{N}\left(y(t,\boldsymbol{x}_A^{(j)})\right)^2\right) - \left(\frac{1}{N}\sum_{j=1}^{N} y(t,\boldsymbol{x}_A^{(j)})\right)^2}. \end{aligned} \qquad (11)$$

with $\boldsymbol{x}_{A,B,C_i}^{(j)}$ are row vectors of the sampling matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, respectively.

4 Results and discussion

The bridge crane configuration has the follow parameters [14, 15]: L = 6 m, m_b = 163.2 kg/m, $EI = 4.50\times10^4$ Nm^2, $m_c = m_p$ = 97.9 kg/m, $l = L/3$ (see Fig. 1a). Here, the initial swing angle of the payload is $\theta(0)$ = - 0.01 rad, and trolley speed is v = 0.4 m/s.

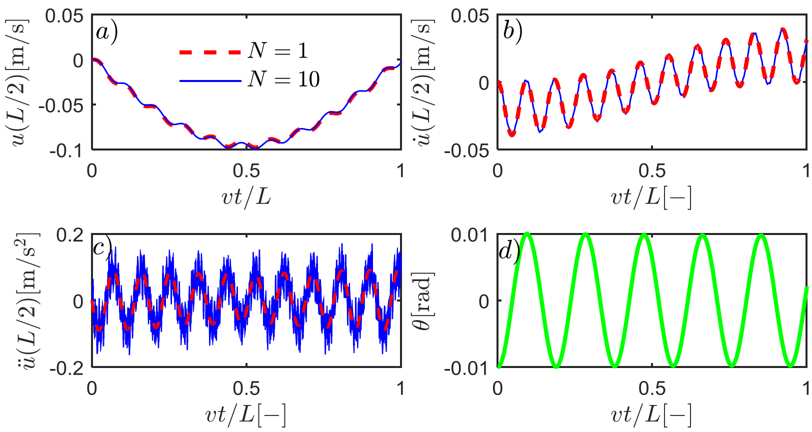

Fig. 3. Dynamic response of the crane system: (a-c) beam midpoint, and (d) payload.

From the curves in Fig. 3 that the solution obtained by considering only the first mode (N=1) is very close to that obtained by considering ten first modes (i.e., N=10) only for displacement and velocity responses (Fig. 3a-b), while there is a great difference in terms of acceleration behavior (Fig. 3c). This suggests that the high modes cannot be neglected for such factor. The reference data for NN model are produced within the dynamic behavior of ten first modes. Using the fast Fourier transfer for the swing angle of the payload, the obtained swing frequency of ~ 0.333 Hz is slightly lower than $f_p = 1/2\pi\sqrt{g/l}$. The above observations are consistent with Refs. [10, 15].

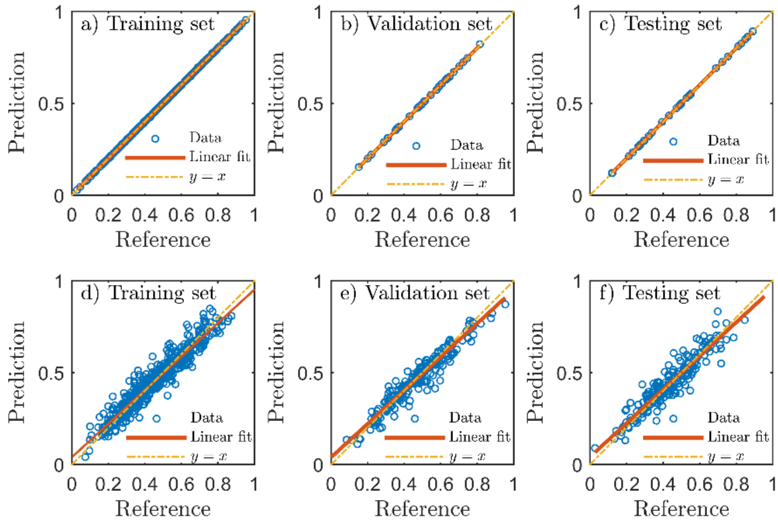

Fig. 4. Performance of the proposed NN model within the training dataset.

In the established NN architecture, both hidden layers have ten neurons. To generate data for the NN model, the sampling dataset of 2×10^4 data points. For training and testing purposes, the data is randomly divided into three distinct sets: a training set (80 % of the data), a validation set (10 %), and a test set (10 %). Here, as a demonstration, we consider two output metrics as the maximum displacement and acceleration of the girder. For the first output u_{max}, regression graphs for the dataset shown in Fig. 4a-c clearly demonstrate good predictability of the proposed NN model, in which the output values perfectly track the target ones (i.e., root mean squared error is approximately 1). It can be noticed that we only use a sampling of 10^3 points for such case. However, in Fig. 4d-f for $\ddot{u}_{\max}$, the results show a lower degree of correlation (i.e., ~0.956). This confirms again the nonlinear dynamic feature of the considered system.

Now, we perform the sensitivity analysis on the established NN model. Fig. 5 presents the calculation results of sensitivity index S and S_T with the output metric of $u_{\max}$ (left panel) and $\ddot{u}_{\max}$ (right panel), respectively. Based on the investigated configurations, the properties of the girder (EI) and the trolley mass (m_c) are the most important factors, while others related to the payload (i.e., m_p, l) and mass m_b should be neglected except in the effect of interaction between m_b with others. In addition, for velocity output with $\sum S_i > 0.994$ it is seen that all high order sensitivity indices can be ignored, while for acceleration behavior with $\sum S_i < 0.745$, we should consider high order effects or interactions of the girder properties including its mass per length unit m_b.

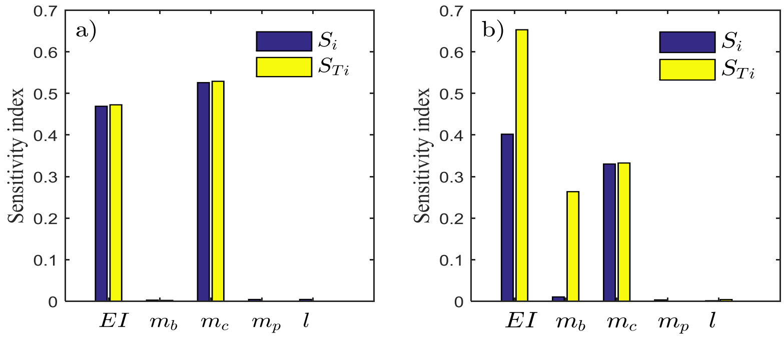

Fig. 5. Results of sensitivity index S and S_T with the output of u_{max} (a) and $\ddot{u}_{max}$ (b).

5 Conclusion

This work deals with the sensitivity analysis for a dynamic model of a crane system using the surrogate technique and MC simulations. The global sensitivity analysis method is applied to the dynamic model through the constructed NN model. The constructed NN model offers an accurate prediction of dynamic behavior, whereas time consumption for NN simulations is significantly reduced compared with numerical simulations (e.g., a computing six-hour task can be undertaken by the NN model in less than a second). The results reveal that the sensitivity functions of input variables are highly dependent on output metrics (i.e., vertical displacement and acceleration). Based on the investigated configurations, the properties of the girder (EI) and the trolley mass (m_c) are the most important factors, while others (e.g., related to the payload) play an important role. In addition, the system acceleration behavior as the output, we should consider high order effects or interactions of the girder properties including its mass distribution. For both cases, the uncertainties in factors related to the payload seem to have no effect on the system response.

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