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A Novel Image Watermarking Scheme Using LU Decomposition

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***Abstract*—In recent years, protecting copyright of digital images is an indispensable requirement for owners. To against with rapidly increasing of attacks, many techniques have been proposed in transform domain for ensuring quality of watermarked image, robustness of extracted watermark and execution time. Among these techniques, LU decomposition is considered as an outstanding technique in term of computation. However, it is that not all square matrices have an LU decomposition. Therefore, the suitable blocks need to be chosen before factorizing pixel matrices into lower and upper triangular matrix. In addition, in order to improve the invisibility of watermarked image, watermark should be embedded on one element of L matrix instead of two elements as the previous proposals. In this paper, we propose a novel image watermarking scheme which is based on strategy of LU blocks selection and an improved embedding method. Beside that, the extraction time is significantly sped up by a new solution to get out $L(2,1)$ and $L(3,1)$ elements of L matrix without performing LU decomposition in the extracting stage. According to the experimental results, our proposed method not only has the much better visual quality of watermarked images, but also can effectively extracts the watermark under some attacks.**

***Index Terms*—image watermarking, LU decomposition, block selection strategy, embedding formula, extracting formula**

I. Introduction

A. Background

Nowadays, copyright protection is more and more becoming important because the digital data is very easy to modify or fake information of owner in support of modern tools. To protected ownership, a technique called watermarking has been researched by many researchers in recent years. Watermarking is a technique with similarities to steganography. It is the operation of hiding watermark into digital data where exists a relationship between the watermark and the carrier signal [1].

Depending on the watermark embedding domain, we can separate digital watermarking methods in form of spatial domain and transform domain [2]. Spatial domain based methods have low computational complexity, but they are not often robust against almost image attacks. On the contrary, in transform domain methods, the host image is first transformed into the frequency domain by several transformation methods such as discrete cosine transform (DCT) [3], discrete wavelet transform (DWT) [4] or matrix decomposition such as singular value decomposition (SVD), QR decomposition, LU decomposition, Schur decomposition. Although these watermarking methods have high calculation time, they are often stronger than spatial domain based schemes.

While the time required to conduct SVD computation is about $11n^3$ flops, the Schur decomposition needs fewer number of flops which is approximately $8n^3/3$ for $n \times n$ matrix [6]. That is the reason why some researchers focused on kind of this matrix analysis [5], [6]. Su [6] proposed a new Schur decomposition based algorithm where the $U(2,k)$ and $U(3,k)$ elements of U unitary matrix are chosen for embedding (with k is a row index of D triangular matrix that contains the biggest value).

In order to strengthen the robustness of watermark, many authors combined different transforms such as DWT and SVD [7]-[12], DCT and SVD [13], DWT and DCT [14], DWT and QR [15], [16], or DWT and LU [17]. In 2016, Dongyan Wang *et. al.* [17] combined DWT and LU decomposition to produce a novel scheme. In this research, the author executed one-level DWT transformation on G channel of original image, divided into 4×4 blocks and applied LU computation on LH and HL subbands after that. Watermark, which is encoded by Arnold algorithm before it is converted to binary sequence, is embedded on $U(1,4)$ element of U upper triangular matrix. Furthermore, there is a combination of DWT and SVD which is proposed by Lou in 2020 [7]. In this proposal, Lou decomposed the host image into four subbands by DWT transform, and LL is splitted up non-overlapping 4×4 blocks at fisrt. For each block, SVD decomposition is performed on LL subband and the suitable SVD blocks will be chosen with an optimal selection policy. After that, adaptive embedding factor is calculated based on information entropy for each block. The experimental results of these proposals showed that robustness of extracted watermark is more improved than previous research. Normalized Correlation (NC) values, which measures robustness, are often up to 90% for almost image attacks. However, the invisibility of watermarked images is only around 40dB by Peak Signal to Noise Ratio (PSNR) value.

Difference from above methods, LU decomposition, which often hides information on the 2^{nd} and 3^{rd} elements of the first column of L lower triangular matrix, has a big advantage in term of computational complexity. Su *et. al.* [2] found a certain similarity between any two elements in the first column of the lower triangular matrix L after performing LU decomposition on 4×4 image pixel blocks. After that, Normalized Correlation (NC) value between these elements is computed to find out needed ones. Experimental results showed that LU computing is extremely easier than other transformations, but watermarked image often has bit worse invisibility. This is because Su [2] embedded on two elements ($L(2,1)$ and $L(3,1)$), so it led to modified two rows of pixel matrix. This change impacted on the quality of watermarked image. Therefore, we need a better solution for watermarking scheme based on LU decomposition and our contributions in this paper include:

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- Propose a block selection strategy for LU decomposition.
- Improve the embedding formula to have the better invisibility of watermarked image.
- Propose a new way to get out $L(2,1)$ and $L(3,1)$ of L matrix without calculating LU decomposition in order to speed up execution time in the extracting process.

B. Roadmap

The rest of this paper is organized as follows. First, improved ideas will be represented in Section II. In Section III, a proposed image watermarking scheme is figured in detail. The experiments and comparisons are also discussed in Section IV. Finally, Section V will conclude the paper in form of a paragraph.

II. Improved Ideas

A. LU decomposition

A square matrix A can be decomposed in form of a product of two matrices as (1):

$$A = LU, \tag{1}$$

where L is a lower triangular matrix and U is an upper triangular matrix. Then A has an LU decomposition. Because LU decomposition is not always unique, we consider L matrix where all diagonal elements are set to 1.

B. A noticeable point of LU decomposition

In linear algebra, LU decomposition is an approach designed to exploit triangular systems. However, some researchers found a remarkable point of this factorization. In [17], Dongyan noticed that "LU decomposition does not exist in all cases and the condition when the decomposition can be conducted is the determinants $det(A(1:k, 1:k)) \neq 0$, with $k = 1:n$". Beside that, Taboga [18] emphasized "Sometimes it is impossible to write a matrix in the form of "lower triangular"×"upper triangular"". In addition, two propositions are represented in [18] and [19] as follows:

Proposition 1: Not all square matrices have an LU factorization.

Proof: It is sufficient to provide a single counter example. Take a 2×2 invertible matrix as

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Suppose A has an LU factorization $A = LU$ with factors

$$L = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \text{ and } U = \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} \tag{2}$$

Compute the product

$$LU = \begin{bmatrix} L_{11}U_{11} & L_{11}U_{12} \\ L_{21}U_{11} & L_{21}U_{12} + L_{22}U_{22} \end{bmatrix} \tag{3}$$

Now, $A_{11} = (LU)_{11}$ implies $L_{11}U_{11} = 0$, which in turn implies that at least one of L_{11} and U_{11} must be zero. As a consequence, at least one of L and U is not invertible (because triangular matrices are invertible only if their diagonal entries are nonzero). This is in contradiction with the fact that A is invertible and, as a consequence, L and U must be invertible (see the proposition about the invertibility of products). Thus, we have proved by contradiction that A cannot have an LU decomposition.

TABLE I
Statistical Data of the Unsuitable Blocks for Some Images

Image (.bmp)	Number of unsuitable blocks	Percentage (%)	NC
baboon	79	0.48	0.9952
lena	517	3.15	0.9714
avion	1040	6.34	0.9444
peppers	1414	8.63	0.9157
Girl	4959	30.26	0.8522
blueeye	8239	50.29	0.7918
anhinga	12488	76.22	0.7267

Proposition 2: An invertible matrix A has an LU decomposition provided that all its leading submatrices have nonzero determinants.

To illustrate for these arguments, we surveyed on some digital images. The experiments are performed on seven color images with size of 512×512. Each image is divided into 4×4 blocks, so the total blocks are 16,384. We calculate the number of unsuitable blocks which do not agree with the above proposition, the rate and the Normalized Correlation index (NC) for each image. The results are expressed in Tab. I. The figures show that there is a relationship between the number of unsuitable blocks with NC index. It means that the image has fewer the number of unsuitable blocks, it has higher NC value and vice versa. Therefore, in order to improve the robustness of the extracted watermark, it is necessary to chose suitable blocks before embedding. We need to embed the watermark on suitable blocks which have LU decomposition instead of the whole pixel blocks.

C. Considering the element to embed

For image watermarking scheme, selecting the element(s) to embed is extremely important. In [2], Su *et. al.* computed Normalized Cross-Correlation (NCC) between the first column elements of two lower triangular matrices. The result showed that L_{21} and L_{31} are the closest elements, so they can be used to embed information. However, because Su embedded the watermark bits on these two elements at the same time for each block, the values of the matrix after embedding are changed on two rows. Thus, the pixel values of watermarked image are modified significantly. This causes the quality of the watermarked image is reduced. To address this issue, we propose a better solution which only embeds on L_{21} or L_{31} at once for each block. Our proposal makes a change on one row of the matrix after embedding, so the invisibility (the quality of the embedded image) is improved. Fig. 1 gives a comparison between formula of Su [2] and our method for both embedding and extracting stage.

D. An idea for improving execution time

For a watermarking method, the execution time consists of embedding time and extracting time. In almost published image watermarking schemes, LU decomposition is always needed to perform in the extracting process [2], [17]. In fact, this is completely unnecessary due to the special feature of LU decomposition. In the extracting stage, we only need to get out

Stage	The formula of Su [2]	The proposed formula
Embedding	If $L_{21} \le L_{31}$ and w =1, then $\begin{cases} L'_{21} = L_{avg} + T \\ L'_{31} = L_{avg} - T \end{cases}$ If $L_{21} > L_{31}$ and w =0, then $\begin{cases} L'_{21} = L_{avg} - T \\ L'_{31} = L_{avg} + T \end{cases}$ where $L_{avg} = (L_{21} + L_{31})/2$ and T is the embedding strength of watermark.	If $L_{21} \ge L_{31}$ and w =1, then $L'_{21} = L_{31} - T$ If $L_{21} < L_{31}$ and w =0, then $L'_{31} = L_{21} - T$ where T is the embedding strength of watermark.
Extracting	$w^*_{ij} = \begin{cases} 0, & if\ L^*_{21} \le L^*_{31} \\ 1, & if\ L^*_{21} > L^*_{31} \end{cases}$	$w^*_{ij} = \begin{cases} 0, & if\ L^*_{21} \ge L^*_{31} \\ 1, & if\ L^*_{21} < L^*_{31} \end{cases}$

Fig. 1. A comparison between the formula of Su and the proposed formula

of L_{21} and L_{31} of L matrix. Let consider the below illustration to see the new solution. We suppose that

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} = LU, \quad (4)$$

where

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ L_{21} & 1 & 0 & 0 \\ L_{31} & L_{32} & 1 & 0 \\ L_{41} & L_{42} & L_{43} & 1 \end{bmatrix} \ and\ U = \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix} \quad (5)$$

By multiplying L matrix with U matrix and setting a equation to A matrix, we have:

$$U_{11} = A_{11}\ ,\ U_{12} = A_{12}\ and\ U_{13} = A_{13} \quad (6)$$

Consider the first entry of the second row

$$L_{21}U_{11} = A_{21} \Longrightarrow L_{21} = \frac{A_{21}}{U_{11}} = \frac{A_{21}}{A_{11}}, \quad (7)$$

and the first entry of the third row

$$L_{31}U_{11} = A_{31} \Longrightarrow L_{31} = \frac{A_{31}}{U_{11}} = \frac{A_{31}}{A_{11}}. \quad (8)$$

Therefore, this is a wonderful way to find L_{21} and L_{31} without using LU factorization. As a result, this idea can reduce significantly calculation time in the extracting stage.

III. The Proposed Image Watermarking Method

In this section, a novel image watermarking scheme is represented in which the appropriate blocks for LU decomposition are chosen to embed the information. Before embedding, a watermark preprocessing operation is executed by utilizing Arnold transform to enhance the security of the proposed algorithm. In the embedding stage, only one element of L matrix ($L(2,1)$ or $L(3,1)$) is modified for each suitable block. In the extracting stage, we calculate $L(2,1)$ and $L(3,1)$ elements which are based on (7) and (8) without using LU factorization.

A. The embedding stage

The embedding stage includes steps as follows:

- Step 1
  - The host image is divided into 4×4 non-overlapping blocks.
- Step 2
  - The watermark is permuted by Arnold Transform. The key for this operation is considered as Key1.
  - The permuted image is convert to a one-dimensional binary array.
- Step 3
  - For each block, assign B components to a matrix A.
- Step 4
  - Calculate determinants of submatrices of A matrix.
  - If one of the determinants = 0, come back to Step 3.
  - If all determinants $\neq 0$, go to Step 5.
- Step 5
  - Decompose $A = LU$
  - Embed the corresponding watermark value into $L(2,1)$ or $L(3,1)$ of L matrix as follows:
  - Case $L(2,1) \ge L(3,1)$ and w_i = "1"

$$L'(2,1) = L(3,1) - T, \quad (9)$$

  - Case $L(2,1) \le L(3,1)$ and w_i = "0"

$$L'(3,1) = L(2,1) - T, \quad (10)$$

  where T is the embedding strength of watermark.
- Step 6
  - Update A to $A' = L'U$ and assign A' back to B components of the block.
  - Mark and save the suitable block to a file. This is considered as Key2 of the scheme.
  - Repeat steps from 3 to 6 until all suitable blocks are embedded.
- Step 7
  - Reconstruct watermarked B components to receive the watermarked image.

The flowchart of the embedding process is illustrated in Fig. 2.

B. The extracting stage

- Step 1
  - The watermarked image is divided into 4×4 non-overlapping blocks.
- Step 2
  - For each block, assign B components to a matrix A.
- Step 3
  - Check the block by comparing to the saved file.
  - If the block is not in the file, it means that it is unsuitable, come back to Step 2
  - If the block is in the file, it means that it is suitable, go to Step 4
- Step 4
  - Get out the elements of L matrix by formula (7) and (8).

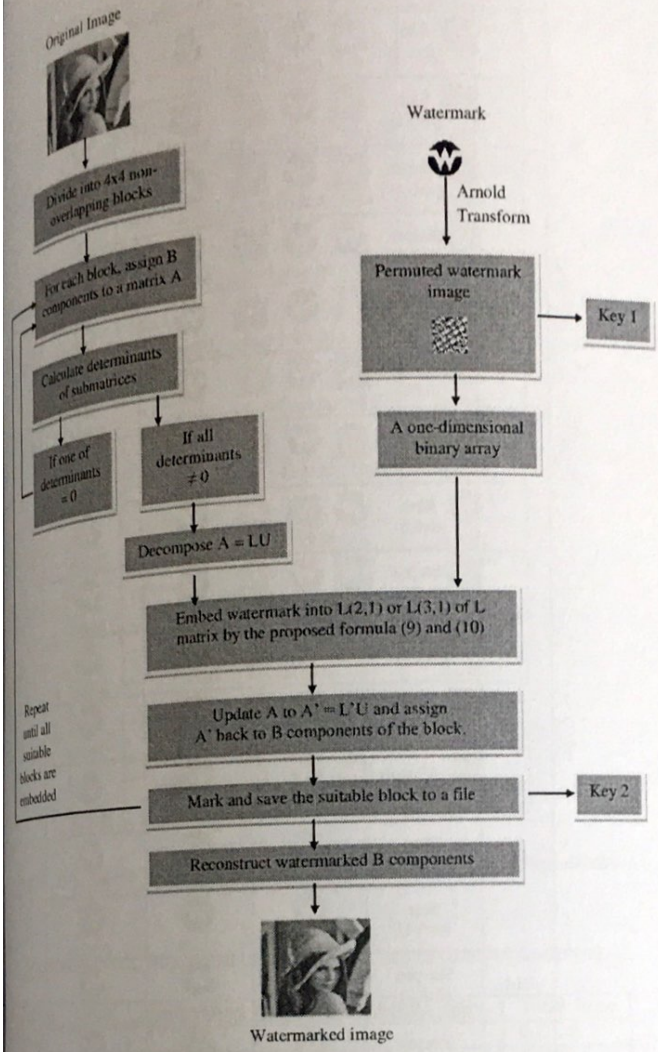

Fig. 2. The embedding process

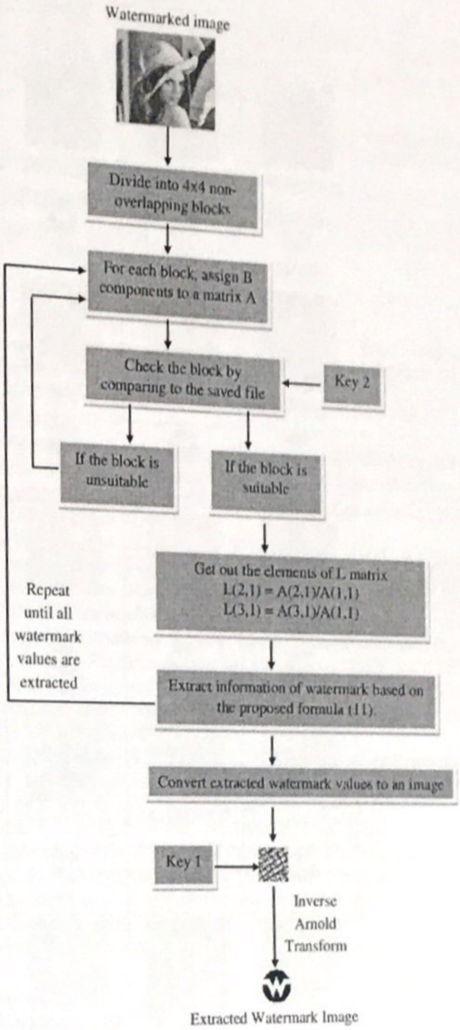

Fig. 3. The extracting process

- Extract information of watermark as follows:

$$w_i^* = \begin{cases} \text{“0”}, & L^*(2,1) \geq L^*(3,1) \\ \text{“1”}, & \text{elsewhere} \end{cases} \quad (11)$$

- Repeat steps from 2 to 4 until all watermark bits are extracted.

• Step 5
  - Convert extracted watermark values to an image.
  - Apply Inverse Arnold Transform to get final extracted watermark image.

The flowchart of the extracting process can be designed as in Fig. 3.

IV. Experimental Results and Discussion

A. Imperceptibility experiments

In our experiments, five standard color 512×512 images from the CVG-UGR[1] image database are chosen as the host images, while a binary 32×32 image is used to be the watermark. The embedding strength of watermark T is designated as 0.0275 which is the same to the method of Su [2]. This value is to ensure a balance between the quality of the watermarked image and the robustness of the watermark after extracting.

[1] https://decsai.ugr.es/cvg/dbimagenes/

The results of imperceptibility experiments are shown in Fig. 4. The figures show that values of PSNR and SSIM of the proposed watermarking algorithm are higher than the both method of Su [2] and Lou [7] for all five images. Therefore, the proposed method has better invisibility of watermarked images. This can be explained as follow: the method of Su [2] and Lou [7] embedded on two elements for each block. In [7], Lou embedded on $U(2,1)$ and $U(3,1)$ of U orthogonal matrix after doing SVD decomposition. It is similar to [2] where Su modified $L(2,1)$ and $L(3,1)$ of L lower triangular matrix at the same time. Thus, this leads a big change the pixel values after embedding. As a result, the quality of the watermarked image will be pulled down. In contrast, our method impacts on one element for each block, so the pixel matrix is only changed on one row instead of two row as the other methods. In addition, because of selecting suitable blocks before embedding of the proposed scheme, this improvement has a part in pulling up the imperceptibility. Fig. 4 also points that the watermark after extracting of the proposed method is clearer than method of Su [2] and Lou [7].

B. Robustness experiments

Robustness is a necessary evaluation principle for building watermarking schemes, so image processing operations are often added to assess the efficiency of this criterion. In our

Method	Su [2]	Luo [7]	Proposed method
Watermarked image (PSNR (dB)/SSIM)	 43.4108/0.9893	 53.9879/0.9978	 54.5432/0.9991
Extracted watermark (NC)	0.8522	0.8977	0.9952
Watermarked image (PSNR (dB)/SSIM)	 44.3169/0.9931	 47.9291/0.9927	 51.9394/0.9985
Extracted watermark (NC)	0.9714	0.9828	0.9962
Watermarked image (PSNR (dB)/SSIM)	 45.9144/0.9959	 47.6466/0.9953	 48.0773/0.9961
Extracted watermark (NC)	0.9158	0.9527	0.9971
Watermarked image (PSNR (dB)/SSIM)	 34.9126/0.9650	 52.9882/0.9985	 54.0974/0.9993
Extracted watermark (NC)	0.9444	0.9827	0.9933
Watermarked image (PSNR (dB)/SSIM)	 39.3293/0.9910	 46.0408/0.9976	 43.7385/0.9963
Extracted watermark (NC)	0.9952	0.9035	0.9956

Fig. 4. The results of invisibility tests

Images	Attacks	Su [2]	Lou [7]	The proposed method
avion	Blur (0x0.2)	0.9609	0.9827	0.9933
	Blur (0x0.5)	0.9330	0.9722	0.9628
	Sharpen (0x0.2)	0.9609	0.9827	0.9933
	Sharpen (0x0.5)	0.9406	0.9557	0.9628
	Salt&Pepper (0.002)	0.9569	0.9663	0.9857
	Salt&Pepper (0.01)	0.9390	0.9134	0.9485
lena	Blur (0x0.2)	0.9415	0.9827	0.9962
	Blur (0x0.5)	0.9292	0.9618	0.9753
	Sharpen (0x0.2)	0.9415	0.9827	0.9962
	Sharpen (0x0.5)	0.9274	0.9743	0.9732
	Salt&Pepper (0.002)	0.9396	0.9751	0.9895
	Salt&Pepper (0.01)	0.9245	0.9342	0.9628
Girl	Blur (0x0.2)	0.8552	0.8977	0.9952
	Blur (0x0.5)	0.8436	0.8591	0.9923
	Sharpen (0x0.2)	0.8522	0.8977	0.9952
	Sharpen (0x0.5)	0.8421	0.8968	0.9952
	Salt&Pepper (0.002)	0.8470	0.8882	0.9904

Fig. 5. The results of robustness tests under blurring, sharpening and Salt&Pepper noise attacks

experiments, three watermarked images are chosen to test under five basic attacks which include blurring, sharpening, salt&pepper noise, rotation and scaling.

The results in Fig. 5 shows that the proposed method is very effective under attacks such as blurring, sharpening and salt&pepper because NC values are above 0.9 for all these attacks. Moreover, our method overcomes the schemes of Su [2] and Lou [7] under these attacks.

For geometric attacks such as rotation and scaling, the robustness of all three methods is not really attractive. In three methods, the method of Lou [7] is better than others, particularly under scaling $\times 2$ operation. The reason for this is because Lou used a combination of DWT and SVD where only LL subband is chosen for SVD decomposition and after that the watermark is embedded into u_5 and u_9 elements of U matrix. Although the proposed method is more improved than the method of Su [2], NC values are only around from 0.6 to 0.8 and the watermark cannot be recognized for almost cases. This happens due to particularity of LU decomposition. As described in Section I, not all square matrices have an LU factorization. Therefore, geometric attacks can make a change to this property of blocks. As a result, the watermark after extracting will be much modified. This is considered as a disadvantage of the schemes based on LU decomposition. The detail of the results is displayed in Fig. 6.

C. A comparison of execution time

In these experiments, a computer with $Intel^{@}$ $Core^{TM}$ i5-6200U CPU at 2.30GHz, 4.00GB RAM, 64-bit OS and Visual Studio $v15$ is used as the computing platform. Tab. II shows a comparison of the execution time between different methods. For embedding stage, the method of Su [2] consumes the least time, while the scheme of Lou [7] costs the most time for calculating. The reason for this is that Lou used a combination between DWT and SVD decomposition, which have a big computational complexity. Although the both the method of Su [2] and the proposed method uses LU decomposition, the proposed method spends more execution time than the one of Su [2]. This is because that the proposed method needs to check and select suitable blocks. However, this expense can be completely accepted when we consider to effects that it brings in improving the quality of the watermarked image as well as the robustness.

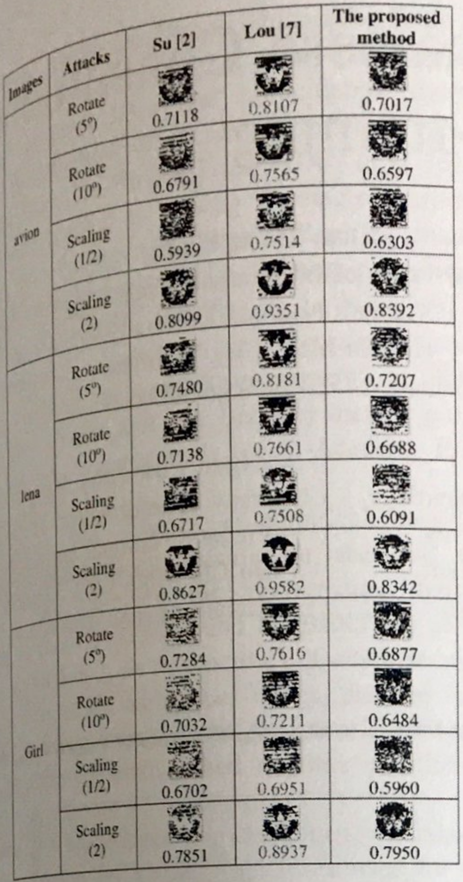

Fig. 6. The results of robustness tests under rotation and scaling attacks

TABLE II
AVERAGE EXECUTION TIME OF DIFFERENT METHODS (IN SECOND)

Method	Embedding time	Extracting time	Total time
Su [2]	0.1774	0.0294	0.2068
Lou [7]	2.4566	0.1730	2.6296
Proposed method	0.2750	0.0060	0.2810

Furthermore, there is an accretion in extracting stage where the proposed method do not need to perform LU decomposition. This reduces significantly extracting time. Overall, the execution time of the proposed method can satisfy with requirement of a real time applications.

V. Conclusion

In this paper, a novel image watermarking scheme is represented which is based on LU decomposition. The both embedding and extracting stages are improved to enhance the quality of the watermarked image, the robustness of extracted watermark, and execution time. In the embedding stage, the host image is divided into 4×4 blocks. For each block, it is checked to satisfy with the condition of LU composition. After that, a watermark preprocessing operation is performed by applying Arnold Transform before embedding information on $L(2,1)$ or $L(3,1)$ of L matrix of suitable blocks. The watermarked image is received after all selected blocks are embedded. In the extracting stage, instead of calculating LU factorization, $L(2,1)$ and $L(3,1)$ are gotten out easily by using formula (7) and (8). This helps to save much time for the extracting process. The experimental results illustrate that the proposed algorithm not only overcomes the method of Su [2] and Lou [7] in term of the quality of the watermarked image, but also improves significantly the robustness under some attacks as well as the execution time. In the future, a combination of DWT and LU can be further studied to improve the disadvantages of this proposal under geometric attacks.

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