

Advanced Method for Detecting multi-component LFM Signals in a Complex Interference Environment

1st Van Minh Duong

Falcuty of Radio-Electronic Engineering
Le Quy Don Technical University
 Hanoi, Vietnam
 minhktqs2008@gmail.com

2nd Thi Phuong Nguyen

Falcuty of Radio-Electronic Engineering
Le Quy Don Technical University
 Hanoi, Vietnam
 phuongnt@lqdtu.edu.vn

3rd Nhat Giang Phan

Falcuty of Radio-Electronic Engineering
Le Quy Don Technical University
 Hanoi, Vietnam
 pngiang20000@gmail.com

4nd Van Hai Nguyen

Falcuty of Radio-Electronic Engineering
Le Quy Don Technical University
 Hanoi, Vietnam
 nvhai77@gmail.com

5nd Trung Thanh Nguyen

Falcuty of Radio-Electronic Engineering
Le Quy Don Technical University
 Hanoi, Vietnam
 thanhmta@gmail.com

Abstract—This paper describes the challenges of detection and parameter estimation priori unknown multi-component radar signals with linear frequency modulation (LFM) in intense noise and complex jamming conditions. A method including two stages is proposed: The first stage is to detect signals, or in other words, estimate the chirp rate, and second is to estimate the pulse width of signals. The proposed approach is firstly examined by identifying the LFM signals in the environment of intense noise and mixing that noise and interference with the continuous wave (CW) signal on MATLAB. An experiment with real-time LFM signals confirms that the proposed method is able to detect and estimate the parameters of multi-component LFM signals in intense noise and in combining CW signal and noise with a signal-to-noise ratio (SNR) $\geq -18\text{dB}$ and $\text{SNR} \geq -12\text{dB}$, respectively.

Index Terms—Linear frequency modulation, cross-correlation function, probability of detection, probability of correct estimation.

I. INTRODUCTION

Low probability of intercept (LPI) radars take advantages of low power, wide frequency bandwidth, and high-frequency variability that make it difficult to be detected by passive surveillance systems (PSS) [1], [2]. Using linear frequency modulation (LFM) has been a typical technique, resulting in spreading the signal over a wide frequency range in a manner that is initially unknown to the PSS [3], [4].

In recent years, the problem of detecting and estimating multi-component LFM signals has attracted much more attention. Many techniques have been proposed to estimate these signals' chirp rate μ and pulse width τ . In [5], a method based on the Hough and Chirplet transform requires obtaining a perfect probability of detection (Pd). Next, a technique using simplified linear canonical transform (SLCT) [6] was proposed, but it only gets the high Pd with $\text{SNR} \geq -4\text{dB}$. Another one based on the extended forms of standard

cubic phase function (CPF) [7] can reach $\text{SNR} \geq -13\text{dB}$ with mono-component LFM detection but with no consideration for multi-component LFM. The latest techniques based on deep learning (DL) and artificial intelligence (AI), e.g., convolution neural network (CNN), deep convolution neural network (DCNN), and deep neural network (CLDN), have been developed [8], [9]. They can process multiple radar signals like LFM and M-PSK with $\text{SNR} \geq -8\text{dB}$. However, the accuracy here depends considerably on the size of the training and testing signal sets. A signals database is required, meaning signals' parameters must be known. More importantly, they are not able to recognize multiple crossed LFM signals. Finally, a method closest to ours using an auto-correlation receiver [10] can estimate mono-component LFM with $\text{SNR} \geq -7\text{dB}$. The approaches mentioned above can effectively identify

TABLE I: Performance Comparison of Methods

Method	Noise Type	SNR[dB]	Signal Type
Hough Chirplet transform [5]	White noise	3	Mono-, multi-component LFM
SLCT [6]	White noise	-4	Mono-, multi-component LFM
CPF [7]	White noise	-13	Mono-component LFM
CNN [8]	White noise	-6	LFM, BPSK, FM, AM, SSB
DCNN [9]	White noise	-8	LFM, BPSK, FM, AM, SSB
Auto-correlation [10]	White noise	-7	LFM, BPSK
Proposed Method	White noise, CW signal	-14	Mono-, multi-component LFM

LFM signals in white noise, but they have problems with lower SNR or LFM signals in high-interference environments.

Also, most of them did not consider multi-component LFM signals. To overcome those weaknesses, a technique based on the auto-correlation function for estimating multi-component LFM signals in high-interference environments is developed in this paper. A performance comparison is indicated in Table I.

A theoretical description of the method for detecting and extracting multi-component LFM signals is described in section 2. Section 3 presents a simulation on MATLAB under different conditions: i) white noise, ii) a CW signal, and white noise to determine SNR thresholds at which LFM signals can still be identified. After that, a verification with real-time LFM signals is performed in section 4. Finally, section 5 will draw the main conclusion based on the simulation results.

II. THE PROPOSED METHOD

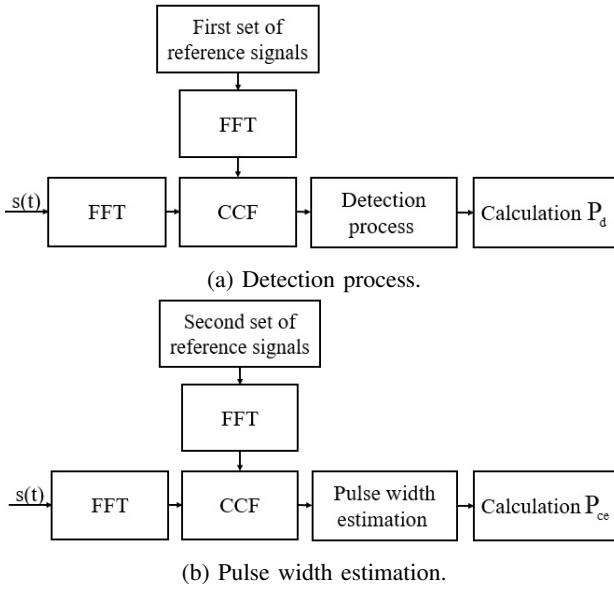


Fig. 1: Block diagram of the proposed technique.

In radar signal processing [11], [12], the cross-correlation function (CCF) helps measure the similarity of two signals as a function of time for one signal relative to the other. Thus, this technique is applied to pattern recognition, single particle analysis, and time series. The CCF between two signals, $x(t)$ and $h(t)$, is defined by the following equation:

$$R(\nu) = \int_{-\infty}^{\infty} x^*(t)h(t + \nu) d\nu \quad (1)$$

where $x^*(t)$ is the complex conjugate of the signal $x(t)$, $h(t+\nu)$ is the second signal, and ν is the time delay. In addition, the CCF could be expressed by FFT as (2).

$$R(\nu) = FFT^{-1}\{X^*(\omega) \times H(\omega)\} \quad (2)$$

where $X(\omega)$ and $H(\omega)$ are the spectra of signals $x(t)$ and $h(t)$, respectively, $X^*(\omega)$ is the complex conjugate of $X(\omega)$, and FFT^{-1} is the inverse of FFT. A new technique, including two stages, is proposed based on CCF. The first is to estimate the chirp rate μ of two LFM signals, and the second is to calculate

their pulse width τ . Note that, in signal reconnaissance, the chirp rate is the most critical parameter that needs to be identified first. The chirp rate is estimated means the signal is detected. Also, when it is known, we can determine the type of modulation and operating frequency range of the signal (of radar). The block diagram of the proposed technique is shown in Fig. 1. Two sets of reference LFM signals are used: The first set is generated with the same pulse width and varying chirp rates μ_{ref} . On the other hand, the second set is the same

TABLE II: The Parameters of All Signals

Signal	Parameter	Value
1 st LFM signal	$\mu_1 (GHz.s^{-1})$	100
	$\tau_1 (\mu s)$	10
2 nd LFM signal	$\mu_2 (GHz.s^{-1})$	200
	$\tau_2 (\mu s)$	15
1 st set of reference LFM signals	$\mu_{ref1} (GHz.s^{-1})$	10 ÷ 250
	$\tau_{ref1} (\mu s)$	20
2 nd set of reference LFM signals	$\mu_{ref2} (GHz.s^{-1})$	100, 200
	$\tau_{ref2} (\mu s)$	1 ÷ 21
CW signal	$f_c (MHz)$	1.6

chirp rate, estimated in the first stage, but with different pulse widths τ_{ref} . Details of the process are listed below:

• Input:

- μ_{ref} and τ_{ref} : parameter vectors of the reference LFM signals,
- ρ_1 and ρ_2 ([13], [14]): thresholds for estimating μ and τ of received LFM signals.

• Output:

- μ and τ of received LFM signals.

* Estimate the chirp rate μ :

- **Step 1:** Generate the first reference LFM signal set $s_{ref1}(t)$ at the same pulse width and with varying chirp rates μ_{ref} .
- **Step 2:** Calculate the spectrum of the received signal $S(\omega)$ and reference signals $S_{ref1}(\omega)$ using FFT.
- **Step 3:** Calculate $R_1(\nu)$ as a function of $\mu_{ref}(f(\mu))$ between the received signal and the first reference signal set using (2) and find out the maximum of $R_1(\nu)$.
- **Step 4:** Estimate μ . If $f(\mu) \geq \rho_1$ and $\mu \in [0.1 \div 0.9]\mu_{ref}$, μ is estimated or detects the LFM signal successfully.

* Estimate the pulse width τ :

- **Step 5:** Generate the second reference LFM signal set $s_{ref2}(t)$ at the estimated chirp rate μ and with varying pulse width τ_{ref} .
- **Step 6:** Calculate the spectrum of the received signal $S(\omega)$ and reference signals $S_{ref2}(\omega)$ using FFT.
- **Step 7:** Calculate $R_2(\nu)$ as a function of $\tau_{ref}(f(\tau))$ between the received signal and the second reference signal set using (2), finding out the maximum of $R_2(\nu)$.
- **Step 8:** Estimate τ . If $f(\tau) \geq \rho_2$ and $\tau \in [0.1 \div 0.9]\tau_{ref}$, μ is estimated.

The parameters of two simulated LFM signals and two reference sets are described in Table II.

III. SIMULATION RESULTS

This section investigates the technique with complex cases in which two LFM signals are crossing in both time and frequency domains. Firstly, the method is studied by analyzing two crossed LFM signals presenting intense white noise (Fig. 3). Secondly, a test of these signals in the interference environment of a CW signal and white noise is performed (Fig. 4). The method's efficiency is evaluated by determining the lowest value of SNR, at which the technique can still achieve 90% of the probability of correct estimation. All tests are done in MATLAB by running the system on 300 loops in the range of SNR from -22dB to -5dB. The test condition is that the parameters of the multi-component LFM signals are in the observed parameter range of the reference signals ($\mu \in \mu_{ref}, \tau \in \tau_{ref}$).

A. Test with Two Crossed LFM Signals in White Noise

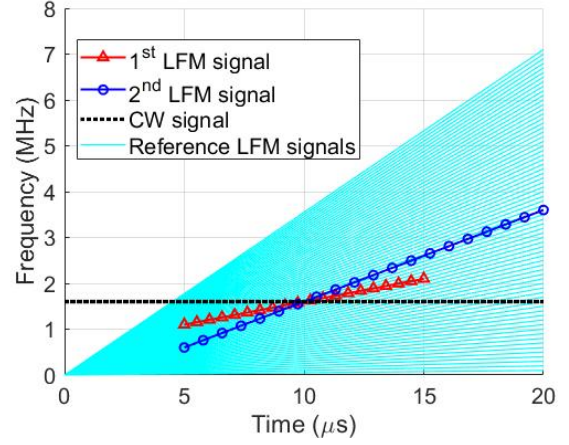
The time-frequency characteristics of the received signals (red line for the first LFM signal, blue line for the second LFM signal) and the reference signals (light blue line) and their spectrum are shown in Fig. 2(a) and (b), respectively. The classic spectrum analyzer based on FFT can not detect LFM signals.

Fig. 3(a) shows the probability of detection (Pd) of LFM signals in white noise as a function of SNR. It shows that the performance of this technique depends directly on the chirp rates of the LFM signals. For the exact value of SNR = -18dB, the highest accuracy was obtained for the second LFM signal (Pd = 94.25%, its chirp rate is higher, blue line), followed by the first LFM signal Pd = 90.18% (red line). Briefly, the required SNR for detecting both LFM signals in the white noise is -18dB with Pd \geq 90%.

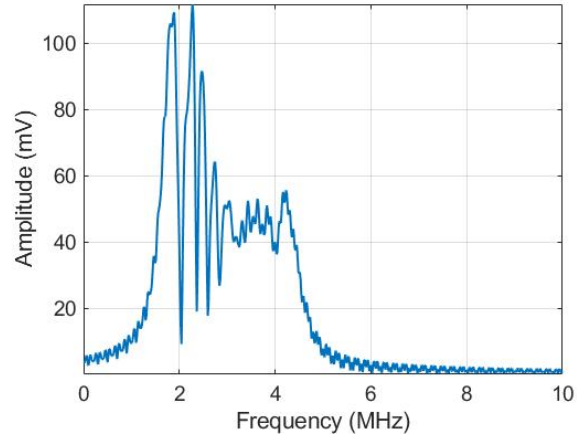
Using the same process as the detection, the probability of correct pulse width estimation (Pce) as a function of SNR is drawn in Fig. 3(b). The proposed method gives the best result for the second LFM signal (Pce = 96.27%, blue line), followed by the first LFM signal (Pce = 91.48%, red line) at SNR = -15dB. Also, it shows that the lowest value of SNR for estimating the pulse width of both LFM signals is SNR \geq -15dB with Pce \geq 90%.

B. Test with Two Crossed LFM Signals in Jamming of a CW Signal and White Noise

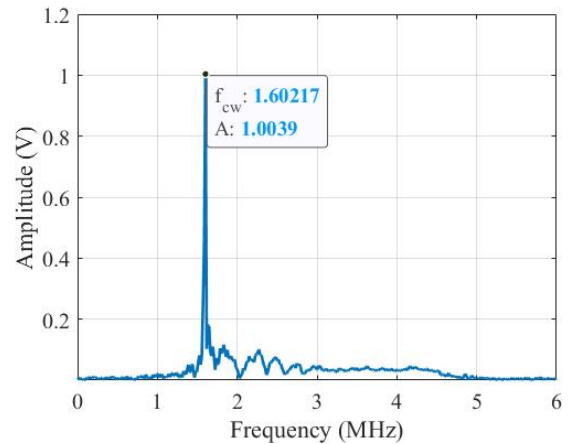
The efficiency of the proposed technique is analyzed in the same setup as the above, using two crossed LFM signals in the jamming of a CW signal and white noise. The carrier frequency of the CW signal is equal to the carrier frequency of LFM signals. The time-frequency characteristics of all signals and their spectrum are shown in Fig. 2(a) and (c). The classic spectrum analyzer based on FFT can only detect the CW signal but not the LFM signals. The simulation results are shown in Fig. 4. In detection (Fig. 4(a)), the proposed technique gets the best Pd = 98.56% for the second LFM signal (blue line) while Pd = 90.18% for the first LFM signal (red line) at SNR = -14dB. Overall, the SNR required for detecting all LFM is SNR = -14dB.



(a) Time-frequency characteristics.



(b) Spectrum of LFM signals.



(c) Spectrum of LFM and CW signals.

Fig. 2: Signals without noise.

Similar to the detection process, P_{ce} is drawn in Fig. 3(b). It is clear that for the same value of SNR = -12dB, the highest accuracy ($P_{ce} = 97.35\%$) is for the second LFM signal (blue line) and followed by $P_{ce} = 93.01\%$ by the first LFM signal (blue line). In conclusion, $SNR \geq -12\text{dB}$ is required at the pulse widths of two LFM signals to obtain a perfect probability of correct estimation ($P_{ce} \geq 93\%$).

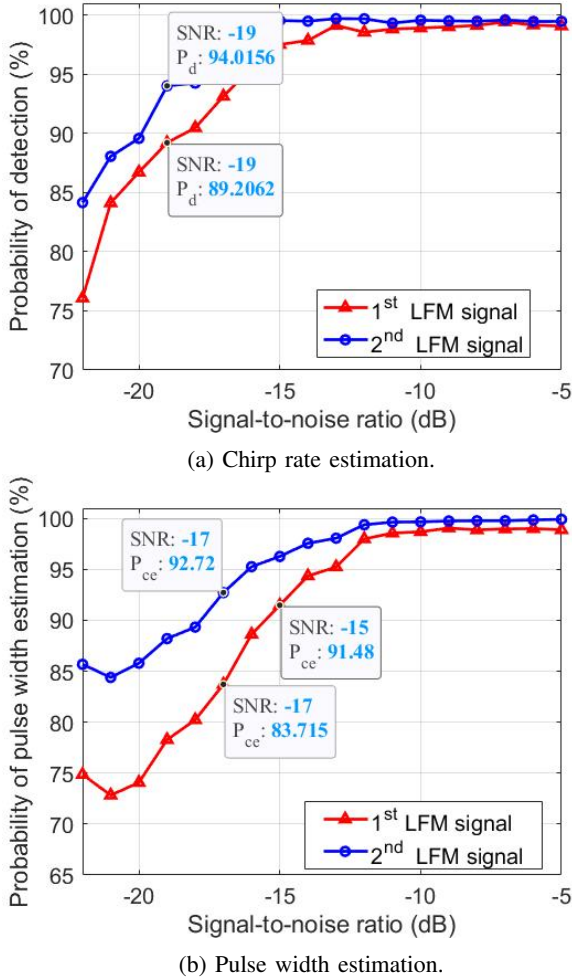
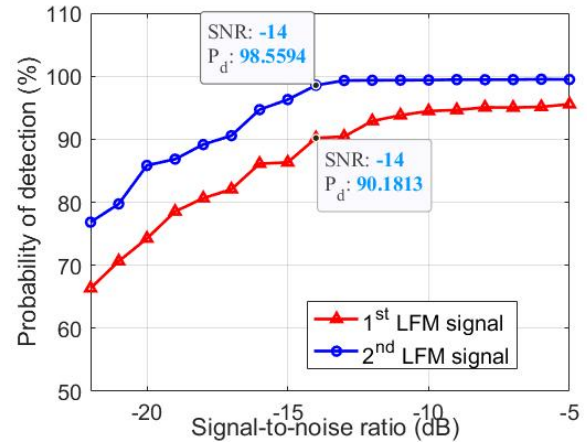


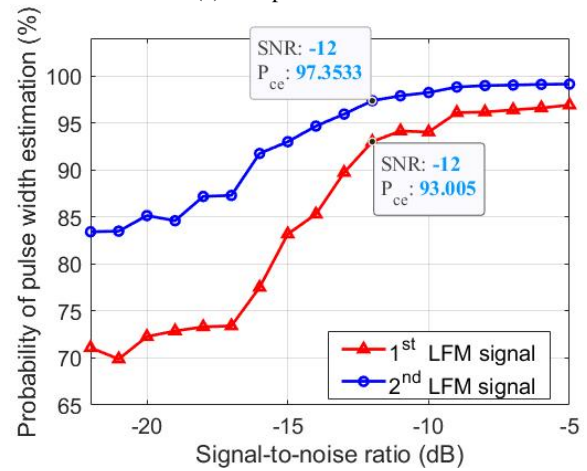
Fig. 3: Estimation probabilities in white noise environments.

IV. EXPERIMENTAL RESULTS

In this section, firstly, the technique is verified using two crossed LFM signals generated in white noise for SNR = -15dB. Next, a verification using two crossed LFM signals generated in the mixture of the white noise and CW signal with SNR = -12dB is performed. A measurement is set up to evaluate the performance of detecting and estimating the parameters of multi-component LFM signals (Fig. 5). A real-time data generator PSG E8267C, which operates in the frequency range from 250 kHz to 20 GHz, produces a CW signal and LFM signals; an oscilloscope RTO 1044 and a spectrum analyzer RIGOL DSA 814 are used to verify LFM signals generated from the MATLAB environment.



(a) Chirp rate estimation.



(b) Pulse width estimation.

Fig. 4: Estimation probabilities in white noise and CW.

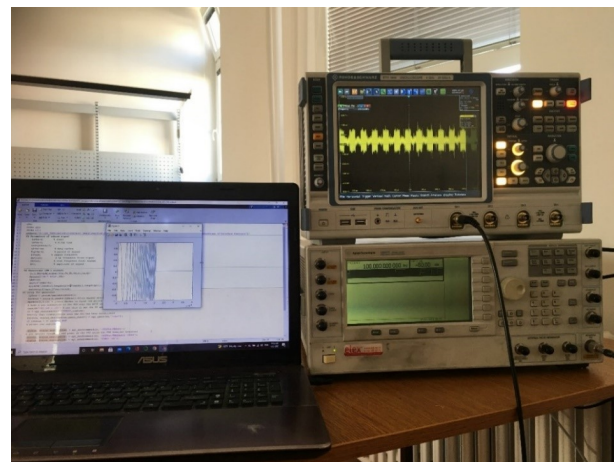


Fig. 5: Experimental Setup.

A. Detecting and Estimating Two Crossed LFM Signals in Strong White Noise

In this part, two crossed LFM signals with SNR = -15dB in white noise are used to verify the method. Their spectrum is shown in Fig. 6. Again, the spectrum analyzer is not able to detect LFM signals. The experimental results are shown

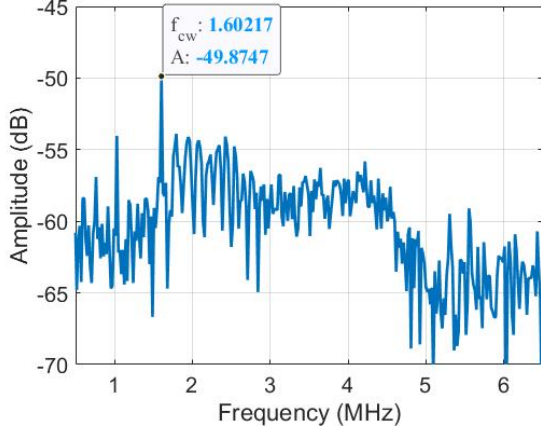


Fig. 6: Spectrum of LFM signals in white noise.

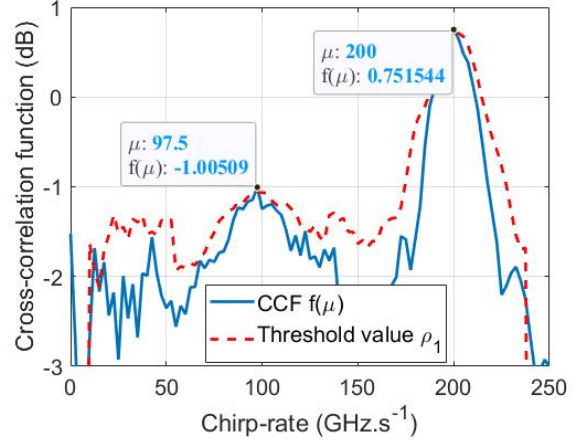
in Fig. 7(a). This figure shows that two LFM signals can be detected. Their chirp rates are $\mu_1 = 97.5 \text{ GHz}\cdot\text{s}^{-1}$ with $f(\mu) = -1.01\text{dB}$ and $\mu_2 = 200 \text{ GHz}\cdot\text{s}^{-1}$ with $f(\mu) = 0.75\text{dB}$. The same process is applied to the chirp rate estimation, and the results are shown in Fig. 7(b). It shows that two pulse widths are estimated ($\tau_1 = 9.8 \mu\text{s}$ with $f(\tau) = -0.04\text{dB}$ and $\tau_2 = 15.6 \mu\text{s}$ with $f(\tau) = 1.12\text{dB}$). The estimated parameters of the two real-time LFM signals are listed in Table III. This table shows that the technique was verified with two crossed LFM signals in strong white noise with SNR = -15dB.

TABLE III: Estimated Parameters of Two Crossed LFM Signals in White Noise

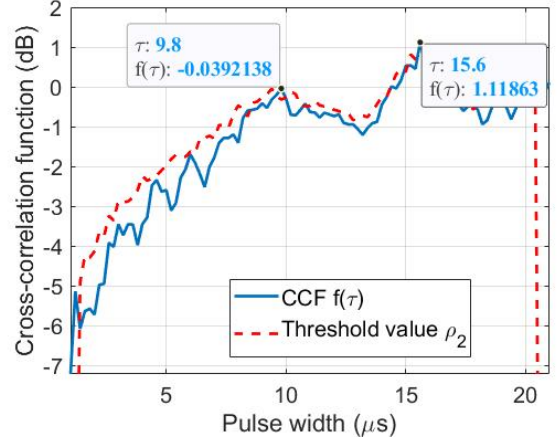
Parameter	Simulation	Estimation	Relative Error (%)
$\mu_1 (\text{GHz}\cdot\text{s}^{-1})$	100	97.5	2.5
$\tau_1 (\mu\text{s})$	10	9.8	2.0
$\mu_2 (\text{GHz}\cdot\text{s}^{-1})$	200	200	0.0
$\tau_2 (\mu\text{s})$	15	15.6	4.0

B. Detecting and Estimating Two Crossed LFM Signals in an Interference Environment

In this section, the proposed technique is verified using two crossed LFM signals in the jamming of a CW signal and white noise with SNR = -12dB. Their spectrum is shown in Fig. 8. Fig. 9(a) shows the results of the first stage of the proposed method. It clearly shows that the first estimated chirp rate is $\mu_1 = 95 \text{ GHz}\cdot\text{s}^{-1}$ with $f(\mu) = 0.43\text{dB}$ and $\mu_2 = 200 \text{ GHz}\cdot\text{s}^{-1}$ with $f(\mu) = 1.90\text{dB}$. It means that all LFM signals were detected. The same process is applied to the second stage; the result is shown in Fig. 9(b). It is clear that the pulse widths of the LFM signals at SNR = -12dB are $\tau_1 = 10.2 \mu\text{s}$ with $f(\tau) = -1.78\text{dB}$



(a) Chirp rate estimation.



(b) Pulse width estimation.

Fig. 7: Estimation probabilities in white noise.

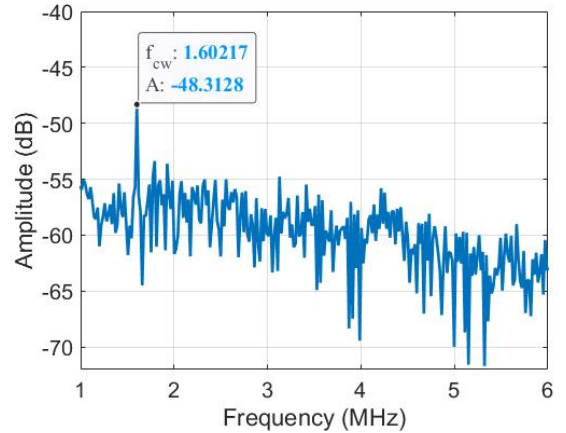
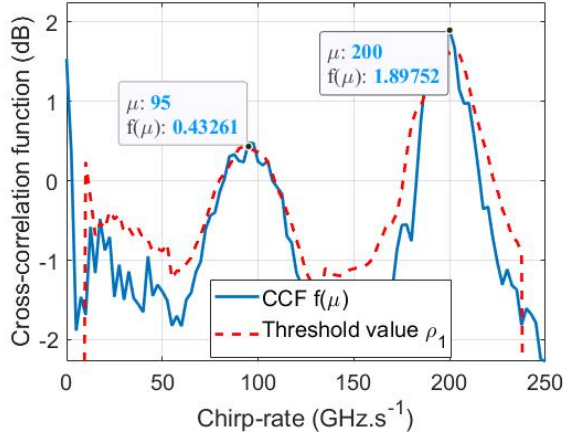
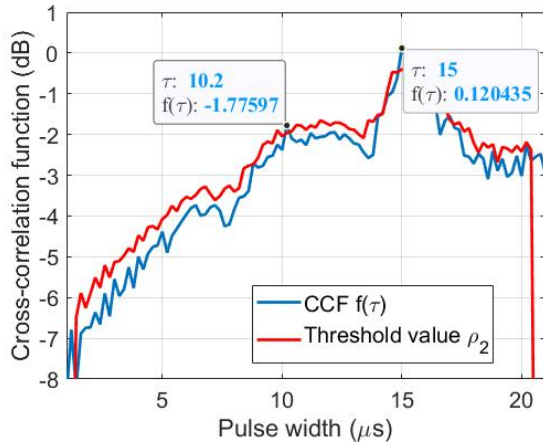


Fig. 8: Spectrum of LFM signals in white noise and CW.

and $\tau_2 = 15 \mu s$ with $f(\tau) = 0.12dB$. The real-time signals' estimated parameters are listed in Table IV. This table confirms that the proposed method can detect and estimate two crossed LFM signals in an interference environment of CW signal and white noise with $SNR = -12dB$.



(a) Chirp rate estimation.



(b) Pulse width estimation.

Fig. 9: Estimation probabilities in white noise and CW.

TABLE IV: Estimated Parameters of Two Crossed LFM Signals in White Noise and CW

Parameter	Simulation	Estimation	Relative Error (%)
$\mu_1 (GHz.s^{-1})$	100	95	5.0
$\tau_1 (\mu s)$	10	10.2	2.0
$\mu_2 (GHz.s^{-1})$	200	200	0.0
$\tau_2 (\mu s)$	15	15	0.0

V. CONCLUSION

A new method for detecting multi-component LFM signals without knowing their parameters in intense white noise and even in an additional CW signal was presented in this paper. Firstly, the CFF between the received and first set of reference signals was calculated to detect LFM signals or estimate their chirp

rate. Then, the CCF between the received signal and the second set of reference signals was used to estimate the pulse width of LFM signals.

In the beginning, the possibility of analyzing the LFM signals in the white noise, then in a more complex case, combining a CW signal and that noise, was investigated in MATLAB. The simulation results showed that the method was able to detect and estimate the parameters of two LFM signals with $SNR \geq -12dB$. Next, verification was performed with real-time generated LFM signals. The experimental results confirmed the method's performance against LFM signals with $SNR = -15dB$ and $SNR = -12dB$ in the white noise and the mixture of white noise and CW signal, respectively. Our method is more effective than the existing ones. Notably, we require the $SNR \geq -14dB$ for detecting LFM signals, while existing methods need the $SNR \geq -8dB$ by DCNN and $SNR \geq -6dB$ by CNN. Also, 3000 samples are necessary for each signal in their work, so the network must be recreated for each training time, while ours only needs a flexible set of 100 reference signals. Or with [7], although they have a reasonable threshold, their method is significantly complex due to two main (time-frequency) computations.

REFERENCES

- [1] Nadav, L., Eli, M.: 'Radar signals' (John Wiley and Sons, Inc., Hoboken, New Jersey, Canada, 2004, ISBN: 0471473782).
- [2] Jonathan, Y., S.: 'Digital signal processing: A computer science perspective'. (Wiley-Interscience, 2000, ISBN: 0-471-29546-9), pp. 354.
- [3] Phillip, E. P.: 'Detecting and Classifying Low Probability of Intercept Radar'. (Boston, London, Artech House, 2009, p. 1-35, ISBN: 1580533221)
- [4] Allen, R. L., and Mills, D. W.: 'Signal Analysis: Time, Frequency, Scale, and Structure'. (Wiley- Interscience, NJ, 2004).
- [5] Zhang, Xueqin, and Ruolun, L.: 'Time-frequency analysis of multi-component LFM signals based on Hough and Chirplet transform'. In MATEC Web of Conference 173 (11), 2018.
- [6] Moghadasian, S. S.: "A Fast and Accurate Method for Parameter Estimation of Multi-Component LFM Signals," in IEEE Signal Processing Letters, vol. 29, pp. 1719-1723, 2022.
- [7] Swiercz, E, Janczak, D. and Konopko K.: "Detection of LFM Radar Signals and Chirp Rate Estimation Based on Time-Frequency Rate Distribution". Sensors (Basel). 2021 Aug 10;21(16):5415.
- [8] Wan, Jian; Yu, Xin; Guo, Qiang. (2019). LPI radar waveform recognition based on CNN and TPOT. Symmetry. 11. 725. 10.3390/sym11050725.
- [9] Shunjun, W.: 'Intra-pulse modulation radar signal recognition based on CLDN network'. IET radar sonar and navigation 14, 2020, pp. 803-810.
- [10] Nhan, N.T.: "Study of Detection Characteristics in Recognition of Simple Radio Pulses and Signals with LFM and PSK in the Autocorrelation Receiver". In: Galinina, Y. (eds) Internet of Things, Smart Spaces, and Next Generation Networks and Systems. NEW2AN ruSMART 2020 2020. Lecture Notes in Computer Science(), vol 12525. Springer, Cham.
- [11] Quan, D., Tang, Z., Wang, X.: 'LPI Radar Signal Recognition Based on Dual-Channel CNN and Feature Fusion'. Symmetry 2022,14(3), doi: 10.3390/sym14030570.
- [12] Mark, R.: 'Fundamentals of Radar Signal Processing'. McGraw Hill, 2005.
- [13] Dejan, I., Milenko, A., and Bojan, Z.: 'A new Model of CFAR Detector', Frequenz, vol. 68, no. 3-4, 2014, pp. 125-136.
- [14] Shrivathsa, V. S.: 'Cell Averaging – Constant False Alarm Rate Detection in Radar'. International Research Journal of Engineering and Technology (IRJET), 2018, vol. 05 (07), e-ISSN: 2395-0056.