Eur. Phys. J. Plus (2021) 136:1199
https://doi.org/10.1140/epjp/s13360-021-02191-4

THE EUROPEAN
PHYSICAL JOURNAL PLUS

Regular Article

Propagation of non-stationary kinematic disturbances from a spherical cavity in the pseudo-elastic cosserat medium

Lai Thanh Tuan[1], **Nguyen Thai Dung**[1], **Do Van Thom**[2,a], **Phung Van Minh**[2], **Ashraf M. Zenkour**[3]

[1] Faculty of Special Equipment, Le Quy Don Technical University, Hanoi City 100000, Vietnam
[2] Faculty of Mechanical Engineering, Le Quy Don Technical University, Hanoi City 100000, Vietnam
[3] Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Received: 13 August 2021 / Accepted: 17 November 2021

Abstract In this work, the Cosserat model is used to simulate non-stationary processes in various structures of composite materials. A non-stationary axisymmetric problem of the propagation of kinematic perturbations from a spherical cavity in a space filled with a homogeneous isotropic pseudo-elastic Cosserat medium is considered. The motion of the medium is represented by a set of three equations written in a spherical coordinate system with the origin at the center of the cavity and nonzero components of the displacement vector and rotation field potentials. At first, it is supposed that the plane wave or spherical wave's front makes contact with the hollow surface. The initial-boundary value issue is mathematically formulated in dimensionless form. A serial expansion of Legendre and Gegenbauer polynomials, as well as the Laplace transform in time, is utilized to obtain the solution. The issue is simplified to a set of independent ordinary differential equations with the Laplace operator applied to the series coefficients. Due to the complexity of images of the series coefficients, to determine the originals in the linear approximation, the Laurent series for images in the vicinity of the start time is employed. The findings indicate that the solutions found using limit techniques are consistent with previously published results for the classical elastic medium. For the granular composite material of aluminum fractions in the epoxy matrix, examples of computations are presented.

1 Introduction

The majority of non-stationary disturbance propagation research [1, 2] is now done in a traditional elastic medium. As a result, there are very few papers on this subject for material models that account for angular momentum. Therefore, these models are used to investigate the functioning of various composite material constructions, including various kinds of weapons .

[a] e-mail: thom.dovan@lqdtu.edu.vn (corresponding author)

Published online: 01 December 2021

The Cosserat brothers investigated the general asymmetrical elastic theory [3]. When analyzing the stress state of a solid deformable continuum, it is essential to include moment stresses that create asymmetric tensors, according to the Cosserat brothers' idea, which takes into consideration the rotating interaction of material particles. There are many fascinating scientific studies connected to this innovation published, readers may discover them in the papers [4–9]. The current scenario of generalized continuum mechanics and its future possibilities are addressed in the works [2] and [3], in which the monograph [11] is concerned with the formulation, analysis, and development of the moment model of a linearly deformable medium with strong effects. The possibility of the wave nature of changes in stresses and deformations has been established, the gradients of which are manifested at the level of simple stresses and strains in the form of new generalized concepts—bulk moment stresses and the corresponding all-round tension deformations—compression and rotation, while Smolin et al. [10] provided an overview of models for the mechanics of generalized medium, with a focus on the Cosserat medium. This model is given to describe the plastic deformation of metallic materials with sub-microcrystalline and nanostructures. The papers [12–14] explore nonlinear moment theories of elasticity. The equations of nonlinear dynamics, energy laws, and wave impulse change for mediums with moment stresses are obtained in [12, 13]. The propagation characteristics of planar periodic and solitary waves are studied. The issue of their stability in the face of transverse disturbances is addressed. Cao and colleagues [14] discussed the application of Cosserat medium dynamics to the nonlinear issue of out-of-plane dynamics of elastic rods. In works [15–17], the issues of the Cosserat thermo-elastic medium are addressed, where Bîrsan [15] addressed the initial-boundary value issue of the linear dynamics of thermo-elastic Cosserat shells with cavities. The mutual and unique theorems of the solution are established. The problem's ongoing reliance on external volumetric forces, temperature effects, and starting circumstances are also examined. Kumar and Gupta [16] investigated wave propagation in a transversally isotropic moment-thermo-elastic space. In Nistor [17], acceleration waves are investigated as moving surfaces on which acceleration discontinuities and temperature gradients suffer within the context of the moment theory of thermoelasticity for an anisotropic body. Liu et al. [18] addressed stress concentration issues that take into consideration instant effects. The issue of a plane with a circular hole uniform in infinite tension in one direction is addressed in the article within the context of the Cosserat theory. The system of equations for the stress function and the torque stress function is solved by isolating the variables. The solution is given in finite form, and it includes certain functions that satisfy ordinary differential equations and are responsible for the stress concentration at the hole's border.

Kumar and Sharma [19] also considered the steady-state mechanisms of wave propagation in the Cosserat medium. This paper looks at plane waves in a moment-thermo-elastic medium without dissipating the energy that fills the half-space. The coefficients of reflection of waves corresponding to various angles of incidence at isothermal half-space borders are calculated. The papers [20–22] addressed non-stationary issues of moment elastic medium. The dynamic issue of the moment theory of elasticity concerning a finite length crack under normal stress on the banks is reduced to a set of singular integral equations for displacements and rotations in [20], which are solved numerically. Furthermore, in the paper [21], the dynamic issue for micro-polar elastic bodies is studied utilizing the eigenvalues technique. In this instance, Fourier transforms in spatial coordinates and Laplace transforms in temporal coordinates are used to address the issues. Saxena and Dhaliwal [22] considered a dynamically linked axisymmetric issue of the micro-polar theory of elasticity for an isotropic medium infinite in the radial direction. In recent years, numerous scientists have investigated the computation

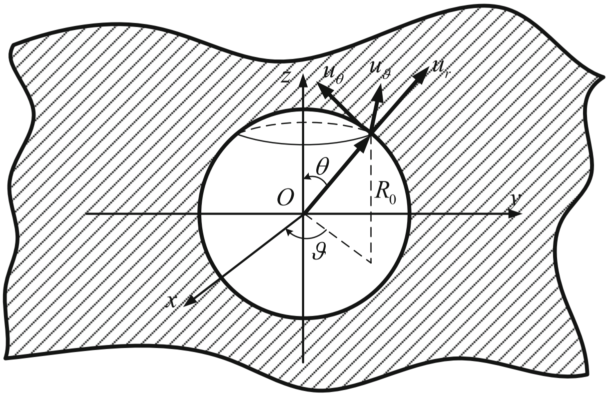

Fig. 1 The model of space filled with the Cosserat pseudo-continuum with a spherical cavity

of plate and shell structures using a variety of various methodologies, and they have obtained some useful discoveries [23–29].

However, several non-stationary problems for moment medium (particularly moment medium with constrained rotation) in the case of spherical interfaces have not yet been sufficiently studied, including the propagation of non-stationary perturbations from a spherical cavity, from the boundary of a solid sphere, and the diffraction of non-stationary waves by a spherical cavity.

The purpose of this paper is to formulate and establish analytical solutions to two-dimensional problems involving the propagation of unsteady axisymmetric boundary disturbances in a "non-classical" elastic medium with spherical boundaries, the model of which is one of the variants of the asymmetric theory of elasticity—the Cosserat pseudo-continuum.

The remainder of this paper is organized as follows. Section 2 delves into the governing equations of the suggested issues. Section 3 introduces the method for defining original functions. The simulation results and comments are presented in Sect. 4. The novel findings of this work are drawn in Sect. 5.

2 Governing equations

2.1 Formulations

The space filled with an isotropic homogeneous pseudo-continuum and a spherical cavity of radius R_0 and center O is considered as shown in Fig. 1. The momentum vector equation for displacement in the absence of mass force has the following form [30, 31]:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{grad}\,div\,\mathbf{u} + 1/4(\gamma+\varepsilon)\mathrm{rot}\,rot\,\Delta\mathbf{u} \quad (1)$$

in which ρ, λ, and μ are the density and the elastic coefficients, respectively; γ and ε are the additional physical properties of the medium; Δ is Laplace operator; t is the time. A spherical coordinate system $r, \theta,$ and υ with center O is used in this proposed model.

It assumes that the model is axially symmetric, which implies that the unknown functions are solely dependent on time, radius r, and angle θ. Using the decomposition of the displacement field into potential and vortex components, the tangential ν, normal w displacements through the scalar potential φ, and the nonzero component of the vector potential ψ are expressed as follows:

$$u_r = w = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\left(\frac{\partial\psi}{\partial\theta} + \psi\,\text{ctg}\theta\right),\quad u_\theta = v = \frac{1}{r}\left(\frac{\partial\varphi}{\partial\theta} - \psi\right) - \frac{\partial\psi}{\partial r},\quad u_\vartheta \equiv 0 \tag{2}$$

and Eq. (1) is replaced by the following equivalent system:

$$\begin{aligned}
&\ddot{\varphi} = \Delta\varphi\\
&\ddot{\psi} - \frac{1-\kappa}{2}\left(\Delta\psi - \frac{\psi}{r^2\sin^2\theta}\right) + \frac{\eta+\xi}{4}\left(\Delta\psi_* - \frac{\psi_*}{r^2\sin^2\theta}\right) = 0\\
&\psi_* = \Delta\psi - \frac{\psi}{r^2\sin^2\theta}\\
&\Delta = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\frac{\partial}{\partial\theta}\right)\right]
\end{aligned} \tag{3}$$

The coordinates of the vector of the angle of rotation are related to the displacements as follows:

$$\omega_r = \omega_\theta \equiv 0,\quad \omega_\vartheta = \omega = \frac{1}{2r}\left[\frac{\partial}{\partial r}(rv) - \frac{\partial w}{\partial\theta}\right] \tag{4}$$

The components of the tensor of deformations and bending-torsion are defined by the following equations:

$$\begin{aligned}
&\varepsilon_{rr} = \frac{\partial w}{\partial r},\quad \varepsilon_{r\theta} = \frac{\partial v}{\partial r} - \omega,\quad \varepsilon_{\theta r} = \frac{1}{r}\left(\frac{\partial w}{\partial\theta} - v\right) + \omega,\\
&\varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v}{\partial\theta} + w\right),\quad \varepsilon_{\vartheta\vartheta} = \frac{1}{r}(w + v\text{ctg}\theta);
\end{aligned} \tag{5}$$

$$\chi_{r\vartheta} = \frac{\partial\omega}{\partial r},\quad \chi_{\theta\vartheta} = \frac{1}{r}\frac{\partial\omega}{\partial\theta},\quad \chi_{\vartheta r} = -\frac{\omega}{r},\quad \chi_{\vartheta\theta} = -\frac{\omega}{r}\text{ctg}\theta. \tag{6}$$

Physical relations for the considered medium are defined as follows:

$$\begin{aligned}
&\mu_{r\vartheta} = \xi_+\chi_{r\vartheta} + \xi_-\chi_{\vartheta r},\quad \mu_{\theta\vartheta} = \xi_+\chi_{\theta\vartheta} + \xi_-\chi_{\vartheta\theta},\quad \mu_{\vartheta r} = \xi_+\chi_{\vartheta r} + \xi_-\chi_{r\vartheta},\\
&\mu_{\vartheta\theta} = \xi_+\chi_{\vartheta\theta} + \xi_-\chi_{\theta\vartheta},\quad \xi_+ = \eta + \xi,\quad \xi_- = \eta - \xi
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\sigma_{rr} = \varepsilon_{rr} + \kappa(\varepsilon_{\theta\theta} + \varepsilon_{\vartheta\vartheta}),\quad \sigma_{r\theta} = \sigma_{r\theta s} - \sigma_{r\theta *},\quad \sigma_{\theta r} = \sigma_{r\theta s} + \sigma_{r\theta *},\\
&\sigma_{\theta\theta} = \kappa(\varepsilon_{rr} + \varepsilon_{\vartheta\vartheta}) + \varepsilon_{\theta\theta},\quad \sigma_{\vartheta\vartheta} = \kappa(\varepsilon_{rr} + \varepsilon_{\theta\theta}) + \varepsilon_{\vartheta\vartheta},\quad \sigma_{r\theta s} = \frac{1-\kappa}{2}(\varepsilon_{r\theta} + \varepsilon_{\theta r}),\\
&\sigma_{r\theta *} = \frac{1}{2}\left\{\frac{\partial\mu_{r\vartheta}}{\partial r} + \frac{1}{r}\left[\frac{\partial\mu_{\theta\vartheta}}{\partial\theta} + 2\mu_{r\vartheta} + \mu_{\vartheta r} + (\mu_{\theta\vartheta} + \mu_{\vartheta\theta})\text{ctg}\theta\right]\right\}
\end{aligned} \tag{8}$$

where $\sigma_{\alpha\beta}$ and $\mu_{\alpha\beta}$ are the components of stress tensors and moment stress tensors; $\sigma_{\alpha\beta s}$ and $\sigma_{\alpha\beta *}$ are the symmetric and asymmetric components of the stress tensor. It assumes that there are no perturbations at infinity. Therefore, the boundary conditions are specified at the surface of the cavity. In the general case, the boundary conditions will be in the form of displacements (u_r, u_θ and $u_\vartheta \equiv 0$) or are given in the form of stresses, where we use two of the following conditions on stresses (σ_{rr}, σ_{rr}, $\sigma_{\theta r}$, $\sigma_{\theta\theta}$, $\sigma_{\vartheta\vartheta}$, $\mu_{r\vartheta}$, $\mu_{\theta\vartheta}$, $\mu_{\vartheta r}$, $\mu_{\vartheta\theta}$) and $u_\vartheta \equiv 0$. However, in this work, the author has chosen the boundary conditions in the simple to reduce the computational volume:

$$w|_{r=1} = w_0(\theta,\tau),\quad v|_{r=1} = 0,\quad \omega|_{r=1} = 0 \tag{9}$$

The initial conditions are defined as follows:

$$\varphi|_{\tau=0} = \psi|_{\tau=0} = \dot{\varphi}|_{\tau=0} = \dot{\psi}|_{\tau=0} \equiv 0. \tag{10}$$

in relations (2)–(10) and for further investigations, dimensionless quantities are used, which are indicated by strokes omitted in the subsequent presentation as

$$r' = \frac{r}{R_0}, \quad w' = \frac{w}{R_0}, \quad v' = \frac{v}{R_0}, \quad \varphi' = \frac{\varphi}{R_0^2}, \quad \psi' = \frac{\psi}{R_0^2}, \quad \chi'_{\alpha\beta} = R_0 \chi_{\alpha\beta}, \quad \sigma'_{\alpha\beta} = \frac{\sigma_{\alpha\beta}}{\lambda + 2\mu},$$
$$\mu'_{\alpha\beta} = \frac{\mu_{\alpha\beta}}{(\lambda + 2\mu) R_0} (\{\alpha, \beta\} = \{r, \theta, \vartheta\}), \quad \eta = \frac{\gamma}{(\lambda + 2\mu) R_0^2}, \quad \xi = \frac{\varepsilon}{(\lambda + 2\mu) R_0^2},$$
$$\kappa = \frac{\lambda}{\lambda + 2\mu}, \quad \tau = \frac{c_1 t}{R_0}, \quad c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_2 = \sqrt{\frac{\mu}{\rho}}, \quad \gamma_m = \frac{c_1}{c_m} (m = 1, 2),$$

herein c_1 and c_2 are the speeds of propagation of tension-compression waves and deformation in a classical elastic medium.

2.2 Theoretical formulation

2.2.1 Representation of solutions in the form of rows

To construct a solution to the initial-boundary problem that given in Eqs. (2)–(10), the method of incomplete separation of variables, expanding the required functions, and the right part of the boundary conditions (9) into series in terms of Legendre $P_n(\cos\theta)$, and Gegenbauer polynomials $C_{n-1}^{3/2}(\cos\theta)$ are employed [32–34]:

$$\begin{pmatrix} \varphi \\ w \\ \varepsilon_{rr} \\ \sigma_{rr} \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \varphi_n(r,\tau) \\ w_n(r,\tau) \\ \varepsilon_{rrn}(r,\tau) \\ \sigma_{rrn}(r,\tau) \end{pmatrix} P_n(\cos\theta), \quad \begin{pmatrix} \psi \\ v \\ \omega \\ \varepsilon_{r\theta} \\ \varepsilon_{\theta r} \end{pmatrix} = -\sin\theta \sum_{n=1}^{\infty} \begin{pmatrix} \psi_n(r,\tau) \\ v_n(r,\tau) \\ \omega_n(r,\tau) \\ \varepsilon_{r\theta n}(r,\tau) \\ \varepsilon_{r\theta n}(r,\tau) \end{pmatrix} C_{n-1}^{3/2}(\cos\theta) \tag{11}$$

$$\begin{pmatrix} \varepsilon_{\theta\theta} \\ \varepsilon_{\vartheta\vartheta} \\ \chi_{\theta\vartheta} \\ \chi_{\vartheta\theta} \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \varepsilon_{\theta\theta n}(r,\tau) \\ \varepsilon_{\vartheta\vartheta n}(r,\tau) \\ \chi_{\theta\vartheta n}(r,\tau) \\ 0 \end{pmatrix} P_n(\cos\theta) + \frac{\cos\theta}{r} \sum_{n=1}^{\infty} \begin{pmatrix} v_n(r,\tau) \\ -v_n(r,\tau) \\ \omega_n(r,\tau) \\ r\chi_{\vartheta\theta n}(r,\tau) \end{pmatrix} C_{n-1}^{3/2}(\cos\theta),$$
$$\begin{pmatrix} \mu_{\theta\vartheta} \\ \mu_{\vartheta\theta} \\ \sigma_{\theta\theta} \\ \sigma_{\vartheta\vartheta} \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \mu_{\theta\vartheta n}(r,\tau) \\ \mu_{\vartheta\theta n}(r,\tau) \\ \sigma_{\theta\theta n}(r,\tau) \\ \sigma_{\vartheta\vartheta n(r,\tau)} \end{pmatrix} P_n(\cos\theta) + \frac{\cos\theta}{r} \sum_{n=1}^{\infty} \begin{pmatrix} 2\eta\omega_n(r,\tau) \\ 2\eta\omega_n(r,\tau) \\ (1-\kappa)v_n(r,\tau) \\ (\kappa-1)v_n(r,\tau) \end{pmatrix} C_{n-1}^{3/2}(\cos\theta) \tag{12}$$

The functions w_0 and $\chi_{r\vartheta}$, $\chi_{\vartheta r}$, $\mu_{r\vartheta}$, $\mu_{\vartheta r}$, $\sigma_{r\theta}$, $\sigma_{\theta r}$ are represented similarly to Eq. (2.1???) in the form of series in the Legendre and Gegenbauer polynomials.

By substituting Eqs. (11) and (12) into Eq. (3), the following equations for the coefficients of the series for the potentials are obtained:

$$\begin{aligned} \ddot{\varphi}_n &= \Delta_n \varphi_n \ (n \geq 0), \\ \ddot{\psi}_n &= \frac{1-\kappa}{2}\Delta_n \psi_n - \frac{1}{4}(\eta + \xi)\Delta_n^2 \psi_n \ (n \geq 1), \\ \Delta_n &= \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{n(n+1)}{r^2} \end{aligned} \tag{13}$$

The series coefficients for the remaining components of the stress–strain state corresponding to Eqs. (2) and (4)–(8) are determined as follows:

$$w_n = \frac{\partial \varphi_n}{\partial r} - \frac{n(n+1)}{r}\psi_n, \quad v_n = \frac{\varphi_n - \psi_n}{r} - \frac{\partial \psi_n}{\partial r}, \quad \omega_n = \frac{1}{2}\left(\frac{v_n - w_n}{r} + \frac{\partial v_n}{\partial r}\right) \tag{14}$$

$$\varepsilon_{rrn} = \frac{\partial w_n}{\partial r}, \quad \varepsilon_{\theta\theta n} = \frac{1}{r}[w_n - n(n+1)v_n], \quad \varepsilon_{\vartheta\vartheta n} = \frac{w_n}{r}, \quad \varepsilon_{r\theta n} = \varepsilon_{\theta r n} = \frac{1}{2}\left(\frac{\partial v_n}{\partial r} + \frac{w_n - v_n}{r}\right) \tag{15}$$

$$\chi_{r\vartheta n} = \frac{\partial \omega_n}{\partial r}, \quad \chi_{\theta\vartheta n} = -n(n+1)\frac{\omega_n}{r}, \quad \chi_{\vartheta\theta n} = -\chi_{\vartheta r n} = \frac{\omega_n}{r} \tag{16}$$

$$\begin{aligned} \mu_{r\vartheta n} &= \xi_+ \frac{\partial \omega_n}{\partial r} - \xi_- \frac{\omega_n}{r}, \quad \mu_{\theta\vartheta n} = -n(n+1)\xi_+ \frac{\omega_n}{r} \\ \mu_{\vartheta r n} &= -\xi_+ \frac{\omega_n}{r} + \xi_- \frac{\partial \omega_n}{\partial r}, \quad \mu_{\vartheta\theta n}(r,\tau) = -n(n+1)\xi_- \frac{\omega_n}{r} \end{aligned} \tag{17}$$

$$\begin{aligned} \sigma_{rrn} &= \frac{\partial w_n}{\partial r} + \frac{\kappa}{r}[2w_n - n(n+1)v_n], \\ \sigma_{r\theta n} &= \sigma_{r\theta ns} - \sigma_{r\theta n*}, \ \sigma_{\theta r n} = \sigma_{r\theta ns} + \sigma_{r\theta n*} \sigma_{r\theta ns} = \frac{1-\kappa}{2}(\varepsilon_{r\theta n} + \varepsilon_{\theta r n}), \\ \sigma_{r\theta n*} &= \frac{1}{2}\left(\frac{\partial \mu_{r\vartheta n}}{\partial r} + \frac{\mu_{\theta\vartheta n} + \mu_{\vartheta r n} + 2\mu_{r\vartheta n}}{r}\right) + \eta \frac{\omega_n}{r^2} \\ \sigma_{\theta\theta n} &= \kappa \frac{\partial \omega_n}{\partial r} + \frac{1}{r}[(1+\kappa)\omega_n - n(n+1)v_n] \\ \sigma_{\vartheta\vartheta n} &= \kappa \frac{\partial \omega_n}{\partial r} + \frac{1}{r}[(1+\kappa)\omega_n - n(n+1)\kappa v_n] \end{aligned} \tag{18}$$

The corresponding initial and boundary conditions according to Eqs. (9) and (10) have the following form:

$$w_n|_{r=1} = w_{0n}(\theta,\tau), \quad v_n|_{r=1} = 0, \quad \omega_n|_{r=1} = 0; \tag{19}$$

$$\varphi_n|_{\tau=0} = \psi_n|_{\tau=0} = \dot{\varphi}_n|_{\tau=0} = \dot{\psi}_n\big|_{\tau=0} \equiv 0. \tag{20}$$

2.2.2 Determination of coefficients of series in the Laplace transformations

The Laplace transform in time to differential Eqs. (13) and taking into account conditions (19) is applied as follows:

$$\frac{\partial^2 \varphi_n^L(r,s)}{\partial r^2} + \frac{2}{r}\frac{\partial \varphi_n^L(r,s)}{\partial r} - \left[\frac{n(n+1)}{r^2} + s^2\right]\varphi_n^L(r,s) = 0; \tag{21}$$

$$(\eta + \xi)\Delta_n^2 \psi_n^L(r,s) - 2(1-\kappa)\Delta_n \psi_n^L(r,s) + 4s^2 \psi_n^L(r,s) = 0. \tag{22}$$

in which s is the parameter of transform, and the subscript "L" denotes a transformant.

The general solution of Eq. (21) has the form:

$$\varphi_n^L(r,s) = r^{-1/2}\left[C_{n1}^{(1)}(s)K_{n+1/2}(rs) + C_{n2}^{(1)}(s)I_{n+1/2}(rs)\right] \tag{23}$$

where $C_{n1}^{(1)}$ and $C_{n2}^{(1)}$ are arbitrary constants of integration; $I_\nu(z)$ and $K_\nu(z)$ are modified functions Bessel of the order of the first and second kind [35].

To solve Eq. (22), the following equation is considered:

$$\Delta_n \psi_n^L = \lambda \psi_n^L \tag{24}$$

Then, the following characteristic equation is obtained as followed:

$$(\eta + \xi)\lambda^2 - 2(1-\kappa)\lambda + 4s^2 = 0. \tag{25}$$

and the roots of Eq. (25) are gotten as follows:

$$\lambda_{1,2} = \frac{(1-\kappa) \pm \sqrt{(1-\kappa)^2 - 4s^2(\eta+\xi)}}{\eta+\xi}, \quad \mathrm{Re}\sqrt{\cdot} > 0. \tag{26}$$

By taking into account the fundamental system of solutions to Eq. (24) consisting of modified Bessel functions, the general solution of Eq. (22) can be found:

$$\sum_{m=1}^{2} C_{nm}^{(2)}(s) K_{n+1/2}\left(r\sqrt{\lambda_m}\right) + \sum_{m=1}^{2} C_{n,m+2}^{(2)}(s) I_{n+1/2}\left(r\sqrt{\lambda_m}\right) \tag{27}$$

where $C_{nj}^{(2)}$ $(j = \overline{1,4})$ is the arbitrary constants of integration.

By taking into account the properties of the modified functions Bessel (at infinity $I_\nu(z)$ is unbounded and $K_\nu(z)$ is bounded) and the condition that there are no perturbations at infinity, $C_{n2}^{(1)}(s) = C_{n3}^{(2)}(s) = C_{n4}^{(2)}(s) \equiv 0$ can be obtained. That means:

$$\varphi_n^L(r,s) = \frac{1}{\sqrt{r}} C_{n1}^{(1)}(s) K_{n+1/2}(rs), \quad \psi_n^L(r,s) = \frac{1}{\sqrt{r}} \sum_{m=1}^{2} C_{nm}^{(2)}(s) K_{n+1/2}\left(r\sqrt{\lambda_m}\right) \tag{28}$$

By using the connection of the modified functions Bessel of a half-integer index with elementary functions ($n = 0,\ 1,\ 2, \ldots$) [32], the following expressions are obtained:

$$\begin{aligned} K_{n+1/2}(z) &= \sqrt{\frac{\pi}{2}} \frac{e^{-z}}{z^{n+1/2}} R_{n0}(z), \quad R_{n0}(z) = \sum_{k=0}^{n} A_{nk} z^{n-k} \\ A_{nk} &= \frac{(n+k)!}{(n-k)!\,k!\,2^k}\ (0 \le k \le n), \quad A_{nk} = 0\,(k < 0, \quad k > n) \end{aligned} \tag{29}$$

then the following expressions for the images of the potential coefficients can be found as follows:

$$\begin{aligned} \varphi_n^L(r,s) &= \frac{1}{r^{n+1}} A_n^L(s) R_{n0}(rs) e^{-(r-1)s} \\ \psi_n^L(r,s) &= \frac{1}{r^{n+1}} \sum_{m=1}^{2} B_{nm}^L(s) R_{n0}\left(r\sqrt{\lambda_m}\right) e^{-(r-1)\sqrt{\lambda_m}} \end{aligned} \tag{30}$$

with $A_n^L(s) = \frac{\pi}{s^n\sqrt{2\pi s}} C_{n1}^{(1)}(s) e^{-s}$ and $B_{nm}^L(s) = \frac{\pi C_{nm}^{(2)}(s)}{\sqrt{2\pi}\lambda_m^{n/2+1/4}} e^{-\sqrt{\lambda_m}}$ $(m = 1, 2)$.

Substituting these equalities into the Laplace transformed formulas (14), the following representations of the series coefficients for the displacements and the angle of rotation are expressed as

$$\begin{aligned} w_n^L &= -\frac{1}{r^{n+2}}\left[A_n^L(s)R_{n1}(rs)e^{-(r-1)s} + n(n+1)\sum_{i=1}^{2} B_{ni}^L(s)R_{n0}\left(r\sqrt{\lambda_i}\right)e^{-(r-1)\sqrt{\lambda_i}}\right] \\ v_n^L &= \frac{1}{r^{n+2}}\left[A_n^L(s)R_{n0}(rs)e^{-(r-1)s} + \sum_{i=1}^{2} B_{ni}^L(s)R_{n3}\left(r\sqrt{\lambda_i}\right)e^{-(r-1)\sqrt{\lambda_i}}\right] \\ \omega_n^L &= -\frac{1}{2r^{n+3}}\left[\sum_{i=1}^{2} B_{ni}^L(s)Q_n\left(r\sqrt{\lambda_i}\right)e^{-(r-1)\sqrt{\lambda_i}}\right] \end{aligned} \tag{31}$$

in which

$$\begin{aligned} &R_{n1}(z) = R_{n+1,0}(z) - nR_{n0}(z),\ R_{n3}(z) = R_{n+1,0}(z) - (n+1)R_{n0}(z) \\ &Q_n(z) = R_{n+2,0}(z) - (2n+3)R_{n+1,0}(z)\ R_{n1}(z) = \sum_{k=0}^{n+1} B_{nk}z^{n+1-k}, \\ &R_{n3}(z) = \sum_{k=0}^{n+1} C_{nk}z^{n+1-k}\ Q_n(z) = \sum_{k=0}^{n+2} D_{nk}z^{n+2-k}, \\ &B_{nk} = A_{n+1,k} - nA_{n,k-1},\ C_{nk} = A_{n+1,k} - (n+1)A_{n,k-1}\ D_{nk} = A_{n+2,k} - (2n+3)A_{n+1,k-1} \end{aligned} \tag{32}$$

Substituting Eq. (31) into the Laplace transformed boundary conditions (19) and determining the integration constants, the following expressions for the images of the series coefficients for the displacements and the angle of rotation are obtained as follows:

$$\begin{aligned} w_n^L(r,s) &= \frac{w_{n0}^L(s)}{r^{n+2}}\left[W_{n0}^L(r,s)e^{-(r-1)s} + n(n+1)\sum_{m=1}^{2} W_{nm}^L(r,s)e^{-(r-1)\sqrt{\lambda_m}}\right] \\ v_n^L(r,s) &= \frac{w_{n0}^L(s)}{r^{n+2}}\left[V_{n0}^L(r,s)e^{-(r-1)s} + \sum_{m=1}^{2} V_{nm}^L(r,s)e^{-(r-1)\sqrt{\lambda_m}}\right] \\ \omega_n^L(r,s) &= \frac{w_{n0}^L(s)}{2r^{n+3}}\sum_{m=1}^{2} \Omega_{nm}^L(r,s)e^{-(r-1)\sqrt{\lambda_m}} \end{aligned} \tag{33}$$

where

$$\begin{aligned} &X_n(s)W_{n0}^L(r,s) = -R_{n1}(rs)S_{n1}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right),\ X_n(s)W_{n1}^L(r,s) = R_{n0}\left(r\sqrt{\lambda_1}\right)S_{n2}\left(s,\sqrt{\lambda_2}\right) \\ &X_n(s)V_{n0}^L(r,s) = R_{n0}(rs)S_{n1}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right),\ X_n(s)V_{n1}^L(r,s) = -R_{n3}\left(r\sqrt{\lambda_1}\right)S_{n2}\left(s,\sqrt{\lambda_2}\right) \\ &X_n(s)\Omega_{n1}^L(r,s) = Q_n\left(r\sqrt{\lambda_1}\right)S_{n2}\left(s,\sqrt{\lambda_2}\right) \\ &X_n(s) = -R_{n1}(s)S_{n1}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right) + n(n+1)R_{n0}(s)\left[S_{n2}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right) - S_{n2}\left(\sqrt{\lambda_2},\sqrt{\lambda_1}\right)\right] \\ &S_{n1}(x,y) = R_{n3}(x)Q_n(y) - R_{n3}(y)Q_n(x),\ S_{n2}(x,y) = R_{n0}(x)Q_n(y) \end{aligned} \tag{34}$$

Formulas for the functions $W_{n2}^{L}(r,s)$, $V_{n2}^{L}(r,s)$ and $\Omega_{n2}^{L}(r,s)$ are obtained from the equalities for $W_{n1}^{L}(r,s)$, $V_{n1}^{L}(r,s)$ and $\Omega_{n1}^{L}(r,s)$ by multiplying by (-1) and swapping λ_1 and λ_2.

The equalities similar to Eq. (31) for the remaining components of the stress–strain state can be obtained.

2.2.3 Limiting transition of symmetric elasticity theory

For the indicated transition in the relations obtained above, it is necessary to pass to the limit when $\eta \to 0$ and $\xi \to 0$. In this case, for the roots appeared in Eq. (26) of the characteristic equation, the following relations are obtained as

$$\lambda_1 \to \infty, \quad \lambda_2 \to \frac{2s^2}{(1-\kappa)} = (\gamma_2 s)^2$$

As a result of the images of the coefficients of the series in the Legendre and Gegenbauer polynomials for displacements, the following results are gotten as

$$w_n^L = \frac{w_{n0}^L(s)}{r^{n+2}}\left[W_{n0}^L(r,s)e^{-(r-1)s} + W_{n2}^L(r,s)e^{-(r-1)\gamma_2 s}\right],$$
$$v_n^L = \frac{w_{n0}^L(s)}{r^{n+2}}\left[V_{n0}^L(r,s)e^{-(r-1)s} + V_{n2}^L(r,s)e^{-(r-1)\gamma_2 s}\right],$$

in which

$$Y_n(s)W_{n0}^L(r,s) = R_{n1}(rs)R_{n3}(\gamma_2 s)D_{n0},\ Y_n(s)W_{n2}^L(r,s) = -n(n+1)R_{n0}(r\gamma_2 s)R_{n0}(s)D_{n0},$$
$$Y_n(s)V_{n0}^L(r,s) = -R_{n0}(rs)R_{n3}(\gamma_2 s)D_{n0},\ Y_n(s)V_{n2}^L(r,s) = R_{n3}(r\gamma_2 s)R_{n0}(s)D_{n0},$$
$$Y_n(s) = R_{n1}(s)R_{n3}(\gamma_2 s)D_{n0} - n(n+1)R_{n0}(s)R_{n0}(\gamma_2 s)D_{n0},$$

This finding is consistent with the one provided in [32].

3 Algorithm for the definition of original functions

The Laplace translation of differential equations from a time function t to an image function s is a reasonably straightforward operation that may be performed on any complex differential equations. However, it is not always feasible to perform the Laplace inverse transformation from the image function to the original function. It is difficult to obtain analytical expressions for the originals of functions $w_n^L(r,s)$, $v_n^L(r,s)$ and $\omega_n^L(r,s)$ because the expression (31) contains the terms radicals $\sqrt{\lambda_{1,2}}$. Therefore, the asymptotic representations of the required functions at the initial moment should be constructed, which corresponds to the expansion of the images in a series in powers s^{-1} in the vicinity of the point at infinity. For roots in Eq. (26), these series have the following form:

$$\sqrt{\lambda_1} = \sqrt{s}\sum_{l=0}^{\infty}\beta_l s^{-l},\ \sqrt{\lambda_2} = \sqrt{s}\sum_{l=0}^{\infty}\overline{\beta}_l s^{-l},\ \beta_0 = \alpha a_0,\ \beta_1 = \overline{\alpha} a_1,\ \beta_2 = \alpha a_2,\ \alpha = 1+i$$
$$a_0 = \frac{1}{(\eta+\xi)^{1/4}}, a_1 = \frac{1-\kappa}{4(\eta+\xi)^{3/4}}, a_2 = -\frac{(1-\kappa)^2}{32(\eta+\xi)^{5/4}} \tag{35}$$

where i is the imaginary unit, and bar is the sign of complex conjugation.

Then, the series for the exponentials in Eqs. (31)–(33) and containing radicals using the well-known Maclaurin series are obtained as

$$
\begin{aligned}
&e^{r\sqrt{\lambda_1}} = e^{-r\beta_0\sqrt{s}} \sum_{k=0}^{\infty} A_k s^{-k/2}, \\
&e^{r\sqrt{\lambda_2}} = e^{-r\overline{\beta}_0\sqrt{s}} \sum_{k=0}^{\infty} \overline{A}_k s^{-k/2} \\
&A_0 = 1,\ A_1 = r\beta_1,\ A_2 = (r\,\beta_1)^2/2, \\
&A_3 = r\,\beta_2,\ A_4 = r^2\beta_1\beta_2 = 2r^2 a_1 a_2
\end{aligned}
\tag{36}
$$

Before expanding polynomials, Eq. (32), with similar arguments, first, the series for the powers of the radicals using Eq. (35) must be constructed as

$$
\begin{aligned}
&\left(\sqrt{\lambda_1}\right)^k = s^{k/2} \sum_{m=0}^{\infty} \frac{b_{km}}{s^m}, \quad \left(\sqrt{\lambda_2}\right)^k = s^{k/2} \sum_{m=0}^{\infty} \frac{\overline{b}_{km}}{s^m} \\
&b_{k0} = \beta_0^k,\ b_{k1} = k\beta_1\beta_0^{k-1}, \quad b_{k2} = k\beta_0^{k-1}\left(\frac{k-1}{2}\beta_1 + \beta_2\right)
\end{aligned}
\tag{37}
$$

As result, it comes to the following results:

$$
\begin{aligned}
&R_{n0}\left(r\sqrt{\lambda_1}\right) = s^{\frac{n}{2}} \sum_{m=0}^{\infty} E_{nm}(r)s^{-m/2},\ R_{n0}\left(r\sqrt{\lambda_2}\right) = s^{\frac{n}{2}} \sum_{m=0}^{\infty} \overline{E}_{nm}(r)s^{-m/2} \\
&R_{n1}\left(r\sqrt{\lambda_1}\right) = s^{\frac{n}{2}} \sum_{m=0}^{\infty} F_{nm}(r)s^{-m/2},\ R_{n1}\left(r\sqrt{\lambda_2}\right) = s^{\frac{n+1}{2}} \sum_{m=0}^{\infty} \overline{F}_{nm}(r)s^{-m/2} \\
&R_{n3}\left(r\sqrt{\lambda_1}\right) = s^{\frac{n+1}{2}} \sum_{m=0}^{\infty} G_{nm}(r)s^{-m/2},\ R_{n3}\left(r\sqrt{\lambda_2}\right) = s^{\frac{n+1}{2}} \sum_{m=0}^{\infty} \overline{G}_{nm}(r)s^{-m/2} \\
&Q_{n}\left(r\sqrt{\lambda_1}\right) = s^{\frac{n+2}{2}} \sum_{m=0}^{\infty} H_{nm}(r)s^{-m/2},\ Q_{n}\left(r\sqrt{\lambda_2}\right) = s^{\frac{n+2}{2}} \sum_{m=0}^{\infty} \overline{H}_{nm}(r)s^{-m/2}
\end{aligned}
\tag{38}
$$

where

$$
\begin{aligned}
&E_{nm}(r) = \sum_{k=k_{nm}}^{[m/2]} A_{n,m-2k} r^{n+2k-m} b_{n+2k-m},\ F_{nm}(r) = \sum_{k=k_{n+1,m}}^{[m/2]} B_{n,m-2k} r^{n+1+2k-m} b_{n+1+2k-m}, \\
&G_{nm}(r) = \sum_{k=k_{n+1,m}}^{[m/2]} C_{n,m-2k} r^{n+1+2k-m} b_{n+1+2k-m},\ H_{nm}(r) \\
&= \sum_{k=k_{n+2,m}}^{[m/2]} D_{n,m-2k} r^{n+2+2k-m} b_{n+2+2k-m}, \\
&k_{nm} = \begin{cases} 0 & \text{when } m \le n, \\ \left[\frac{m-n}{2}\right] & \text{when } m \le n. \end{cases}
\end{aligned}
$$

The expansions for the functions $S_{n1}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right)$, $S_{n2}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right)$, $S_{n2}\left(\sqrt{\lambda_2},\sqrt{\lambda_1}\right)$, $S_{n2}\left(s,\sqrt{\lambda_1}\right)$, $S_{n2}\left(s,\sqrt{\lambda_2}\right)$ and $X_n(s)$ are obtained by using Eqs. (34) and (38).

$$\begin{aligned} &S_{n1}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right)=s^{\frac{2n+3}{2}}\sum_{k=0}^{\infty} i\gamma_{n1k}s^{-k/2}, \quad S_{n2}\left(\sqrt{\lambda_1},\sqrt{\lambda_2}\right)=s^{n+1}\sum_{k=0}^{\infty}\gamma_{n2k}s^{-k/2} \\ &S_{n2}\left(\sqrt{\lambda_2},\sqrt{\lambda_1}\right)=s^{n+1}\sum_{k=0}^{\infty}\overline{\gamma}_{n2k}s^{-k/2}s^{-k/2}, \quad S_{n2}\left(s,\sqrt{\lambda_1}\right)=s^{\frac{3n+2}{2}}\sum_{k=0}^{\infty}\nu_{nk}s^{-k/2} \\ &S_{n2}\left(s,\sqrt{\lambda_2}\right)=s^{\frac{3n+2}{2}}\sum_{k=0}^{\infty}\overline{\nu}_{nk}s^{-k/2}, \quad X_n(s)=s^{\frac{4n+5}{2}}\sum_{k=0}^{\infty} i\xi_{nk}s^{-k/2} \end{aligned} \tag{39}$$

The coefficients γ_{njk} $(j=1,2)$, ν_{nk}, and ξ_{nk} of these series are determined through the coefficients of the series in Eq. (38) using the rule for the product and addition of power series. Adding to these operations the division of power series and using formulas appeared in Eqs. (34) and (36), the following expansions of the terms in Eq. (33) are obtained as

$$\begin{aligned} W_{n0}^{L}(r,s)e^{-(r-1)s} &= e^{-(r-1)s}\sum_{k=0}^{\infty} w_{n0k}(r)s^{-k/2}, \\ V_{n0}^{L}(r,s)e^{-(r-1)s} &= e^{-(r-1)s}\sum_{k=0}^{\infty} v_{n0k}(r)s^{-k/2} \\ W_{n1}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_1}} &= e^{-(r-1)\beta_0\sqrt{s}}\sum_{k=3}^{\infty} w_{n1k}(r)s^{-k/2} \\ W_{n2}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_2}} &= e^{-(r-1)\overline{\beta}_0\sqrt{s}}\sum_{k=3}^{\infty} \overline{w}_{n1k}(r)s^{-k/2} \\ V_{n1}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_1}} &= e^{-(r-1)\beta_0\sqrt{s}}\sum_{k=2}^{\infty} v_{n1k}(r)s^{-k/2} \\ V_{n2}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_2}} &= e^{-(r-1)\overline{\beta}_0\sqrt{s}}\sum_{k=2}^{\infty} \overline{v}_{n1k}(r)s^{-k/2} \\ \Omega_{n1}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_1}} &= e^{-(r-1)\beta_0\sqrt{s}}\sum_{k=1}^{\infty} \omega_{n1k}(r)s^{-k/2} \\ \Omega_{n2}^{L}(r,s)e^{-(r-1)\sqrt{\lambda_2}} &= e^{-(r-1)\overline{\beta}_0\sqrt{s}}\sum_{k=1}^{\infty} \overline{\omega}_{n1k}(r)s^{-k/2} \end{aligned} \tag{40}$$

The originals of the coefficients of the series in (40) are found using theorems of operational calculus and the following tabular relations [32, 35].

$$e^{-ks}s^{-\mu} \doteqdot \frac{(\tau-k)_+^{\mu-1}}{\Gamma(\mu)} \quad (\mu>0)$$

$$e^{-a\sqrt{s}}s^{-m/2} \doteqdot \frac{\tau_+^{m/2-1}}{\sqrt{2^{1-m}\pi}}e^{-\frac{a^2}{8\tau}}D_{1-m}\left(\frac{a}{\sqrt{2\tau}}\right) \quad (m=0,1,2\ldots;\mathrm{Re}a\geq 0)$$

where $\Gamma(\mu)$ is the gamma function; $H(t)$ is the Heaviside function; $D_\nu(x)$ is the function of a parabolic cylinder; and $x_+^\alpha = x^\alpha H(x)$.

Note that, the function of the parabolic cylinder has the property $D_\nu(\bar{z}) = \overline{D_\nu(z)}$. Using this property and also equalities (33) and (40), it follows that the originals of the sought functions are real.

4 Simulation results and discussion

As a material filling the space, consider a granular composite of aluminum shot in an epoxy matrix ($\lambda = 7.59$ GPa; $\mu = 1.89$ GPa; $\gamma + \varepsilon = 2.64$ kN) [36], which corresponds to dimensionless parameters $\kappa = 0.67$, $\eta + \xi = 0.00232$. It assumes that displacements of the type are given on the boundary of the cavity:

$$w_0(\theta, \tau) = \frac{1}{2}(1 + \cos 2\theta)H(\tau)$$

wherein

$$w_{00}^L(s) = \frac{1}{3s}, \; w_{20}^L(s) = \frac{2}{3s}, \; w_{n0}^L(s) \equiv 0 \;\; (n = 1, \; n \geq 3)$$

and in series (11)–(12), only the terms with indices $n = 0$ and $n = 2$ are nonzero.

As a result, one gets:

$$\begin{gathered} w(r, \theta, \tau) = w_0(r, \tau)P_0(\cos\theta) + w_2(r, \tau)P_2(\cos\theta) \\ v(r, \theta, \tau) = -v_2(r, \tau)C_1^{3/2}(\cos\theta)\sin\theta, \; \omega(r, \theta, \tau) = -\omega_2(r, \tau)C_1^{3/2}(\cos\theta)\sin\theta \end{gathered}$$
$$\begin{aligned} & w_0(r, \tau) = \frac{1}{3r^2}\sum_{k=0}^{\infty} w_{00k}(r) f_{1k}(r, \tau), \; \omega_2(r, \tau) = \mathrm{Re}\sum_{k=1}^{\infty} \omega_{21k}(r) f_{2k}(r, \tau) \\ & w_2(r, \tau) = \frac{2}{r^2}\sum_{k=0}^{\infty} w_{20k}(r) f_{1k}(r, \tau) + 2\mathrm{Re}\sum_{k=3}^{\infty} w_{21k}(r) f_{2k}(r, \tau) \\ & v_2(r, \tau) = \frac{2}{r^2}\sum_{k=2}^{\infty} v_{20k}(r) f_{1k}(r, \tau) + 2\mathrm{Re}\sum_{k=2}^{\infty} v_{21k}(r) f_{2k}(r, \tau) \\ & f_{1k}(r, \tau) = \frac{1}{3r^2}\zeta_k(\tau - r + 1)_+^{k/2}, \; f_{2k}(r, \tau) = \frac{1}{3r^4\sqrt{\pi}} 2^{\frac{k+3}{2}} \tau_+^{\frac{k}{2}} e^{-\beta_0^2/(8\tau)} D_{-k-1}\left(\frac{\beta_0}{\sqrt{2\tau}}\right) \\ & \zeta_k = \Gamma^{-1}\left(1 + \frac{k}{2}\right), \; \Gamma\left(1 + \frac{k}{2}\right) = \begin{cases} m! & \text{when } k = 2m, \\ 2^{-m-1}(2m+1)!!\sqrt{\pi} & \text{when } k = 2m+1, \end{cases} \; (m = 0, 1, 2, \ldots) \end{aligned} \tag{41}$$

The graphs of normal displacement versus time at distances $r = 1.01$, $r = 1.03$, $r = 1.05$, and $r = 1.08$ from the center of the cavity at the values of the angles $\theta = \pi/4$ and $\theta = \pi/2$ are shown in Figs. 2. In Fig. 2a, we notice that there are certain edges, implying that some points in graphs seem to be nondifferentiable. It is shown that the difference in value created by the pseudo-elastic Cosserat medium's normal displacement (w) is small in comparison with the boundary value applied to the spherical cavity surface. As seen in Fig. 2b, the graphs first bear little resemblance to one another. It is mentioned that the moment effect has a minimal influence on the changes in the medium's stress–strain state.

Figures 3 and 4 show the graphs of the tangential displacement $v(r, \theta, \tau)$ and the angle of rotation $\omega(r, \theta, \tau)$ at $\theta = \pi/4$. At $\theta = 0$ and $\theta = \pi/2$, these functions are equal to zero since the corresponding equalities contain the factors $\sin\theta$ and $C_1^{3/2}(\cos\theta) = 3\sin\theta$.

All graphs are plotted for four members of the power series. When one more member is taken into account, the results are practically the same.

Fig. 2 **a** Normal displacement at $\theta = \pi/4$, **b** Normal displacement at $\theta = \pi/2$

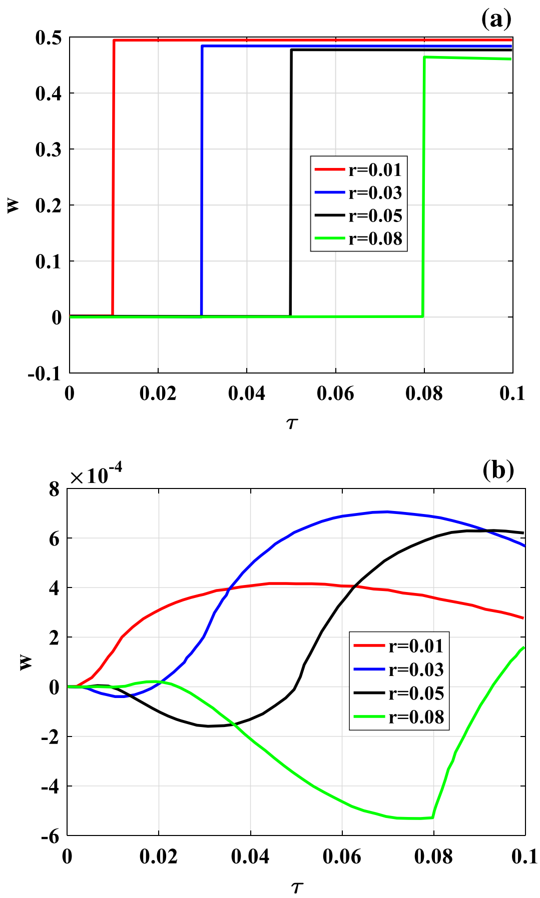

Fig. 3 Tangential displacement, $\theta = \pi/4$

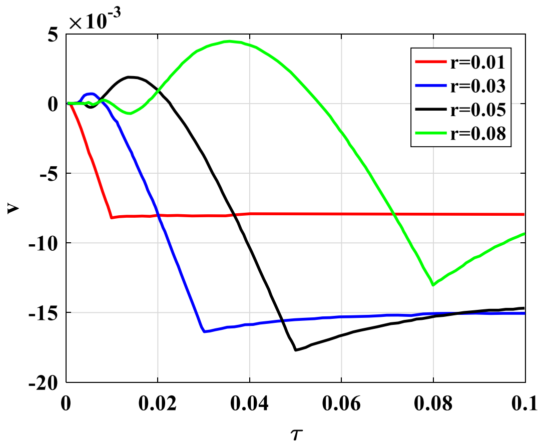

Fig. 4 Angle of rotation

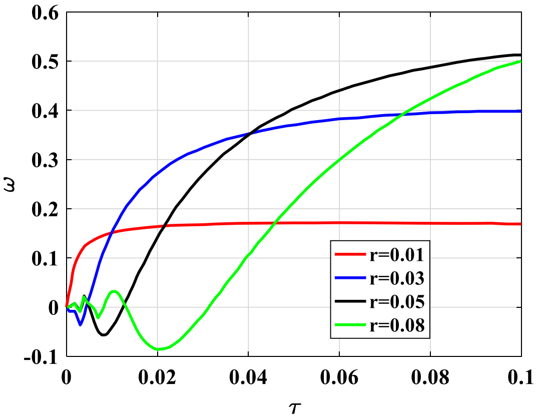

Fig. 5 Comparative graphs of normal displacement for two models of the medium, $\theta = \pi/4$

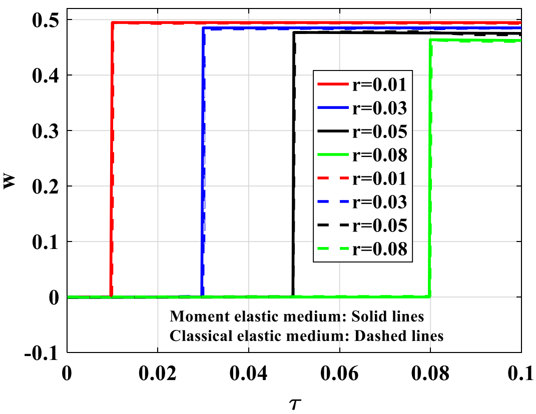

Figure 5 shows comparative graphs of normal displacement for two models of medium (solid lines correspond to the above moment medium, and dashed lines—to the classical version of the same medium) for a given external disturbance.

The simulation results show that at the initial time, the graphs differ little from each other, that the moment effect affects the changes in the stress–strain state of the medium, but it is small.

5 Conclusion

The Cosserat model is used to represent non-stationary processes in composite material constructions in this paper. Unsteady axisymmetric kinematic perturbations propagating across space from an isotropic pseudo-elastic Cosserat medium are studied. The medium's motion is described by three equations, with the origin at the cavity's center and nonzero displacement vector and rotation field potential components. The following main results are given as follows:

- Using the representation of the sought functions in the form of series in Legendre polynomials and the Laplace transform, solutions of new non-stationary axisymmetric problems on the propagation of surface perturbations in the pseudo-continuum Cosserat with spherical boundaries (a space with a spherical cavity) are obtained.

- For images of the Laplace transform containing factors in the form of exponentials with radicals, an inversion algorithm for the coefficients of the series in Legendre polynomials has been developed. This method is based on the expansion of the images into the Laurent series in the vicinity of an infinitely distant point, which corresponds to the power series in the vicinity of the initial moment of time. A method for determining the coefficients of these series has been developed and implemented.
- A numerical study of the convergence in the obtained solutions of series in Legendre polynomials and power series in time is carried out.

Acknowledgements Assoc. Prof. Dr. Nguyen Thai Dung gratefully acknowledges the support of the ministry-level project "Research, design and manufacture torpedo decoy shells for Navy ships."

Declarations

Conflict of interest The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

1. B. K. Panigrahi, S. K. Sinha, Detection and classification of non-stationary power disturbances in noisy conditions, in *2006 International Conference on Power Electronic, Drives and Energy Systems* (2006), pp. 1–5, doi:https://doi.org/10.1109/PEDES.2006.344258
2. I. Aljamaan, D. Westwick, M. Foley, Non-linear closed-loop identification of CSTR in the presence of a non-stationary disturbances. IFAC-PapersOnLine **48**, 1029–1034 (2015). https://doi.org/10.1016/j.ifacol.2015.12.266
3. E. Cosserat, F. Cosserat, Théorie des corps déformables. Nature **81**(2072), 67 (1909). https://doi.org/10.1038/081067a0
4. M. Epstein, M. de Leon, Geometrical theory of uniform cosserat media. J. Geom. Phys. **26**(1), 127–170 (1998). https://doi.org/10.1016/S0393-0440(97)00042-9
5. H. Ramézani, J. Jeong, Z.-Q. Feng, On parallel simulation of a new linear Cosserat elasticity model with grid framework model assumptions. Appl. Math. Model. **35**(10), 4738–4758 (2011). https://doi.org/10.1016/j.apm.2011.03.054
6. J. Till, V. Aloi, D. Rucker, Real-time dynamics of soft and continuum robots based on cosserat-rod models. Int. J. Rob. Res. **38**, 723–746 (2019). https://doi.org/10.1177/0278364919842269
7. P. Neff, A geometrically exact cosserat shell-model including size effects, avoiding degeneracy in the thin shell limit. Part I: formal dimensional reduction for elastic plates and existence of minimizers for positive Cosserat couple modulus. Contin. Mech. Thermodyn. **16**(6), 577–628 (2004). https://doi.org/10.1007/s00161-004-0182-4
8. J. Altenbach, H. Altenbach, V.A. Eremeyev, On generalized Cosserat-type theories of plates and shells: a short review and bibliography. Arch. Appl. Mech. **80**(1), 73–92 (2010). https://doi.org/10.1007/s00419-009-0365-3
9. M. Ostoja-Starzewski, I. Jasiuk, Stress invariance in planar cosserat elasticity. Proc. Math. Phys. Sci. **451**(1942), 453–470 (1995). Available: http://www.jstor.org/stable/52637
10. I. Smolin, R. Bakeev, and P. Makarov, Numerical solution of some two-dimensional problems for an elastoplastic micropolar medium. Vesn. PSTU Math. Model. Syst. Process. **15**, 142–155 (2007). Available: https://cyberleninka.ru/article/n/chislennoe-reshenie-nekotoryh-dvumernyh-zadach-dlya-uprugoplasticheskoy-mikropolyarnoy-sredy
11. V.I. Nikolau, *Moment theory of elasticity: development, analysis, and applications* (AstroPrint, Odessa, 2006)
12. V. Erofeev, A. Potapov, Nonlinear longitudinal waves in elastic media with moment stresses. Akust. Zhurnal **37**(3), 477–483 (1991). Available: https://ui.adsabs.harvard.edu/abs/1991AkZh...37..477E/abstract
13. V.I. Erofeev, Propagation of nonlinear shear waves in a solid with microstructure. Int. Appl. Mech. **29**(4), 262–266 (1993). https://doi.org/10.1007/BF00847023
14. D.Q. Cao, R.W. Tucker, Nonlinear dynamics of elastic rods using the cosserat theory: modelling and simulation. Int. J. Solids Struct. **45**(2), 460–477 (2008). https://doi.org/10.1016/j.ijsolstr.2007.08.016

15. M. Bîrsan, Several results in the dynamic theory of thermoelastic cosserat shells with voids. Mech. Res. Commun. **33**(2), 157–176 (2006). https://doi.org/10.1016/j.mechrescom.2005.08.008
16. R. Kumar, R.R. Gupta, Propagation of waves in transversely isotropic micropolar generalized thermoelastic half space. Int. Commun. Heat Mass Transf. **37**(10), 1452–1458 (2010). https://doi.org/10.1016/j.icheatmasstransfer.2010.08.001
17. I. Nistor, Generalized theory of cosserat thermoelastic media. Bull. Polytech. Inst. Jassy **37**, 89–96 (1991). Available: https://scholar.google.com/scholar?cites=10332940448149240274&as_sdt=2005&sciodt=0,5&hl=vi
18. J. Liu, M. Huang, X.-R. Ge, Solution of stress concentration problem considering influence of couple-stress. J. Shanghai Jiaotong Univ. Chin. Ed **35**, 1481–1485 (2001)
19. R. Kumar, J. Sharma, Reflection of plane waves from the boundaries of a micropolar thermoelastic half-space without energy dissipation. Int. J. Appl. Mech. Eng. **10**, 631–645 (2005)
20. S.Y. Han, M.N.L. Narasimhan, T.C. Kennedy, Finite crack propagation in a micropolar elastic solid. KSME J. **3**(2), 103 (1989). https://doi.org/10.1007/BF02953595
21. R. Kumar, R. Singh, T.K. Chadha, Eigen value approach to second dynamic problem of micropolar elastic solid. Indian J. Pure Appl. Math. **34**, 743–754 (2003)
22. H.S. Saxena and R.S. Dhaliwal, Eigenvalue approach to axially-symmetric coupled micropolar thermoelasticity. Bull. Polish Acad. Sci. Tech. Sci. **38**(1), 7–18 (1990). Available: https://pascal-francis.inist.fr/vibad/index.php?action=getRecordDetail&idt=4952845
23. H.N. Nguyen, T.C. Tan, D.T. Luat, V.D. Phan, D. Van Thom, P. Van Minh, Research on the buckling behavior of functionally graded plates with stiffeners based on the third-order shear deformation theory. Materials (Basel) (2019). https://doi.org/10.3390/ma12081262
24. T.T. Tran, V.K. Tran, P.B. Le, V.M. Phung, V.T. Do, H.N. Nguyen, Forced vibration analysis of laminated composite shells reinforced with graphene nanoplatelets using finite element method. Adv. Civ. Eng. (2020). https://doi.org/10.1155/2020/1471037
25. V.H. Nam, N.H. Nam, P.V. Vinh, D.N. Khoa, D.V. Thom, P.V. Minh, A new efficient modified first-order shear model for static bending and vibration behaviors of two-layer composite plate. Adv. Civ. Eng. (2019). https://doi.org/10.1155/2019/6814367
26. D. Van Thom, D.H. Duc, P. Van Minh, N.S. Tung, Finite element modelling for free vibration response of cracked stiffened fgm plates. Vietnam J. Sci. Technol. **58**(1), 119 (2020). https://doi.org/10.15625/2525-2518/58/1/14278
27. N.C. Tho, N.T. Thanh, T.D. Tho, P. Van Minh, L.K. Hoa, Modelling of the flexoelectric effect on rotating nanobeams with geometrical imperfection. J. Braz. Soc. Mech. Sci. Eng. (2021). https://doi.org/10.1007/s40430-021-03189-w
28. N.T. Dung, P. Van Minh, H.M. Hung, D.M. Tien, The third-order shear deformation theory for modeling the static bending and dynamic responses of piezoelectric bidirectional functionally graded plates. Adv. Mater. Sci. Eng. (2021). https://doi.org/10.1155/2021/5520240
29. D. Nguyen Thai, P. Van Minh, C. Phan Hoang, T. Ta Duc, N. Nguyen Thi Cam, D. Nguyen Thi, Bending of symmetric sandwich FGM beams with shear connectors. Math. Probl. Eng. (2021). https://doi.org/10.1155/2021/7596300
30. V.I. Erofeyev, *Wave Processes in Solids with Microstructure* (World Scientific, Singabore, 2003)
31. V. Novatsky, *Theory of elasticity* (Mir, Moscow, 1975)
32. E. Aero, E. Kuvshinskii, Basic equations in the theory of elasticity of media with rotation interaction of particles. Fiz. Tverd. Tela **7**(2), 1399–1409 (1960). Available: https://scholar.google.com/scholar?hl=vi&as_sdt=2005&sciodt=0,5&cites=16691621741707340366&scipsc=&q=Aero. E.L., Kuvshinskii E.V., Basic equations in the theory of elasticity of media with rotation interaction of particles, Fiz. Tverd. Tela&btnG=
33. J. Boyd, R. Petschek, The relationships between chebyshev, legendre and jacobi polynomials: the generic superiority of chebyshev polynomials and three important exceptions. J. Sci. Comput. (2014). https://doi.org/10.1007/s10915-013-9751-7
34. G. Lohöfer, Inequalities for Legendre functions and gegenbauer functions. J. Approx. Theory **64**(2), 226–234 (1991). https://doi.org/10.1016/0021-9045(91)90077-N
35. G. Bagdasaryan, D. Asanyan, Basic equations and relations of the theory of asymmetric magnetoelasticity of a ferromagnetic body, Problems of the mechanics of thin deformable bodies, in *Dedicated to the 80th Anniversary of Academician of the National Academy of Sciences of Armenia SA Ambartsumyan*, pp. 37–47 (2002). Available: https://scholar.google.com/scholar?hl=vi&as_sdt=0,5&q=. Nistor, Generalized theory of Cosserat thermoelastic media, Bul. Inst. Politech., Jassy 37, pp. 89–96, 1991.&btnG=
36. V. Baskakov, N. Bestuzheva, N. Konchakova, Linear dynamic theory of thermoelastic media with a microstructure. *Vor. State Tech. Univ.* 5–15 (1998). Available: https://scholar.google.com/scholar?q=BaskakovVA%2CBestuzhevaNP%2CKonchakovaNA%282001%29LinearDynamicTheoryofThermoelasticMediawithaMicrostructure%28inRuss.%29