An Effective and Efficient Approach to Classification with Incomplete Data

Cao Truong Tran[a,b,*], Mengjie Zhang[a], Peter Andreae[a], Bing Xue[a], Lam Thu Bui[b]

[a] *School of Engineering and Computer Science, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand*
[b] *Research Group of Computational Intelligence, Le Quy Don Technical University, 236 Hoang Quoc Viet St, Hanoi, Vietnam.*

Abstract

Many real-world datasets suffer from the unavoidable issue of missing values. Classification with incomplete data has to be carefully handled because inadequate treatment of missing values will cause large classification errors. Using imputation to transform incomplete data into complete data is a common approach to classification with incomplete data. However, simple imputation methods are often not accurate, and powerful imputation methods are usually computationally intensive. A recent approach to handling incomplete data constructs an ensemble of classifiers, each tailored to a known pattern of missing data. The main advantage of this approach is that it can classify new incomplete instances without requiring any imputation. This paper proposes an improvement on the ensemble approach by integrating imputation and genetic-based feature selection. The imputation creates higher quality training data. The feature selection reduces the number of missing patterns which increases the speed of classification, and greatly increases the fraction of new instances that can be classified by the ensemble. The results of experiments show that the proposed method is more accurate, and faster than previous common methods for classification with incomplete data.

Keywords: incomplete data, missing data, classification, imputation, feature

*Corresponding author. Tel: +64221587242
Email address: cao.truong.tran@ecs.vuw.ac.nz (Cao Truong Tran)

selection, ensemble learning

1. Introduction

Classification is one of the most important tasks in machine learning and data mining [1]. Classification consists of two main processes: a training process and an application (test) process, where the training process builds a classifier which is then used to classify unseen instances in the application process. Classification has been successfully applied to many scientific domains such as face recognition, fingerprint, medical diagnosis and credit card fraud transaction. Many algorithms have been proposed to deal with classification problems, but the majority of them require complete data and cannot be directly applied to data with missing values. Even when some methods can be applied, missing values often lead to big classification error rates due to inadequate information for the training and application processes [2].

Unfortunately, missing values are a common issue in numerous real-world datasets. For example, 45% of the datasets in the UCI machine learning repository [3], which is one of the most popular benchmark databases for machine learning, contain missing values [2]. In an industrial experiment, results can be missing due to machine failure during the data collection process. Data collected from social surveys is often incomplete since respondents frequently ignore some questions. Medical datasets usually suffer from missing values because typically not all tests can be done for all patients [4, 5]. Financial datasets also often contain missing values due to data change [6, 7].

One of the most common approaches to classification with incomplete data is to use imputation methods to substitute missing values with plausible values [4, 8, 9]. For example, mean imputation replaces all missing values in a feature by the average of existing values in the same feature. Imputation can provide complete data which can then be used by any classification algorithm. Simple imputation methods such as mean imputation are often efficient but they are often not accurate enough. In contrast, powerful imputation methods such as

multiple imputation [10] are usually more accurate, but are computationally expensive [11, 12]. It is not straightforward to determine how to combine classification algorithms and imputation in a way that is both effective and efficient, particularly in the application process.

Ensemble learning is the process of constructing a set of classifiers instead of a single classifier for a classification task, and it has been proven to improve classification accuracy [13]. Ensemble learning also has been applied to classification with incomplete data by building multiple classifiers in the training process and then applicable classifiers are selected to classify each incomplete instance in the application process without requiring any imputation method [14, 15, 16]. However, existing ensemble methods for classification with incomplete data often cannot work well on datasets with numerous missing values [14, 16]. Moreover, they usually have to build a large number of classifiers, which then require a lot of time to find applicable classifiers for each incomplete instance in the application process, especially when incomplete datasets contain a high proportion of missing values [14, 15]. Therefore, how to construct a compact set of classifiers able to work well even on datasets with numerous missing values should be investigated.

Feature selection is the process of selecting relevant features from original features, and it has been widely used to improve classification with complete data [17]. Feature selection has also been investigated in incomplete data [18, 19], but the existing methods typically still use imputation to estimate missing values in incomplete instances before classifying them. By removing redundant and irrelevant features, feature selection has the potential of reducing the number of incomplete instances, which could then improve accuracy and speed up classifying incomplete instances. However, this aspect of feature selection has not been investigated. This paper will show how to utilise feature selection to improve accuracy and speed up the application process for classification with incomplete data.

1.1. Goals

To deal with the issues stated above, this paper aims to develop an effective and efficient approach for classification with incomplete data, which uses three powerful techniques: imputation, feature selection and ensemble learning. Imputation is used to transform incomplete training data to complete training data which is then further enhanced by feature selection. After that, the proposed method builds a set of specialised classifiers which can classify new incomplete instances without the need of imputation. The proposed method is compared with other common approaches for classification with incomplete data to investigate the following main objectives:

1. How to effectively and efficiently use imputation for classification with incomplete data; and
2. How to use feature selection for classification with incomplete data to not only improve classification accuracy but also speed up classifying new instances; and
3. How to build a set of classifiers which can effectively and efficiently classify incomplete instances without the need of imputation; and
4. Whether the proposed method can be more accurate and faster than using imputation both in the training process and the application process; and
5. Whether the proposed method can be more accurate and faster than the existing ensemble methods.

1.2. Organisation

The rest of this paper is organised as follows. Section 2 presents a survey of related work. The proposed method is described in Section 3. Section 4 explains experiment design. The results and analysis are presented and discussed in Section 5. Section 6 states conclusions and future work.

2. Related Work

This section firstly introduces traditional approaches to classification with incomplete data. It then discusses ensemble learning for classification with

incomplete data. Finally, it presents typical work on feature selection.

2.1. Traditional Approaches to Classification with Incomplete Data

There are several traditional approaches to classification with incomplete data. The deletion approach simply deletes all instances containing missing values. This approach is limited to datasets with only a few missing values in the training data and no missing values in the application process [20]. A second approach is to use one of classifies such as C4.5 which can directly classify incomplete datasets using a probabilistic approach [21]. However, their accuracy is limited when there are a lot of missing values [22].

The most used approach to classification with incomplete data is to use imputation methods to transform incomplete data into complete data before building a classifier in the training process or classifying a new incomplete instance in the application process. This approach has the advantage that the imputed complete data can be used by any classification algorithm. This approach also can deal with incomplete datasets with a large number of missing values [8, 23].

Fig. 1 shows the main steps using imputation for classification with incomplete data. In the training process, imputation is used to estimate missing values for incomplete training data. After that, imputed training data is put into a classification algorithm to build a classifier. In the application process, complete instances are directly classified by the classifier. With each incomplete instance, its missing values are first replaced by plausible values by using the imputation to generate a complete instance which is then classified by the classifier.

There are two classes of imputation: single imputation and multiple imputation.

2.1.1. Single Imputation

Single imputation estimates a single value for each missing value. Mean imputation is an example of single imputation methods which fills all missing values in each feature by the average of all existing values in the same feature.

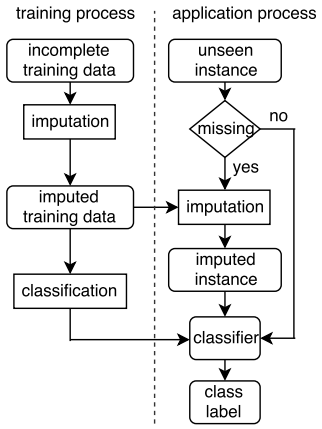


Figure 1: A common approach to using imputation for classification with incomplete data.

kNN-based imputation is one of the most powerful single imputation methods [22]. To estimate missing values in an incomplete instance, it first searches for its k nearest neighbour instances. After that, it replaces missing values of the instance with the average of existing values in the k instances. kNN-based imputation is often more accurate than mean imputation [22]. However, kNN-based imputation is more computationally expensive than mean imputation because it takes time to find the nearest instances, especially with datasets containing a large number of instances and a large value of k [22].

Single imputation has been widely used to estimate missing values for classification with incomplete data [8, 20, 23, 22]. In [20] and [22], kNN-based imputation is shown to outperform C4.5, mean imputation and mode imputation. In [23], the impact of six imputation methods on classification accuracy for six classification algorithms are investigated. Results show imputation on average can improve classification accuracy when compared to without using imputation. However, in [8], fourteen different imputation methods are evaluated on three different classification algorithm groups. The analysis shows that there is no best imputation for all classifiers.

2.1.2. Multiple Imputation

Multiple imputation estimates a set of values for each missing value. Multiple imputation is often better than single imputation because it can better reflect the uncertainty of missing data than single imputation [11]. However, multiple imputation is usually more computationally expensive than single imputation since it takes time to estimate a set of values for each missing value [4].

Multiple imputation using chained equations (MICE) is one of the most flexible and powerful multiple imputation methods [10, 24]. MICE uses regression methods to estimate missing values. Initially, each missing value in each feature is replaced by a random value in the same feature. Each incomplete feature is then regressed on the other features to compute a better estimate for the feature. The process is performed several times (q) for all incomplete features to provide a single imputed dataset. The whole procedure is repeated p times to provide p imputed datasets. Finally, the final imputed dataset is calculated by the average of the p imputed datasets [10, 24].

Multiple imputation has also been applied to classification with incomplete data [11, 12, 25]. In [11], multiple imputation methods are compared with single imputation methods. The study shows that multiple imputation methods are often more accurate, but more expensive than single imputation methods. In [25], MICE is compared to other four single imputation methods. Results show that MICE can outperform the other single imputation methods, especially on datasets containing numerous missing values. In [12], ensemble learning is combined with multiple imputation to construct a set of classifiers. Each imputed dataset which is generated by the multiple imputation method is used to build a classifier. The analysis shows that the proposed method can normally obtain better accuracy than using multiple imputation.

Existing imputation researches for classification often focus on improving classification accuracy [22, 25]. However, using imputation takes time to estimate missing values, especially powerful imputation such as MICE is computationally intensive. Therefore, how to effectively and efficiently use imputation

for classification should be further investigated.

2.2. Ensemble Classifiers for Incomplete Data

Ensemble learning is a learning method which constructs a set of base classifiers for a classification task in the training process. To classify new instances in the application process, the predictions of base classifiers are combined. Ensembles of classifiers have been proven to be more accurate than any of base classifiers making up the ensemble [13].

Ensemble learning also has been used for classification with incomplete data. One of the first work using ensembles for classification with incomplete data appears in [26], where four neural networks are built to address classification with a thyroid disease database consisting of two incomplete features (one classifier that ignores both the missing features, two classifiers that can be used if one of the features is present, and one classifier that requires both features to be present). Experimental results show that the ensemble is superior to an imputation method using neural networks to estimate missing values. The ensemble is also more accurate than an induction algorithm which builds a decision tree able to directly work with incomplete data. The systems described in [14] and [27] tackle incomplete data by learning a set of neural networks, where each neural network is trained on one complete sub dataset extracted from incomplete data by dropping some of missing features. Given a new incomplete instance to classify, the available learnt classifiers are combined to classify the instance without requiring imputation. Empirical results show that the ensembles of neural networks can achieve better accuracy than two other ensemble methods (based on bagging and boosting) combined with mean and mode imputations. A similar approach is proposed in [28], where conditional entropy is used as the weighting parameter to reflect the quality of feature subsets which are used to build base classifiers. The proposal of [28] is extended in [16] by using the mutual information criterion to eliminate redundant feature subsets. As a result, the extended system can not only outperform other methods, but can also reduce the computation time to classify incomplete instances. In [15] and [29],

an ensemble of numerous classifiers is constructed, where each base classifier is trained on a sub dataset by randomly selecting a feature subset from the original features. Thanks to constructing a larger number of base classifiers, the system can cope with incomplete data containing a high proportion of missing values.

Existing ensemble methods for classification with incomplete data can deal with missing values to some extent. However, the ensemble methods usually do not obtain good accuracy when datasets contain a large number of missing values [14, 16, 26]. The underlying reason is that the complete sub datasets often only have a small number of instances when the original incomplete data includes a large number of missing values. Therefore, the base classifiers trained on the complete sub datasets are weak classifiers. To overcome the problem, numerous base classifiers have to be built [15], which requires a long time for exploring available classifiers to classify new incomplete instances. Therefore, how to build an effective and efficient ensemble for classification with datasets containing numerous missing values should be further investigated.

2.3. Feature Selection for Classification

The purpose of feature selection is to select a subset of relevant features from the original features because many datasets often contain irrelevant and/or redundant features which can be removed without losing much information. By removing irrelevant and redundant features, feature selection can reduce the training time, simplify the classifier and improve the classification accuracy [17]. However, feature selection is a hard problem because there are 2^n possible feature subsets where n is the number of original features [30].

A feature selection method consists of two main components: an evaluation measure and a search technique [17]. The evaluation measure is used to evaluate the goodness of selected features while the search technique is used to explore new feature subsets. The quality of the feature selection method strongly depends on both the evaluation measure and the search technique [17, 30].

Evaluation measures for feature selection can be divided into wrapper methods and filter methods [17]. A wrapper method employs a classification algo-

rithm to score feature subsets while a filter method employs a proxy measure such as mutual information to score feature subsets. Filter methods are often more efficient and general than wrapper methods. However, wrapper methods tend to more accurate than many filter methods because wrapper methods directly evaluate feature subsets using classification algorithms while filter methods are independent of any classification algorithm [17].

Search techniques for feature selection can be categorised into deterministic search techniques and evolutionary search techniques [17]. Sequential forward selection (SFS) and sequential backward selection (SBS) are typical examples of deterministic search techniques [31]. In recent years, Evolutionary Computation (EC) techniques such as Genetic Algorithms (GAs), Genetic Programming(GP) and Particle Swarm Optimisation (PSO) have been successfully applied to feature selection [17]. The underlying reason is that EC techniques are good at searching for global best solutions. EC techniques also do not require domain knowledge and do not require any assumption related to the search space [17].

Feature selection has also been used for incomplete data [18, 32, 19]. However, these methods still provide incomplete data which cannot be directly used by the majority of classification algorithms. Therefore, how to apply feature selection to improve the performance of such classification algorithms when facing with missing values should be investigated.

3. The Proposed Method

This section presents the proposed method in detail. It starts with showing the definitions used in the method. The section then presents the overall structure and the underlying ideas of the method. After that, it describes the details of the training process and the application process.

3.1. Definitions

Let $D = \{(X^i, c^i)|i = 1, .., m\}$ denote a dataset, where each X^i represents an input instance with its associated class label c^i, and m is the number of instances

in the dataset. The input space is defined by a set of n features $F = \{F_1, ..., F_n\}$. Each instance X^i is represented by a vector of n values $(x_1^i, x_2^i, ...x_n^i)$, where an x_j^i is either a valid value of the j^{th} feature F_j, or is the value "?", which means that the value is unknown (a missing value).

An instance X^i is called an incomplete instance if it contains at least one missing value. A dataset, D, is called an incomplete dataset if it contains at least one incomplete instance. A feature, F_j, is called an incomplete feature for a dataset if the dataset contains at least one incomplete instance, X^i with a missing value x_j^i. For example, the incomplete dataset shown in Table 1 contains five incomplete instances: X^2, X^4, X^5, X^6, and X^7. It has four incomplete features: F_1, F_3, F_4 and F_5.

Table 1: An example dataset with missing values.

	F_1	F_2	F_3	F_4	F_5	c
X^1	5	67	3	5	3	1
X^2	4	43	1	1	?	1
X^3	4	28	1	1	3	0
X^4	5	74	1	5	?	1
X^5	4	56	1	?	3	0
X^6	4	70	?	?	3	0
X^7	?	66	?	?	1	1

A subset of features, $S \subset F$, is called a *missing pattern* in a dataset D if there is at least one instance, X^i in D, such that the value in X^i for each feature in S is missing, and the value in X^i for all the other features are known. That is, $S \subset X$ is a missing pattern in D if there exists an instance X^i in D such that if $F_j \in S$, $x_j^i =$? otherwise, x_j^i is not missing. For example, the dataset shown in Table 1 has five missing patterns: $\{\emptyset\}$, $\{F_5\}$, $\{F_4\}$, $\{F_3, F_4\}$ and $\{F_1, F_3, F_4\}$.

Algorithm 1 shows the steps to identify all missing patterns of a dataset. We use $\mathcal{MP}$ to denote the all missing patterns. At the beginning of the algorithm, $\mathcal{MP}$ is empty. The outer loop in the algorithm iterates over all instances. For each instance, all features with missing values are combined to form a missing pattern. If the missing pattern is not yet in $\mathcal{MP}$, it will be added in $\mathcal{MP}$. By the end of the algorithm, $\mathcal{MP}$ contains all missing patterns.

Given a dataset D and a feature subset S, we use D_S to represent the

Algorithm 1: $MissingPatterns(D)$

Input:
D, a dataset with m instances and n features
Output:
$\mathcal{MP}$, a set of missing patterns

```
MP ← {}
for i ← 1 to m do
    temp ← {∅}
    for j ← 1 to n do
        if x_j^i =? then
            temp ← temp ∪ F_j
        end
    end
    MP ← MP ∪ {temp}
end
return MP
```

projected dataset D onto the features in S, i.e. the dataset D reduced to the feature subset S. That is, each instance in D is replaced by the projected instance in which values for features not in S are removed. For example, given the dataset shown in Table 1 with five features, the data subset $D_{\{F_1,F_2,F_3\}}$ is shown in Table 2.

Table 2: The dataset in Table 1 reduced to the feature subset $\{F_1, F_2, F_3\}$.

	F_1	F_2	F_3	c
X^1	5	67	3	1
X^2	4	43	1	1
X^3	4	28	1	0
X^4	5	74	1	1
X^5	4	56	1	0
X^6	4	70	?	0
X^7	?	66	?	1

3.2. Overall Proposed Method

The proposed method has two main processes: a training process and an application process. The training process constructs an ensemble of classifiers which is then used to classify new instances in the application process. Fig. 2 shows the flowchart of the method.

The method is based on three key ideas. The first idea is that the method

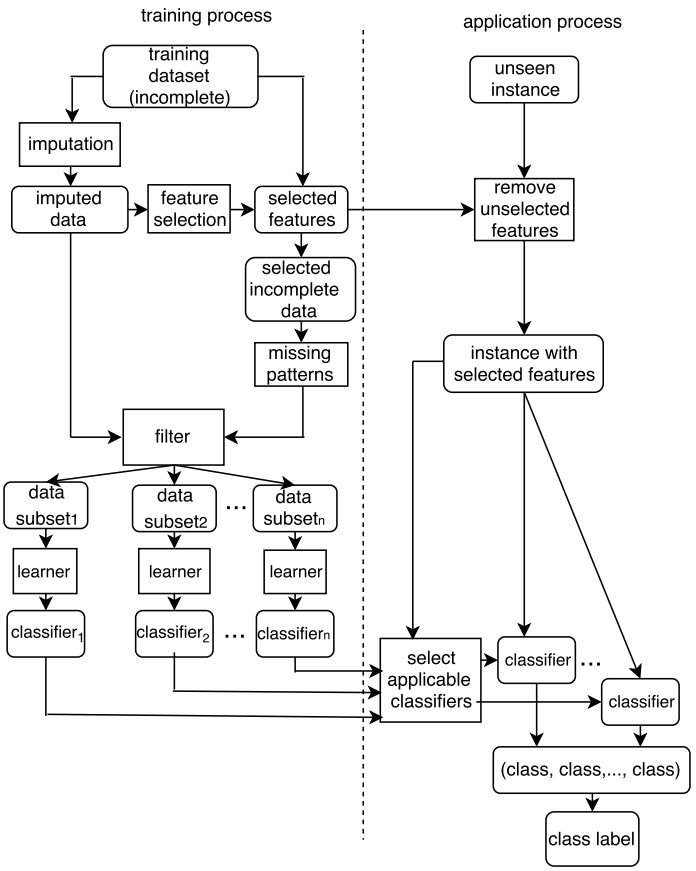


Figure 2: The proposed method builds an ensemble of classifiers then used to classify new incomplete instances without imputation.

constructs an ensemble of classifiers to cover possible missing patterns, each classifier being built on one missing pattern. Therefore, new incomplete instances can be classified by the ensemble without requiring imputation. This is not a complete novel—it has also been used in [14, 16]. The second idea is to use imputation in the training process, but not in the application process. Using a powerful imputation method to generate high quality complete training data for building classifiers results in more accurate classifiers. In contrast, existing ensemble methods based on missing patterns [14, 16] do not use any imputation, so the training set for each classifier may be as small as a single instances which leads to low accuracy. However, good imputation methods such as multiple

imputation are computationally expensive. For the training process, there is no time limit for many applications, and the high cost of multiple imputation is not a problem; in the application process, there may be tight time limits on the process of classifying a new instance, and using multiple imputation may be infeasible. The third idea is to use feature selection to further improve the training data. By removing redundant and irrelevant features, feature selection not only produces a high quality feature set, but also reduces the number of missing patterns and removes missing values of incomplete instances in the application process.

3.3. Training Process

The purpose of the training process is to build a set of classifiers, one classifier for each missing pattern in the data. Algorithm 2 shows the main steps of the training process. The inputs of the algorithm are an original training dataset D and a classifier learning algorithm $\mathcal{L}$.

Algorithm 2: The training process

Input:
D, an original training dataset
$\mathcal{L}$, a classifier learning algorithm
Output:
$\mathcal{C}$, a set of learnt classifiers
$\mathbf{w}$, the weighting of classifiers
$\mathcal{SF}$, a set of selected features

```
1  ImpD ← Imputation(D)
2  SF ← FeatureSelection(ImpD)
3  MP ← MissingPaterns(D_SF)
4  C ← {}
5  foreach MP_i ∈ MP do
6  |   CP_i ← SF − MP_i
7  |   Divide ImpD_CP_i into ImpTrain and ImpValidation
8  |   classifier_i ← L(ImpTrain)
9  |   C ← C ∪ classifier_i
10 |   w_i ← classifier_i(ImpValidation)
11 end
12 return C, w and SF ;
```

The algorithm starts by using an imputation method to estimate missing

values in the original dataset D to generate an imputed dataset $ImpD$ which is complete. After that, a feature selection method is applied to the complete dataset $ImpD$ to select the subset $\mathcal{SF}$ of important features which is then applied to the original training dataset to construct a projected (incomplete) dataset with just the features in $\mathcal{SF}$. The missing patterns algorithm (Algorithm 1) is then used to search for all missing patterns, $\mathcal{MP}$, in the reduced dataset $D_{\mathcal{SF}}$. For each missing pattern $\mathcal{MP}_i$, a "complete pattern" $\mathcal{CP}_i$ is generated by selecting features which are in $\mathcal{SF}$, but not in $\mathcal{MP}_i$. After that, the imputed dataset is reduced to the features in the complete pattern and then split into a training dataset and a validation dataset. The training dataset is used to construct a classifier which is evaluated using the validation dataset. The average accuracy on the validation set becomes the score (or weight) of the classifier. As a result, the application process generates a set of classifiers and their scores, one classifier for each missing pattern.

The four main components in the application process are imputation, feature selection, identifying missing patterns and learning the classifiers. Either single imputation or multiple imputation can be used to transform the incomplete training data to the imputed training data. Multiple imputation is generally more accurate than single imputation, especially when the data contains a large number of missing values [25, 33]. Therefore, a multiple imputation method such as MICE should be used to estimate missing values for the training data where possible. However, multiple imputation is usually much more expensive than single imputation, especially when data contains a large number of features such as in gene expression datasets [34]. Therefore, with datasets containing numerous features, a good single imputation method such as kNN-based imputation to estimate missing values for the training data can be used which makes the imputation cost in the training process feasible.

Feature selection can also be expensive, and the choice of both the evaluation method and the search method must be made carefully. Wrapper evaluation methods are often more accurate than filter methods, but generally more expensive, especially with large training datasets or if the wrapper methods use

expensive classifiers such as multiple layer perceptron. There exist fast filter methods such as CFS [31] and mRMR [35] have comparable accuracy to wrapper methods, and these filter methods could be used to evaluate feature subsets efficiently, even when the training data contains a large number of instances and features. For search techniques, evolutionary techniques have been proven to be effective and efficient for feature selection. Therefore, using evolutionary techniques such as GAs and PSO to search for feature subsets and CFS or mRMR to evaluate them enables feature selection to be done efficiently.

Searching for all missing patterns is not time-consuming. The computation time of Algorithm 1 is *O(m*n)* where m is the number of instances, and n is the number of features which is no more than the cost of reading the dataset (assuming that *temp* is represented as a bitset, and $\mathcal{MP}$ as a hash table).

If there are a large number of missing patterns, then the cost of training a set of classifiers for all missing patterns may be very expensive. Existing ensemble methods search for missing patterns in the original training data [14, 28, 16]; therefore, they often get a very large number of missing patterns when the training data contains numerous missing values. In contrast, the proposed method searches for missing patterns in the training data after it has been reduced to the selected features. This reduces the number of missing values, often by a large fraction. Therefore, the proposed method often generates a much smaller number of missing patterns even when the original training data contained numerous missing values. Therefore, the cost of the classifier learning is much less than in other ensemble methods.

3.4. Application Process

The application process is to classify new instances using the learnt classifiers. Algorithm 3 shows the main steps of the application process. The inputs of the algorithm are an instance needing to be classified X, a set of selected features $\mathcal{SF}$, and an ensemble of learnt classifiers $\mathcal{C}$ along with their weights $\mathbf{w}$. The algorithm will output the most suitable class label for the instance.

The algorithm starts by removing features in the instance X which are not

Algorithm 3: The application process

Input:
X, an instance to be classified
$\mathcal{SF}$, a set of selected features
$\mathcal{C}$, a set of learnt classifiers
$\mathbf{w}$, weights of classifiers
Output:
the class of x

```
1  Reduce X to only containing the features in SF
2  AC ← C
3  foreach missing values x_j =? in reduced X do
4  |  foreach classifier ∈ AC do
5  |  |  if classifier requires F_j then
6  |  |  |  AC ← AC − classifier
7  |  |  end
8  |  end
9  end
10 Apply each classifier in AC to reduced X
11 return majority vote of classifiers, weighted by w;
```

in the set of selected features $\mathcal{SF}$. Next, the algorithm searches for all classifiers which are applicable to the instance—classifiers which do not require any incomplete features in the instance. Subsequently, each applicable classifier is used to classify the instance, and the algorithm returns a class by taking a majority vote of the applicable classifiers' predictions, weighted by the quality of the classifiers measured in the training process.

Typical methods for classification with incomplete data as shown in Figure 1 perform imputation on the new instance. In order get adequate accuracy, it is particularly important to use a high quality imputation method such as MICE, which is very expensive. The proposed method, on the other hand, does not require any imputation method to estimate missing values for unseen incomplete instances. Therefore, the proposed method is expected to be faster than the common approach.

To classify an incomplete instance, the proposed method also reduces the instance to contain only selected features. After this reduction step, the incomplete instance frequently becomes a complete instance, which in turn removes

the need to search for applicable classifiers for the instance. Moreover, because of the feature selection, the proposed method often generates a smaller number of classifiers than existing ensemble methods which reduces the cost of the search if the instance is incomplete. Therefore, the proposed method is expected to be considerably faster than existing ensemble methods for classification with incomplete data.

4. Experiment Setup

This section discusses the aim and design of the experiments. The discussion consists of methods for comparison, datasets and parameter settings.

4.1. Benchmark Methods for Comparison

In order to investigate the effectiveness and efficiency of the proposed method, namely *NewMethod*, its accuracy and computation time were compared with five benchmark methods. The first two methods are common approaches to classification with incomplete data by using imputation as shown in Fig. 1. The other three methods are ensemble methods for classification with incomplete data without requiring imputation. The details of the five methods are as follows:

- The first benchmark method, namely *kNNI*, is to use kNN-based imputation, which is one of the most common single imputation methods, to estimate missing values for both training data and unseen instances. This benchmark method provides complete training data and complete unseen instances which can be used by any classification algorithm. Comparing the proposed method with this benchmark method can show the proposed method's advantages compared to one of the most common methods for classification with incomplete data.

- The second benchmark method, namely *MICE*, is to use MICE, which is a powerful multiple imputation method, to estimate missing values for both training data and unseen incomplete instances. Both the proposed method and this benchmark method use multiple imputation to estimate

missing values for training data; the key difference is that this benchmark method requires multiple imputation to estimating missing values for incomplete unseen instances in the application process which is very expensive. However, the proposed method can classify unseen instances by constructing a set of classifiers instead of requiring multiple imputation. Therefore, comparing the proposed method with this benchmark method can show the proposed method's effectiveness and efficiency in classifying incomplete instances.

- The third benchmark method, namely *Ensemble[14]*, is a recent ensemble method for classification with incomplete data in [14]. Both the proposed method and this benchmark method search for missing patterns and build one classifier for each missing pattern. One difference is that this benchmark method does not use any imputation method to fill missing values in the training data, while the proposed method uses a powerful imputation method to estimate missing values and provide complete data for the training process. Another difference is that this benchmark method does not use any technique to reduce the number of missing patterns; hence it may have to build a large number of classifiers. In contrast, the proposed method uses feature selection to reduce the number of missing patterns, so it is expected to speed up classifying unseen instances by building a compact number of classifiers. Therefore, comparing the proposed method with this benchmark method can show the proposed method's benefits due to using multiple imputation and feature selection.

- The fourth benchmark method, namely *Ensemble[16]*, is a very recent extension of the third benchmark method, using the mutual information to reduce the number of missing patterns [16]. Comparing the proposed method with this benchmark method can shows the proposed method's advantages due to using feature selection to not only reduce the number of missing patterns, but also reduce missing values in unseen incomplete instances.

- The final benchmark method, namely *Ensemble[15]*, is an ensemble method for classification with incomplete data in [15]. This benchmark method randomly generates missing patterns rather than exploring missing patterns from the training data; hence it has to build a large number of classifiers. Therefore, comparing the proposed method with this benchmark method can show the proposed method's effectiveness and efficiency thanks to searching for missing patterns from training data.

4.2. Datasets

Ten real-world incomplete classification datasets were used in the experiments. These datasets were selected from the UCI machine learning repository [3]. Table 3 summarises the main characteristics of the datasets including name, the number of instances, the number of features and their types (Real/Integer/Nominal), the number of classes and the percentage of incomplete instances.

Table 3: Datasets used in the experiments

Name	**#Inst**	**#Features (R/I/N)**	**#Classes**	**Incomplete inst(%)**	**Abbrev**
Chronic	400	24(11/0/13)	2	60.5	Chr
Credit	690	15(3/3/9)	2	5.36	Cre
Heart-c	303	13(13/0/0)	5	1.98	Hec
Heart-h	294	13(6/0/7)	2	100	Heh
Hepatitis	155	19(2/17/0)	2	48.39	Hep
Housevotes	435	16(0/0/16)	2	46.67	Hou
Mammographic	961	5(0/5/0)	2	13.63	Mam
Marketing	8993	13(0/13/0)	9	23.54	Mar
Ozone	2536	73(73/0/0)	2	27.12	Ozo
Tumor	339	17(0/0/17)	22	61.01	Tum

These benchmark datasets were carefully chosen to cover a wide-ranging collection of problem domains. These tasks have various percentages of incomplete instances (incomplete instances range from 1.98% in the *Hec* dataset to 100% in the *Hed* dataset). These problems range from a small number of instances (*Hep* only has 155 instances) to a large number of instances (*Mar* has 8993

instances). These datasets also range between low to high dimensionality (*Mam* only has 5 features while *Ozo* has 73 features). These problems encompass binary and multiple-class classification tasks. We expect that these datasets can reflect incomplete problems of varying difficulty, size, dimensionality and type of features.

Ten-fold cross-validation [36] was used to separate each dataset into different training and test sets. The ten-fold cross-validation process is stochastic, so it should be performed multiple times to eliminate statistical variations. Hence, the ten-fold cross-validation was independently performed 30 times on each dataset. As a result, there are 300 pairs of training set and test set for each dataset.

4.3. Parameter Settings

4.3.1. Imputation

The experiments used two imputation methods: kNN-based imputation and MICE, representing two types of imputation, single imputation and multiple imputation, respectively. With kNN-based imputation, for each incomplete dataset, different values for the number of neighbors k (1, 5, 10, 15, 20) were checked to find the best value. The implementation of MICE using R language in [24] was used to run MICE where random forest was used as a regression method. The number of cycles was set as five and the number of imputed datasets was set 20 following the recommendation in [10].

4.3.2. Feature Selection

The proposed approach is a framework, so any feature selection method can be used to select relevant features. The experiments used a filter-based feature selection method because a filter method is often quicker and more general than a wrapper method. The Correlation Feature Selection (CFS) measure [31] was used to evaluate feature subsets. The main reason is that CFS not only can evaluate the correlation between each feature with the class, but also can evaluate the uncorrelation between features in the feature subset. Moreover,

in many cases, CFS is as accurate as wrapper methods and it executes much faster than the wrapper methods [31]. An GA was used to search for feature subsets because GA has been successfully applied to feature selection. Moreover, GA's individuals can be represented by bitstrings which are suitable for feature selection, where 1 reflects selected features and 0 reflects unselected features. The parameters of GA for feature selection were set as follows. The population size was set to 50 and the maximum number of generations was set to 100. The crossover probability was set 0.9 and the mutation probability was set 0.1. CFS and GA were implemented under the WEKA [37].

4.3.3. Classifier learning Algorithms

In machine learning, classifier learning algorithms are often categorised into decision trees such as *C4.5* [21], rule-based classifiers such as *PART* [38] , instance-based classifiers such as k nearest neighbour (*kNN*) [39], and function-based classifiers such as a multilayer perceptron (*MLP*) [1]. In recent years, genetic programming (GP) has been successfully applied to classification [40, 41]. Therefore, five classification algorithms (C4.5, PART, kNN, MLP and GP) were used to compare the proposed method with the other benchmark methods. The first four classification algorithms were implemented under the WEKA [37]. The implementation of GP in the experiment used the ECJ [42]. GP using a set of static thresholds as shown in [43] was used to decide class. Table 4 shows the parameters of GP for classification.

The proposed method, *Ensemble[14]* and *Ensemble[16]* automatically identify the number of classifiers from the training data. The number of classifiers in Ensemble[15] was set equally to the number of classifiers explored by *Ensemble[14]*.

5. Results and Analysis

This section presents and discusses the experimental results. It first shows the comparison on accuracy between the proposed method and the benchmark methods. It then presents the comparison between them on computation time.

Table 4: GP parameter settings

Parameter	Value
Function set	+, -, x, / (protected division)
Variable terminals	all features
Constant terminals	Random float values
Population size	1024
Initialization	Ramped half-and-half
Generations	50
Crossover probability	60%
Mutation probability	30%
Reproduction rate	10%
Selection type	Tournament(size=7)

Further analysis is also discussed to demonstrate the advantages of the proposed method.

5.1. Accuracy

5.1.1. Comparison Method

Table 5 shows the mean and standard deviation of classification accuracies of the proposed method and the benchmark methods. The first column shows datasets, and the second column shows classification algorithms used in the experiments. "*NewMethod*" refers to the proposed method. The rest five columns are the five benchmark methods. "*kNNI*" and "*MICE*", respectively, are the benchmark methods using kNN-based imputation and MICE to estimate missing values as shown in Fig.1. "*Ensemble*[14]", "*Ensemble*[16]" and "*Ensemble*[15]" refer to the three ensemble benchmark methods in [14], [16] and [15], respectively. The values in the table are the average classification accuracy ± standard deviation resulting from combining a classifier (row) with an approach to classification with incomplete data (column).

It is very important to choose a suitable statistical test to correctly evaluate the significance of the results. A multiple test rather than a pair test should be used to compare the proposed method with the multiple (five) benchmark methods. Moreover, a non-parametric test rather a parametric test should be used because non-parametric tests do not require the normal distribution of

data as parametric tests. Therefore, the Friedman test [44], which is one of the most popular multiple non-parametric tests, is used to test the significance of the results. The test indicates that there exits significant differences between the methods in each dataset and each classifier. Therefore, the Holm procedure [44], which is a post-hoc procedure, is used to perform pair tests between two methods. In Table 5, the symbol ↑ indicates that the proposed method is significantly better than the benchmark method. In contrast, the symbol ↓ shows that the proposed method is significantly worse than the benchmark method.

5.1.2. Compare with Imputation Methods

Fig. 3 shows the fraction of cases that the proposed method is significantly better or worse than the benchmark methods. It is clear from Fig 3 that the proposed method can achieve significantly better accuracy than using imputation (*kNNI* and *MICE*) in most cases. The proposed method is significantly better than both *kNNI* and *MICE* in about 65% cases, and it is only significantly worse than both of them in 2 out of the 50 cases.

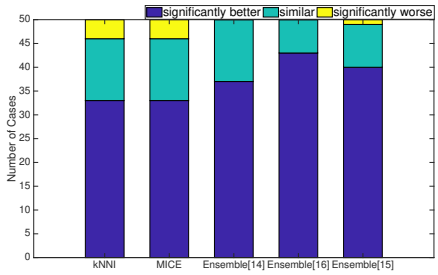


Figure 3: The comparison between the proposed method and each of the benchmark methods on all the classification algorithms.

The proposed method is more accurate than the imputation methods because it incorporates feature selection to remove redundant and irrelevant features, which helps to improve classification accuracy. Furthermore, the proposed method can construct multiple classifiers, which can be more comprehensive and generalise better than constructing a single classifier. Therefore, the proposed method can classify new instances better than the imputation methods.

Table 5: Mean and standard deviation of classification accuracies.

Data	Classifier	The proposed method and benchmark methods					
		NewMethod	**kNNI**	**MICE**	**Ensemble[14]**	**Ensemble[16]**	**Ensemble[15]**
Chr	J48	**99.10±0.33**	99.05±0.35	97.53±0.62↑	94.21±0.46↑	94.18±0.35↑	97.41±0.73↑
	PART	99.30±0.34	99.20±0.53	98.40±0.68↑	94.28±0.49↑	94.20±0.31↑	97.60±0.60↑
	kNN	97.78±0.28	98.40±0.28↓	**98.65±0.30**↓	93.55±0.36↑	93.95±0.31↑	98.06±0.69↓
	MLP	96.92±0.47	98.48±0.44↓	**99.00±0.35**↓	93.93±0.33↑	93.90±0.40↑	96.50±0.67↑
	GP	**98.97±0.29**	97.95±0.48↑	97.95±0.52↑	97.62±0.31↑	97.46±0.34↑	96.20±0.65↑
Cre	J48	85.34±0.54	**85.38±0.52**	85.31±0.58	85.06±0.67↑	84.98±0.76↑	80.24±1.26↑
	PART	**85.33±0.66**	83.92±0.92↑	83.82±0.86↑	85.26±0.71	84.80±0.77↑	79.38±1.58↑
	kNN	**84.20±0.81**	82.75±0.53↑	82.76±0.62↑	83.46±0.61↑	82.46±0.54↑	74.62±1.59↑
	MLP	**86.17±0.59**	83.02±0.94↑	83.25±0.78↑	85.18±0.62↑	85.10±0.56↑	77.19±2.84↑
	GP	**86.19±0.52**	86.15±0.62	85.95±0.62	86.03±0.49	85.55±0.52↑	79.78±1.46↑
Hec	J48	**58.06±1.55**	54.26±2.08↑	54.09±1.98↑	55.17±1.74↑	53.10±1.49↑	56.67±0.96↑
	PART	**57.32±1.77**	53.65±2.13↑	53.75±1.45↑	54.00±1.71↑	51.55±2.08↑	56.57±1.06
	kNN	**55.51±1.83**	54.82±1.13↑	54.57±1.18↑	54.74±1.05↑	54.61±1.35↑	55.11±1.40
	MLP	**58.34±1.45**	53.63±1.53↑	54.19±1.96↑	54.21±1.50↑	52.98±1.57↑	57.24±1.07↑
	GP	**57.83±0.92**	56.73±1.05↑	56.29±1.05↑	56.77±1.17↑	57.21±1.42↑	56.73±0.58↑
Heh	J48	**78.92±1.51**	78.33±1.56	78.25±1.39↑	76.86±1.41↑	76.76±1.49↑	63.50±6.32↑
	PART	**79.02±1.47**	76.92±2.07↑	78.80±1.19↑	77.11±1.44↑	77.61±1.50↑	63.79±7.02↑
	kNN	**80.11±1.50**	76.63±1.33↑	76.21±1.75↑	74.69±1.30↑	74.82±1.24↑	62.38±5.72↑
	MLP	**80.50±1.30**	77.96±1.56↑	78.58±1.63↑	76.93±1.43↑	77.23±1.48↑	63.78±5.95↑
	GP	**81.55±1.13**	79.76±1.70↑	80.34±1.42↑	79.36±0.90↑	79.63±1.10↑	72.83±2.56↑
Hep	J48	**82.19±1.46**	78.55±2.05↑	80.01±2.25↑	79.80±1.76↑	80.98±1.58↑	81.52±1.52
	PART	**82.16±1.64**	79.32±2.75↑	82.11±1.85	80.75±1.83↑	80.69±1.92↑	82.04±1.91
	kNN	80.49±2.68	80.66±1.40	80.21±1.44	80.17±0.92	77.94±1.01↑	**80.82±2.31**
	MLP	83.01±1.88	81.61±1.77↑	82.56±1.24↑	80.16±1.23↑	78.85±2.18↑	**83.16±1.74**
	GP	**82.70±1.70**	80.43±2.81↑	80.45±1.90↑	80.54±1.40↑	78.45±1.33↑	82.59±2.00
Hou	J48	95.00±0.35	**96.31±0.52**↓	96.15±0.54↓	93.69±0.37↑	93.70±0.39↑	90.97±0.72↑
	PART	94.94±0.38	95.59±0.63↓	**95.72±0.79**↓	94.14±0.35↑	94.31±0.44↑	91.26±0.82↑
	kNN	**95.40±0.39**	92.33±0.55↑	93.01±0.34↑	90.83±0.41↑	91.43±0.42↑	91.43±0.77↑
	MLP	94.70±0.47	94.82±0.51	94.72±0.65	93.45±0.53↑	93.98±0.48↑	91.19±0.74↑
	GP	**95.14±0.59**	95.08±0.66	95.11±0.48	94.32±0.33↑	94.24±0.32↑	91.64±0.85↑
Mam	J48	**82.86±0.52**	81.92±0.54↑	82.24±0.65↑	82.57±0.46↑	82.33±0.49↑	79.68±1.53↑
	PART	**82.58±0.52**	81.57±0.52↑	81.71±0.49↑	82.18±0.64	82.13±0.55	79.74±0.79↑
	kNN	**80.31±0.60**	75.47±0.69↑	75.93±0.66↑	78.37±0.66↑	79.64±0.71↑	79.89±1.16
	MLP	**83.11±0.43**	82.72±0.63	82.97±0.49	82.99±0.36	82.95±0.45	79.73±1.05↑
	GP	**82.52±0.53**	79.72±1.12↑	79.99±0.68↑	82.25±0.53↑	82.46±0.69	77.04±2.15↑
Mar	J48	**33.90±0.30**	30.02±0.56↑	30.01±0.41↑	33.32±0.40↑	33.43±0.37↑	31.14±0.51↑
	PART	**33.53±0.34**	28.71±0.33↑	28.83±0.42↑	32.82±0.37↑	32.82±0.42↑	30.84±0.63↑
	kNN	**33.13±0.37**	27.30±0.42↑	27.55±0.39↑	29.34±0.38↑	29.71±0.38↑	30.77±0.76↑
	MLP	**34.35±0.21**	32.15±0.45↑	32.40±0.38↑	34.26±0.22	34.24±0.29	30.18±0.84↑
	GP	**31.45±0.20**	30.51±0.47↑	30.47±0.50↑	31.34±0.24	31.32±0.38	27.87±1.07↑
Ozo	J48	**97.10±0.04**	95.84±0.82↑	95.90±0.40↑	96.44±0.24↑	96.83±0.21↑	83.47±0.95↑
	PART	**97.10±0.02**	95.47±1.23↑	95.86±0.44↑	96.89±0.16↑	96.93±0.17↑	83.43±0.99↑
	kNN	**96.50±0.19**	95.07±0.62↑	95.15±0.32↑	95.35±0.31↑	96.28±0.26↑	83.17±0.74↑
	MLP	**96.35±1.04**	96.07±0.59↑	96.34±0.27	96.28±0.19	96.30±0.17	83.70±1.10↑
	GP	**97.09±0.05**	95.81±0.72↑	95.90±0.40↑	96.44±0.24↑	96.83±0.21↑	83.61±0.97↑
Tum	J48	41.16±2.24	41.08±2.71	41.24±2.05	**42.32±2.12**	38.38±2.59↑	31.26±3.18↑
	PART	**40.55±2.06**	40.17±1.78	40.49±1.76	40.35±1.99	37.46±2.41↑	31.40±3.50↑
	kNN	**39.12±1.96**	38.43±1.74↑	38.41±1.98↑	38.13±1.66↑	36.05±1.86↑	31.62±2.95↑
	MLP	38.85±2.01	39.38±1.62	39.79±2.29	**39.83±2.09**	36.40±1.96↑	32.97±2.20↑
	GP	**31.59±1.56**	30.86±2.41↑	30.83±1.90	31.55±1.88	30.39±1.52↑	27.61±1.49↑

5.1.3. Compare with Other Ensemble Methods

As also can be seen from Fig. 3 that the proposed method also can achieve significantly better accuracy than the benchmark ensemble methods in most cases. The proposed method is significantly more accurate than the benchmark ensemble methods in at least 75% cases, and it is only significantly less accurate than the *ensemble[15]* in 1 out of the 50 cases.

The proposed method is more accurate than the other benchmark ensemble methods because it uses a powerful imputation to provide complete data for the training process rather than working on incomplete training data as *Ensemble[14]* and *Ensemble[16]*. The second reason is that feature selection helps to further improve the training data of the proposed method. Moreover, by removing redundant and irrelevant features, feature selection helps to reduce the number of incomplete instances in the application process as shown in Fig.4. As a result, the proposed method can more frequently choose applicable classifiers to classify incomplete instances than the other ensemble methods.

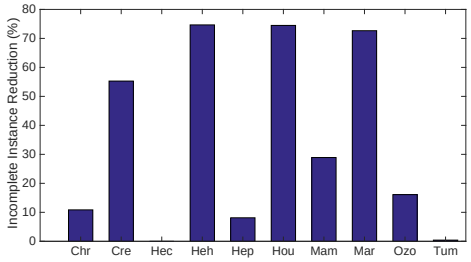


Figure 4: The percentage of incomplete instance reduction by using feature slection.

Table 6 shows the percentage of incomplete instances which can be classified by the ensemble methods. It is clear from Table 6 that the proposed method can classify all incomplete instances on 8 out of 10 datasets and classify almost all incomplete instances on the other 2 datasets. In contrast, the other ensemble methods cannot well classify incomplete instances. Especially, *Ensemble[15]* only can classify 85.85% and 48.85% incomplete instances on the *Heh* and *Ozo* datasets, respectively, because *Ensemble[15]* randomly generates missing patterns instead of finding missing patterns in the training data as the other ensemble methods.

Table 6: The percentage of incomplete instances are classified by ensemble methods.

Data	**NewMethod**	**Ensemble[14]**	**Ensemble[16]**	**Ensemble[15]**
Chr	100	92.53	92.53	99.68
Cre	100	98.95	98.95	100
Hec	100	97.13	97.13	93.59
Heh	100	99.32	99.32	85.85
Hep	98.66	97.02	97.02	95.69
Hou	98.04	96.79	96.79	97.88
Mam	100	99.25	99.25	100
Mar	100	98.95	98.95	100
Ozo	100	99.82	99.82	48.85
Tum	100	99.03	99.03	100

5.1.4. Further Comparison

As can be seen from Table 5 that the proposed method can obtain better accuracy than the benchmark methods not only on datasets with a small number of incomplete instances, but also on datasets with a large number of incomplete instances. For instance, the proposed method achieves the best accuracy on *Hec* dataset containing only 5.36% incomplete instances and also on *Heh* dataset with 99.65% incomplete instances.

Fig. 5 shows the fraction of cases that the proposed method is significantly better or worse than the other methods on each classifier. Fig. 5 shows that with any of the classifiers, the proposed method can significantly outperform the other methods in most cases. Moreover, *J48* and *GP* can get more benefits from the proposed method. The reason is likely that a filter feature selection often removes irrelevant and redundant features, but it may keep redundant features. *J48* and *GP* can perform feature selection while constructing classifiers [45, 46]. Therefore, by further removing redundant and irrelevant features, these classifiers can be more tolerant of missing values [12].

In summary, the proposed method can obtain significantly better accuracy than the benchmark methods in almost all cases when combining with any of the classification algorithms.

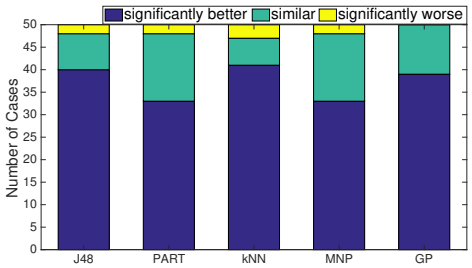


Figure 5: The comparison between the proposed method and all the benchmark methods on each of classification algorithms.

5.2. Computation Time

For most classification tasks, the training time has no constraint, but the computation time to classify an unseen instance should be feasible. Therefore, we focus on the computation time to classify unseen instances in the application process.

The experiments show that different classification algorithms have the same pattern of computation time. Hence, we only report the computation time of one classification algorithm: J48. Table 7 shows the computation time to classify instances in the application process.

Table 7: Time to classify instances in the application process (millisecond).

Data	**NewMethod**	**kNNI**	**MICE**	**Ensemble[14]**	**Ensemble[16]**	**Ensemble[15]**
Chr	1.2×10^{1}	3.1×10^{1}	9.6×10^{5}	6.1×10^{1}	2.3×10^{1}	1.5×10^{2}
Cre	2.0	8.0	5.1×10^{4}	5.0	3.0	8.0
Hec	1.0	6.0	2.7×10^{3}	1.0	1.0	6.0
Heh	1.0	1.9×10^{1}	2.0×10^{5}	2.0	1.0	2.0
Hep	1.0	8.0	8.1×10^{4}	3.0	1.0	2.0
Hou	4.0	3.1×10^{1}	4.6×10^{5}	2.7×10^{1}	1.3×10^{1}	1.7×10^{1}
Mam	3.0	6.4×10^{1}	1.8×10^{5}	7.0	5.0	8.0
Mar	7.9×10^{3}	1.3×10^{4}	7.1×10^{8}	8.8×10^{4}	6.7×10^{4}	4.7×10^{4}
Ozo	1.2×10^{4}	2.7×10^{4}	1.2×10^{9}	1.3×10^{5}	2.9×10^{4}	1.7×10^{5}
Tum	2.1×10^{1}	5.3×10^{1}	8.2×10^{5}	3.1×10^{1}	4.0	6.2×10^{1}

5.2.1. Compare with Imputation Methods

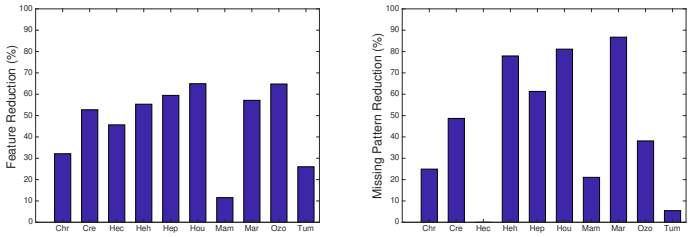
It is clear from Table 7 that the proposed method is considerably more efficient than the methods using imputation (*kNNI* and *MICE*). The proposed method is thousand times faster than *MICE* because it does not take any time

to estimate missing values in the application process. In contrast, *MICE* takes a long time to estimate missing values in the application process because *MICE* needs to rebuild all regression functions when it estimates missing values for each unseen incomplete instance. The proposed method is also remarkably more efficient than *kNNI* because *kNNI* also takes time to estimate missing values for unseen incomplete instances. Especially with big datasets such as the *Mar* and *Ozo* datasets, the proposed method is much more efficient than both *MICE* and *kNNI* because the two methods take a long time to estimate missing values in datasets with numerous instances and features.

5.2.2. Compare with Other Ensemble Methods

As can be seen from Table 7 that the proposed method is also more efficient than the benchmark ensemble methods. The first reason is that the proposed method uses feature selection to remove redundant and irrelevant features before building classifiers, so it can generate simpler classifiers than the other ensemble methods. As demonstrated in Fig.6a that feature selection can remove over half of the features in the majority of datasets. Moreover, by removing redundant and irrelevant features, the proposed method also reduces the number of missing patterns; therefore, it only needs to build a small number of classifiers. As is evident from Fig. 6b that the proposed method can reduce over 50% missing patterns in many datasets. In other words, the proposed method only needs to build half of the number of classifiers compared to other ensemble methods such as the ensemble method in [14]. With a smaller number of classifiers, the proposed method can classify instances quicker than the other methods. Finally, by using feature selection, the proposed method can reduce the number of incomplete instances in the application process. Therefore, the proposed method can save time to search for applicable classifiers for incomplete instances. As can be seen from Fig. 4 that the proposed method can significantly reduce the number of incomplete instances. For example, it can reduce over 70% incomplete instances in the *Heh*, *Hou* and *Mar* datasets.

In summary, the proposed method can not only be more effective, but also



(a) Feature reduction (%) (b) Missing pattern reduction (%)

Figure 6: Feature reduction and missing pattern reduction by using feature slection.

more efficient than the other benchmark methods.

5.3. Further Analysis

This section discusses further analysis to deeply understand the effectiveness and efficiency of the proposed method.

5.3.1. Evaluation of the Proposed Method on Different Imputation Methods

One of the important component in the proposed method is imputation. In order to know the impact of imputation on the proposed method, experiments were designed to compare the proposed method on three multiple imputation methods in MICE (using random forest regression (rf), bayesian linear regression (norm) and linear regression (nob)) and kNN-based imputation (kNNI). Table 8 shows the classification accuracy and the training time of the proposed methods on different imputation methods.

It is clear from Table 8 that the proposed method with multiple imputation is generally more accurate than the proposed method with single imputation. Moreover, the proposed method with multiple imputation using non-linear regression such as random forest is usually more accurate than the proposed method with multiple imputation using linear regression. However, the training time of the proposed method using multiple imputation is much more expensive than using single imputation.

5.3.2. Evaluation of the Proposed Method on Gene Expression Datasets

Gene expression datasets usually contain a large number of features. Moreover, gene expression datasets often contain a large number of missing values

Table 8: Classification accuracy and training time of the proposed method by using different imputation methods.

Data	Classification accuracy				Training time			
	rf	**norm**	**nob**	**kNNI**	**rf**	**norm**	**nob**	**kNN**
Chr	99.10	99.09	**99.12**	99.04	9.1×10^4	8.2×10^3	4.6×10^1	2.8×10^1
Cre	85.34	85.34	**85.45**	85.16	4.2×10^4	3.2×10^3	5.2×10^1	1.2×10^1
Hec	**58.06**	57.82	57.21	57.10	7.7×10^3	6.9×10^2	2.0×10^1	7.6
Heh	**78.92**	78.56	78.43	78.89	7.3×10^4	6.1×10^3	3.4×10^1	1.6×10^1
Hep	**82.19**	81.96	80.88	79.32	5.8×10^4	6.1×10^3	2.1×10^1	9.1
Hou	95.00	94.94	94.66	**95.32**	8.1×10^4	7.3×10^3	3.6×10^1	1.5×10^1
Mam	**82.86**	82.73	82.61	82.05	2.5×10^4	1.7×10^3	4.8×10^1	1.2×10^1
Mar	**33.90**	33.86	32.89	32.21	4.5×10^4	1.7×10^4	1.2×10^3	4.2×10^2
Ozo	**97.10**	**97.10**	97.07	96.45	3.1×10^6	5.2×10^5	7.9×10^2	2.9×10^2
Tum	41.16	**41.27**	40.67	40.34	3.4×10^4	1.8×10^3	5.2×10^1	2.2×10^1

[34]. Therefore, we evaluate the proposed methods on gene expression datasets to further validate the effectiveness and efficiency of the proposed methods. Table 9 shows eight gene expression datasets which were chosen to evaluate the proposed methods.

Table 9: Gene expression datasets used in the experiments.

Name	**#Samples**	**#Genes**	**#Classes**	**Incomplete samples (%)**	**Incomplete genes (%)**
alizadeh-2000-v1	42	1095	2	100	65.66
alizadeh-2000-v2	62	2093	3	100	80.21
bredel-2005	50	1739	3	100	33.12
chen-2002	180	85	2	59.77	81.17
garber-2001	66	4553	4	100	44.47
liang-2005	37	1411	3	100	22.39
tomlins-2006	104	2315	5	100	87.65
tomlins-2006-v2	92	1288	4	100	88.76

Table 10 shows the classification accuracy of the proposed method and the other methods on the gene expression datasets, respectively. It is clear from the Table 10 the proposed method using kNN-based imputation in the training

processes is more accurate than using kNN-based imputation both in the training and application processes. Moreover, the proposed method is much more accurate than existing ensemble methods. For example, in *tomlins* datasets, the accuracy of the proposed method is double that of the ensemble methods in [14, 16].

Table 10: Classification accuracy (using *J48* as a classifier) of the proposed method (using kNN-based imputation) and the other benchmark methods on the gene expression datasets.

Dataset	**NewMethod**	**kNNI**	**Ensemble[[14]]**	**Ensemble[[16]]**	**Ensemble[[15]]**
alizadeh-2000-v1	74.41±6.41	66.81±8.39↑	50.23±5.23↑	51.42±5.54↑	64.83±5.21↑
alizadeh-2000-v2	83.18±5.17	81.24±4.96	67.81±4.24↑	65.87±4.52↑	62.12±3.21↑
bredel-2005	69.43±7.21	66.36±7.48↑	62.01±4.26↑	63.27±4.03↑	59.35±3.69↑
chen-2002	87.72±2.75	82.28±2.93↑	78.86±4.62↑	79.06±4.64↑	88.72±2.45
garber-2001	71.55±4.93	63.62±4.87↑	52.29±4.93↑	53.62±4.78↑	51.17±4.38↑
liang-2005	79.68±6.08	78.42±5.09	75.73±2.84↑	74.65±2.76↑	75.60±3.25
tomlins-2006	65.42±6.03	52.61±5.12↑	30.78±1.42↑	30.78±1.42↑	60.78±3.33↑
tomlins-2006-v2	63.14±3.27	53.78±4.81↑	34.79±1.42↑	35.64±1.38↑	58.72±3.61+

Table 11 shows the computation time to classify new instances in the application process of the proposed method and the benchmark methods. It is clear from Table 11 that although the proposed method is slightly more expensive than kNN-based imputation, but it is much faster than the ensemble methods.

Table 11: Time to classify instances in the application process on the gene expression datasets (millisecond).

Dataset	**NewMethod**	**kNNI**	**Ensemble[[14]]**	**Ensemble[[16]]**	**Ensemble[[15]]**
alizadeh-2000-v1	4.4×10^1	9.3	1.4×10^3	1.3×10^3	1.4×10^4
alizadeh-2000-v2	9.9×10^3	2.9×10^1	1.1×10^4	1.0×10^4	1.2×10^5
bredel-2005	2.1×10^2	3.6×10^1	2.3×10^3	2.1×10^3	1.3×10^5
chen-2002	5.4	6.8	1.1×10^2	9.1×10^1	3.4×10^2
garber-2001	2.4×10^3	8.1×10^1	2.9×10^4	2.7×10^4	8.5×10^5
liang-2005	4.9×10^1	9.9	6.1×10^2	5.8×10^2	8.0×10^3
tomlins-2006	7.4×10^3	8.9	6.3×10^4	6.1×10^4	5.5×10^5
tomlins-2006-v2	8.3×10^2	3.7	1.4×10^4	1.2×10^4	1.2×10^5

In summary, the proposed methods are still able to produce dramatic improvement in efficiency and better accuracy on large datasets.

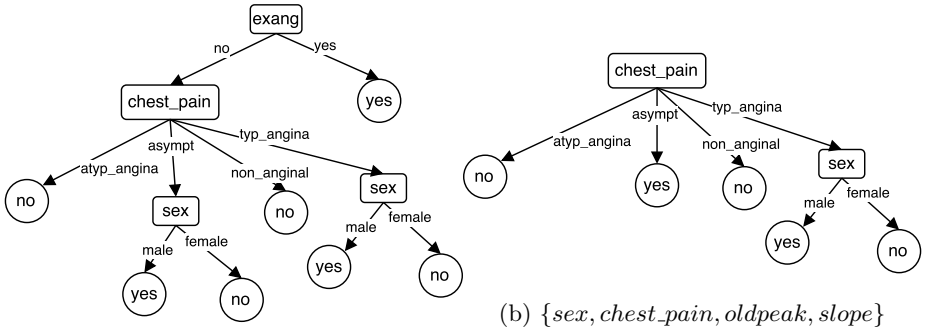
5.3.3. Evaluation of the Proposed Method on Specific Problem

In order to demonstrate how the proposed method works and its effectivenesses and efficiencies, we analysed carefully the proposed method on *Heart-h* using *C4.5*. The *Heart-h* dataset was chosen because it has the largest percentage of incomplete instances (100%) compared to the other datasets. *C4.5* was chosen because decision trees generated by *C4.5* are straightforward to interpret.

Hear-h describes the contents of the heart-disease collected by Hungarian Institute of Cardiology [47]. The dataset has 13 features: {*age, sex, chest_pain, trestbps, chol, fbs, restecg, thalach, exang, oldpeak, slope, ca, thal*}. The values of the 13 features are used to decide the diagnosis of heart disease shown in a class feature, where 0 indicates less than 50% diameter narrowing and 1 indicates more than 50% diameter narrowing. The original dataset has six incomplete features: {*chol, fbs, exang, slope, ca, thal* } and contains 15 missing patterns.

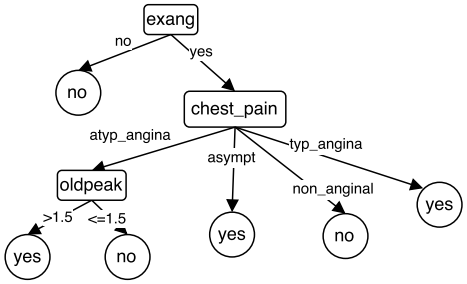
In the application process, *MICE* imputation is firstly used to transform the incomplete dataset into an imputed dataset. *CFS* is then applied on the imputed dataset, and it selects a subset of five features $\mathcal{SF}$ = {*sex, chest_pain, exang, oldpeak, slope*} and removes the other eight features. As a result, the original dataset reduced on $\mathcal{SF}$ has only two incomplete features {*exang, slope*} and it contains only three missing patterns: {*slope*}, {*exang*} and {$\emptyset$}. Therefore, feature selection helps to reduce the number of incomplete features (from 6 to 2) and reduce the number of missing patterns (from 15 to 3).

From the three missing patterns, the proposed method generates three complete patterns: {*sex, chest_pain, exang, oldpeak*}, {*sex, chest_pain, oldpeak, slope*} and {*sex, chest_pain, exang, oldpeak, slope*}. Figs. 7a, 7b and 7c show three decision trees generated by *C4.5* according to the three complete patterns. It is clear from the figures that the decision trees do not require all features, so they are tolerant with missing values because they can be applicable to more



(a) {*sex, chest_pain, exang, oldpeak*}

(b) $\{sex, chest\_pain, oldpeak, slope\}$



(c) $\{sex, chest\_pain, exang, oldpeak, slope\}$.

Figure 7: Decision trees constructed by using different feature subsets.

than one missing pattern. For example, the decision tree in Fig. 7b is build on a dataset with four features (*sex, chest_pain, oldpeak, slope*), but it requires only two features (*sex, chest_pain*). As a result, the decision tree in Fig. 7b is originally designed to classify incomplete instances with only one missing value in feature *exang*; however, it does not require feature *slope*, so it also can be used to classify any incomplete instance with missing value in feature *slope*.

Table 12 shows some incomplete instances in the *Hear-h* dataset, which need to be classified. Table 13 presents instances in Table 12 reduced on the selected features $\mathcal{SF} = \{$*sex, chest_pain, exang, oldpeak, slope*$\}$. As can be seen from Tables 12 and 13 that feature selection can help to reduce the number of missing values and reduce the number of incomplete instances. For example, the second and third instances in Table 12 are incomplete, but by only keeping the selected features, these instances become complete as shown in Table 13.

In Table 13, the second and third instances are complete so they are quickly

Table 12: Incomplete instances in the original *Hear-h* dataset.

N	age	sex	chest_pain	trestbps	chol	fbs	restecg	thalach	exang	oldpeak	slope	ca	thal
1	59	male	asympt	140	?	f	normal	140	no	0	?	0	?
2	46	male	asympt	120	277	f	normal	125	yes	1	flat	?	?
3	54	male	asympt	150	365	f	st_t_wave_abnormality	134	no	1	up	?	?
4	48	male	atyp_angina	100	?	f	normal	100	no	0	?	?	?
5	54	female	atyp_angina	140	309	?	st_t_wave_abnormality	140	no	0	?	?	?
6	48	female	atyp_angina	?	308	f	st_t_wave_abnormality	?	?	2	up	?	?

Table 13: Instances in Table 12 reduced on the selected features.

N	age	chest_pain	exang	oldpeak	slope
1	59	asympt	no	0	?
2	46	asympt	yes	1	flat
3	54	asympt	no	1	up
4	48	atyp_angina	no	0	?
5	54	atyp_angina	no	0	?
6	48	atyp_angina	?	2	up

classified by all the decision trees without requiring time for exploring applicable classifiers. The first, fourth and fifth instances contain a missing value in feature *slope*. Although only the decision tree in Fig. 7a is learned to classify these incomplete instances, the other decision trees in Fig. 7b and 7c also can be used to classify the incomplete instances because they do not require feature *slope* thanks to implicitly performing feature selection of *C4.5*.

In summary, three powerful techniques—multiple imputation, feature selection and ensemble learning—make the proposed method effective and efficient.

6. Conclusions

This paper proposed an effective and efficient approach for classification with incomplete data by integrating imputation, genetic-based feature selection and ensemble learning. The proposed method uses imputation only in the training process to transform incomplete training data into complete training data that is then further improved using feature selection to remove redundant and irrelevant features. Then the proposed method constructs an ensemble of classifiers which can classify new incomplete instances without the need of imputation.

The experiments were designed to compare the classification accuracy and the computation time of the proposed method with five benchmark methods of the two common approaches to classification with incomplete data: using imputation in both the training and application processes and using ensemble for classification with incomplete data. The results and analysis show that the proposed method can achieve better classification accuracy in most cases, and can be much faster than the other methods in almost all cases.

Missing values are also common issues in many regression problems [48]. However, there have not been much work on handling missing data in regression, much less than in classification. In future work, we would like to investigate how the ideas of imputation, feature selection and ensemble can be used for regression with incomplete data.

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