



# Forest parameters inversion by mean coherence set from single-baseline PolInSAR data

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## Abstract

The effectiveness of the forest height inversion methods using polarimetric SAR interferometry data is affected by the estimation accuracy of ground phase, volume only coherence and model prediction for forest canopy layer. Finding the forest model that considers the effects of the signal penetration in the forest medium as well as optimum volume coherence, which is an important purpose of the forest inversion method. This paper suggests a novel inversion algorithm to increase the accuracy of forest height estimation as well as ground phase estimation based on mean of coherence set. For this purpose, an adaptive total least square line fit is proposed to estimate the ground phase and coherence line. Then, the combination between the VE-RVoG model and an optimization volume only coherence algorithm based on mean coherence set and polarization signature is developed to find more accurate forest parameter values. The proposed algorithm applies forest height estimation using L-band PolInSAR data acquired by PolSARProSim software, spaceborne SIR-C and UAVSAR system. The method was further validated by using UAV L-band PolInSAR data and the reference data of LiDAR canopy height model over rainforest Lope National Park, Gabon. Results showed that RMSE of results approximate 2.91 m and  $R^2$  is 0.909. The obtained results demonstrate the potential of proposed method in forest parameters estimation.

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## 1. Introduction

Forest height not only plays a critical role in environmental resource management but also contributes scientific data to consider on the global carbon cycle and climate change (Mette et al., 2004). In microwave remote sensing, polarimetric synthetic aperture radar interferometry (PolInSAR) has been extensively studied in the last two decades and proved to be an effective tool for forest height estimation because of its sensitivity to the vertical structure and physical characteristic of the scattering media (Cai et al., 2007; Papathanassiou and Cloud, 2001; Cloude and

Papathanassiou, 1997). Theoretically, the forest height can be directly retrieved from interferograms of ground and volume mechanisms without models (Neumann et al., 2010; Ky et al., 2018; Varvia et al., 2019). This method is simple to apply in the practical circumstances but its accuracy is not high. On other hand, the inversion approaches for forest height estimation based on the physically scattering model are developed (Cloude and Papathanassiou, 2003; Tayebe et al., 2018; Cuong et al., 2019; Wenxue et al., 2016; Xiao and Feng, 2019; Garestier and Thuy, 2010). One of them, the random volume over ground (RVoG) model is extensively applied to invert the PolInSAR measurements into forest parameters (Treuhaft and Cloude, 1999). In the RVoG model, forest medium is described as a homogeneous volume layer over ground sur-

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face. In general, this model addresses the volume interferometry coherence to be a function of four major parameters: (1) forest height; (2) the volume mean extinction coefficient, which defines the penetration depth of microwave through the canopy layer and it is assumed to be constant; (3) the ground-to-volume amplitude ratio; and (4) ground topography phase.

Based on RVoG model, the 3-stage algorithm was first proposed for tree height retrieval (Cloude and Papathanassiou, 2003). Because of the simplicity of implementation and time saving, this method is widely used for forest height extraction. However, the estimations of forest height and ground phase are relatively inappropriate. It is reasonable that the estimation of volume decorrelation is not very accurate. In order to overcome these shortcomings, some researchers have used the combination of 3-stage inversion algorithm and optimum polarization (Tayebe et al., 2018; Cuong et al., 2019; Wenxue et al., 2016). Although these methods are obtained some results better than original 3-stage method but the effects of the vertically varying extinction coefficient in the determination of the forest height need to be further investigated.

We know that the penetration depth of radar wave is not only inversely proportional to the transmitting frequency but also depends on the vertical forest structure consisting of leaf, branch and trunks layers from the canopy top to the ground (Nghia et al., 2015). Therefore, the extinction coefficient and the contribution of ground scattering component into HV coherence may causes an unpredictable error in forest height estimation. In order to finding the accurate volume only coherence, 3-stage method has also been associated with dual-polarization PolInSAR data for forest height estimation (Wenxue et al., 2016; Khati et al., 2017, 2018). However, determining the complex interference coefficient without ground scattering contribution using two polarization channels is a very difficult task. In recent years, several new methods have been published to improve accuracy in estimating forest parameters such as modified DBPI (Xie et al., 2017), PolInSAR inversion error model (Xiao and Feng, 2019). In which, Xie introduced a method that incorporate the sloped random volume over ground (S-RVoG) model in order to reduce the range terrain slope effect (Xie et al., 2017). Although the results achieved are better, but some disadvantages of the ground phase estimation have not been solved and increase the error due to baseline decorrelation. Moreover, Xiao Wang studied the impact of PolInSAR system errors on performance of coherence optimization technique for forest height estimation (Xiao and Feng, 2019). This method overcomes the shortcoming of the previous forest height conversion methods but it has some limitations such as: the coherence optimization is determined based on DEM differencing, the setting of  $A = 3.67$  may be effective only in temperate forest. Therefore, the obtained results become inappropriate in some case.

The performance of forest height inversion methods depends not only the accuracy of estimated coherence line but also the accuracy of observed volume only coherence, the main purposed of this paper is to seek for a more accurate coherence line and observed complex coherence without the need any assumptions. In other words, the main object of this paper is to find the complex interferometric coherence in ambiguous segment of coherence line that can characterize for the scattering process from crown layer in the nature medium. Therefore, in this framework, a new adaptive total least square line fit is introduced to determine the coherence line and more accurate ground phase. After that, the comprehensive optimization method based on polarization signature and coherence set theory is proposed to improve the accuracy of forest height inversion. In this stage, a vertically varying extinction RVoG model is applied to construct the look-up-table (LUT) of volume only coherence, which considers the forest vertical heterogeneity. The experimental results demonstrate that the proposed algorithm is able to overcome the limitation of previous techniques relating to the forest parameter inversion.

## 2. Model-based forest height inversion methods using PolInSAR images

### 2.1. Complex polarimetric interferometry coherence

A monostatic, fully polarimetric interferometry system is measured for each element in the scene from two slightly different viewing angles. In the general, the complex interferometry coherence can be expressed as (Papathanassiou and Cloud, 2001):

$$\tilde{\gamma}(\vec{\omega}) = \frac{\vec{\omega}_1^H \Omega \vec{\omega}_2}{\sqrt{(\vec{\omega}_1^H T_{11} \vec{\omega}_1)(\vec{\omega}_2^H T_{22} \vec{\omega}_2)}} = \frac{\vec{\omega}^H \Omega \vec{\omega}}{\vec{\omega}^H T \vec{\omega}} \quad (1)$$

where  $\vec{\omega}_1 = \vec{\omega}_2 = \vec{\omega}$  is a unitary vector defining the selection of each polarization channel. The superscript “ $H$ ” denotes complex conjugation and transposition. An equivalent formula to represent the interferometry coherence by using the modified normalized coherence matrix is given by Eq. (2):

$$\tilde{\gamma}(\vec{v}) = |\tilde{\gamma}(\vec{v})| e^{i\phi} = \vec{v}^H \Pi \vec{v}, \quad \vec{v}^H \vec{v} = 1 \quad (2)$$

where  $\Pi = T^{-1/2} \Omega T^{-1/2}$  is the modified normalized coherence matrix,  $\vec{v} = \sqrt{T} \vec{\omega} / \vec{\omega}^H \sqrt{T} \vec{\omega}$  denotes the normalized polarization projection vector.

### 2.2. Varying extinction RVoG model (VE-RVoG)

The RVoG model describes forest as a volume of random oriented particles over an underlying ground, which is characterized by a constant extinction (Cloude and Papathanassiou, 2003). According to the RVoG model,

the complex interferometry coherence of the polarization scattering mechanism can be written as (Treuhaf and Siqueira, 2000).

$$\tilde{\gamma}(\vec{\omega}) = e^{j\phi_0} \left( \tilde{\gamma}_{0v} + \frac{\mu(\vec{\omega})}{1 + \mu(\vec{\omega})} (1 - \tilde{\gamma}_{0v}) \right) \quad (3)$$

where  $\phi_0$  denotes the ground phase under the assumption of no-ground decorrelation,  $\mu(\vec{\omega})$  is the ground-to-volume scattering ratio.  $\tilde{\gamma}_{0v}$  is the volume-only coherence of the canopy layer with thickness  $h_v$  (Cloude and Papathanassiou, 2003).

In reality, the natural forest has a varying vertical structure consisting of leaf, branch and trunk layer from the canopy to ground. The extinction coefficient is significantly influenced by forest vertical structure, and it will affect the forest parameters inversion. Therefore, a more suitable RVoG model for height inversion needs to investigate how to depict the vertical forest structure accurately. Garestier et al introduced the vertically varying extinction RVoG (VE-RVoG) to take into account the heterogeneous vertical structure (Garestier and Thuy, 2010). The process of derivation of the VE-RVoG model is implemented from RVoG model by replacing the constant extinction with the linear varying extinction. In this model, the extinction of microwave at the canopy top layer is set equal zero, and then, it is supposed to increase with depth in the forest medium from a given offset value. Therefore, the extinction value  $\sigma$  varies linear in the  $z$  vertically direction with a slope  $\alpha$

$$\sigma(z) = \alpha z, \quad \text{with } \alpha > 0 \quad (4)$$

The volume only coherence associated with a varying extinction value can be determined by using Gaussian error function, which can be written as (Garestier and Thuy, 2010):

$$\tilde{\gamma}_v(h_v, \alpha) = e^{-\frac{\cos\theta k_z^2}{8\alpha} + jk_z h_v} \frac{\text{erf}\left(\frac{j\cos\theta k_z + 4\alpha h_v}{2\sqrt{2\alpha\cos\theta}}\right) - \text{erf}\left(\frac{jk_z}{2} \sqrt{\frac{\cos\theta}{2\alpha}}\right)}{\text{erf}\left(\sqrt{\frac{2\alpha}{\cos\theta}} h_v\right)} \quad (5)$$

where  $\text{erf}(\cdot)$  denotes the Gaussian error function.  $k_z$  and  $\sigma$  are the vertical wavenumber and mean extinction coefficient, respectively. The angle  $\theta$  is the incidence angle of the SAR system. The LUT for volume only coherence is instituted based on Eq. (5). The forest height is estimated by comparing the LUT with  $\tilde{\gamma}_{HV}$ . The VE-RVoG can reduce the effect of the constant extinction in determination forest height. However, the estimation of volume decorrelation of this method is not appropriate and this might cause error height.

### 3. Forest height inversion based on mean of coherence set

#### 3.1. Adaptive total least square line fit for coherence line and ground phase estimation

One of the indirect solutions to improve the accuracy of forest height estimation is to increase the stability and

reliability of the coherence straight line determination in the complex unit circle (CUC). A great number method of the topographic phase and coherence straight line estimation are proposed. These methods can be divided into two main groups: line fit and finding the optimal coherence coefficient. The first one includes algorithms based on least square (LS) and total least square line fit (TLS) methods (Cloude and Papathanassiou, 2003; Kumar et al., 2017; Xiao and Feng, 2019; Khati et al., 2018). However, the accuracy of these methods significantly depends on the number of used polarization channels. Therefore, these methods often lead to the problem with ill-conditions, time consuming and not accuracy. The second group includes methods based on finding the optimal complex coherence coefficients such as Maximum Likelihood (Seymour and Cumming, 1994), mean coherence set theory (Neumann et al., 2006), Kugler (Kugler et al., 2015), CLSA (Fu et al., 2014). The main idea of these methods is to find two polarization channels that represent the scattering components from canopy and ground. Then the coherence straight line that passing through these two points in the CUC. The method has improved the accuracy of topographic phase estimation and reduced computation time. However, the assumption of two polarized channels without any contribution from other scattering components is not consistent with the actual forest environment. Therefore, the method's efficiency is reduced.

For the above reasons, in this article we propose an adaptive total least square line fit (ATLS) method for estimating the coherence straight line and the topographic phase based on mean coherence set theory. The proposed ATLS method uses three complex interference coherence coefficients obtained from the contraction matrix  $\Pi$ . We can assume that the  $\Pi$  matrix has three complex eigenvalues  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  and  $\arg(\lambda_1) \leq \arg(\lambda_2) \leq \arg(\lambda_3)$ . According to mean coherence set theory, the 3 eigenvalues of  $\Pi$  matrix approximately with the complex interference coefficient exclusively for three scattering components from canopy ( $\tilde{\gamma}_3 \approx \lambda_3$ ), trunk-ground ( $\tilde{\gamma}_2 \approx \lambda_2$ ) and ground ( $\tilde{\gamma}_1 \approx \lambda_1$ ) in the forest environment.

Without generality, we assume that a linear relationship between the real and the imaginary part of the complex coherence coefficient is expressed through a straight line as follow:

$$y = Mx + C \quad (6)$$

The main disadvantage of using Eq. (6) to represent a combined line is that it is not possible to represent a vertical line due to  $M \rightarrow \infty$ . To fix this problem we use the normalized form of the line as follows:

$$l = \langle r, \phi \rangle \quad (7)$$

where  $r$  and  $\phi$  are the length and angle of the normalized line. Applying this form, the combined line can be expressed as follows:

$$y = -\cot(\phi) \cdot x + \frac{r}{\sin\phi} \quad (8)$$

From Fig. 1, we show that the shortest distance from a given point  $Z_i = (x_i, y_i)$  to the coherence line  $l = (r, \phi)$  is  $d_{\perp} = (Z_i, l)$ , denote by  $d_i$  (Leonardo et al., 2014).

$$d_i = \frac{1}{\sqrt{1 + \cot^2\phi}} \cdot \left( y_i + \cot\phi x_i - \frac{r}{\sin\phi} \right) \quad (9)$$

In the TLS problem, the minimum function  $\chi^2 = \sum_{i=1}^N d_i^2$  is used for fitting coherence line. However, the performance of TLS method depends strongly on the number of used polarization channels. To increase the accuracy of the coherence straight line estimation, a coefficient  $a$  is added for the minimum problem. Then the minimum function has the form:

$$\chi^2(a, \phi) = \sum_{i=1}^3 d_i^2 = \sum_{i=1}^3 \frac{a \cot^2(\phi) + 1}{\cot^2(\phi) + 1} \left( y_i + \cot(\phi) x_i - \frac{r}{\sin(\phi)} \right)^2 \quad (10)$$

We see that when  $a = 0$ , the Eq. (10) will correspond to the TLS problem and vice versa when  $a = 1$ , this equation will correspond to the LS problem. Therefore, the Eq. (10) shows that ALTS method is general form for 2 methods: TLS and LS. On the other hand, according to Leonardo (Leonardo et al., 2014) the length of the normalized line is expressed in inclination as follows:

$$r = \bar{x} \cos\phi + \bar{y} \sin\phi \quad (11)$$

where

$$\bar{x} = \sum_{i=1}^3 \frac{\text{Re}(\tilde{\gamma}_i)}{3} \quad \text{and} \quad \bar{y} = \sum_{i=1}^3 \frac{\text{Im}(\tilde{\gamma}_i)}{3} \quad (12)$$

The Eq. (10) can then be re-expressed as follows:

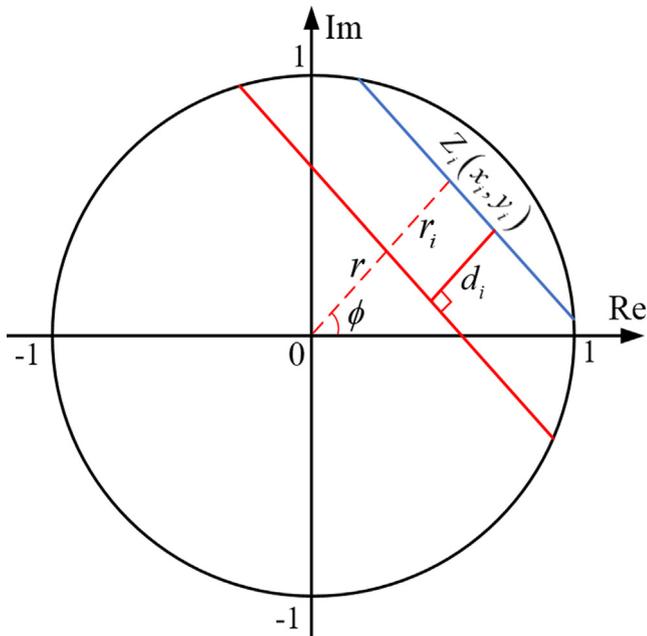


Fig. 1. Orthogonal distance  $d_i$  from point  $z_i$  to line  $l$ .

$$\chi^2(a, \phi) = \sum_{i=1}^3 \frac{a \cot^2(\phi) + 1}{\cot^2(\phi) + 1} (y_i + x_i \cot^2(\phi) - (\bar{x} \tan(\phi) + \bar{y}))^2 \quad (13)$$

The comprehensive search method is proposed to improve the accuracy of coherence straight line estimation as well as topographic phase extraction. In this paper, the input parameters of comprehensive search method for two parameters  $(a, \phi)$  vary in the range  $a \in [0, 1]$  and  $\phi \in (0, \pi)$ , respectively. In this work, we implemented the finding of  $(a, \phi)$  optimal pair, which is considered as more reliable for coherence straight line. This mean, we find  $(a_{opt}, \phi_{opt})$  optimal pair value which satisfy the following condition:

$$\min_{\{a, \phi\}} \|\chi^2(a, \phi)\| \quad (14)$$

After that, the coherence straight line in the CUC can be represented as follow:

$$y = \hat{M}_{opt} x + \hat{C}_{opt} \quad (15)$$

In which

$$\begin{cases} \hat{M}_{opt} = -\frac{\cos \phi_{opt}}{\sin \phi_{opt}} \\ \hat{C}_{opt} = -\frac{\hat{r}_{opt}}{\sin \phi_{opt}} \\ \hat{r}_{opt} = \bar{x} \cos \phi_{opt} + \bar{y} \sin \phi_{opt} \end{cases} \quad (16)$$

The coherence straight line in Eq. (15) intersects the CUC at two points, which are the ground coherence candidates. The ground coherence will be the point with the greatest distance from the HV coherence. Then the surface phase is determined by formula (17), as shown in Fig. 2.

$$\phi_0 = \arg(x_0 + j y_0) \quad (17)$$

### 3.2. Volume only coherence optimization

In the previous forest height inversion methods, the HV channel is assumed that no any contribution of ground scattering component and it is usually applied to compute the volume only coherence (Cloude and Papathanassiou, 2003; Garestier and Thuy, 2010). In practically, the HV channel not only is affected by ground scattering component but also is a mixture of other scattering mechanism, such as: scattering from tree trunk, double bounce scattering mechanism. That mean, the HV phase center is not the upper phase center between all observed coherence (Tayebe et al., 2018). Therefore, the assumption of these methods would cause big error in the forest parameters estimation, especially in estimating forest height. In order to reduce the inversion error, some optimal volume coherence methods are introduced to find out the pure volume only coherence (Tayebe et al., 2018; Cuong et al., 2019). One of them, the Tayebe's method has significantly enhanced the effectiveness of 3-stage method (Tayebe et al., 2018). This method implemented finding the optimal interferometry coherence coefficient by complex unitary vector, which corresponds to the upper phase center on the ambiguous line

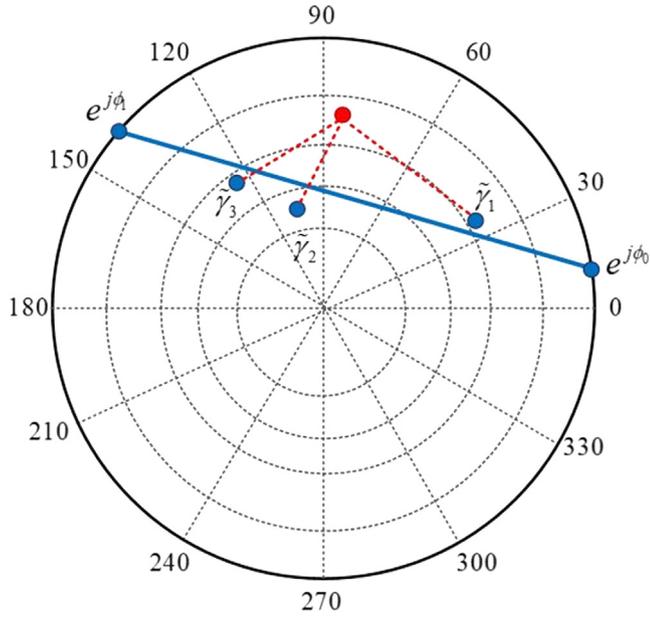


Fig. 2. The PolInSAR coherence line model inside the CUC.

segment. Although, the results of this method is more accurate but it takes a lot of time for forest height estimation.

To overcome the limitations of the previous forest height conversion methods, an exhausted search method based on coherence set theory and polarization signals was proposed to determine the best volume coherence that represents the characteristics of the direct scattering from the canopy. In this paper, we will search all coherence phase by finding possible complex unitary vector  $\vec{w}$ . As a results, we introduce complex interference diagram not only represents in elliptical polarization system. Then the contraction matrix in the elliptical polarization system is as follows:

$$\Pi' = W \cdot \Pi \cdot W^H; \quad W = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]^H \quad (18)$$

where

$$\begin{cases} \vec{v}_1 = \frac{1}{2}[\cos 2\chi + \cos 2\tau - j \sin 2\chi \sin 2\tau & -\sin 2\tau \sin 2\chi + j \sin 2\chi & \sin 2\tau \cos 2\chi + j \sin 2\chi]^T \\ \vec{v}_2 = \frac{\sqrt{2}}{2}[\sin 2\tau + j \cos 2\chi \sin 2\tau & j \cos 2\tau \cos 2\chi & -\sin 2\tau + j \cos 2\chi \sin 2\tau]^T \\ \vec{v}_3 = \frac{1}{2}[\cos 2\tau - \cos 2\chi + j \sin 2\chi \sin 2\tau & \sin 2\tau - \cos 2\chi - j \cos 2\tau \sin 2\chi & \cos 2\tau + \cos 2\chi + j \sin 2\chi \sin 2\tau]^T \end{cases} \quad (19)$$

By replacing the  $\Pi'$  matrix into Eq. (2), the complex interferometric coherence in the elliptical polarization system can be expressed as:

$$\gamma(\vec{v}') = \vec{v}'^H \cdot \Pi' \cdot \vec{v}'; \quad \vec{v}'^H \cdot \vec{v}' = 1 \quad (20)$$

In which

$$\vec{v}' = \frac{W\sqrt{T}W^H \cdot \vec{w}}{\vec{w}^H \cdot W\sqrt{T}W^H \cdot \vec{w}} \quad (21)$$

Each pair of values  $(\chi, \tau)$  in the range of  $[-\pi/4; \pi/4]$ , we perform analysis on eigenvalues and eigenvector for the matrix  $\Pi'(\chi_k, \tau_k)$ . Then, the complex unitary vector can be extracted as follow:

$$\vec{w}_j^k = T^{1/2} \cdot V_j^k; \quad j = 1, 2, 3 \quad (22)$$

In which  $V_j^k; \{j = 1, 2, 3\}$  denote the eigenvector of  $\Pi'(\chi_k, \tau_k)$  matrix. It can be seen that the coherence phase of all polarization states is a function of two polarization variables  $(\chi, \tau)$ . According to the electromagnetic wave scattering theory (Treuhaft and Siqueira, 2000), the direct scatter components from the canopy usually have the largest coherent interference phase and its polarized coherent interference coefficient is on the coherence line. In this paper, we suggest two constraint condition to improve the accuracy of volume coherence estimation. Firstly, the proposed algorithm will look for pairs of values  $(\chi, \tau)$  that satisfy the following conditions:

$$\begin{aligned} \phi_1 &> \arg\left(\left[\omega_j^k(\chi_k, \tau_k)\right]^H \cdot \Omega \omega_j^k(\chi_k, \tau_k)\right) \\ &> \arg(\tilde{\gamma}_{HV}) \end{aligned} \quad (23)$$

where  $\phi_1 = e^{j\phi_1}$  is the remaining intersection point of the combined line with the unit circle (Fig. 2). We assume that  $M$  pairs of values  $(\chi, \tau)$  are found out that satisfy the condition (23). Then, the set of  $M$  complex interferometric coherence will be determined as Eq. (24).

$$\begin{aligned} \tilde{\gamma}(\chi_l, \tau_l) &= \frac{\vec{w}^H(\chi_l, \tau_l) \cdot \Omega \cdot \vec{w}(\chi_l, \tau_l)}{\vec{w}^H(\chi_l, \tau_l) \cdot T \cdot \vec{w}(\chi_l, \tau_l)}; \quad l \\ &= 1 \div M \end{aligned} \quad (24)$$

In the coherence set of PolInSAR theory, the most optimal complex coherence will lie on the straight coherence line in the complex plane. As a results, the optimal pair

of value  $(\chi_{opt}, \tau_{opt})$  can be extracted under following condition:

$$\begin{aligned} \min_{\chi_l, \tau_l} &\quad \|\text{Im}(\tilde{\gamma}(\chi_l, \tau_l)) - \hat{M}_{opt} \text{Re}(\tilde{\gamma}(\chi_l, \tau_l)) - \hat{C}_{opt}\| \\ l &= 1 \div M \end{aligned} \quad (25)$$

where  $\hat{M}_{opt}$  and  $\hat{C}_{opt}$  are the coefficients of the straight coherence line, which defined in Eq. (16). According to Eq. (25) an optimal complex coherence coefficient is determined and denoted by  $\tilde{\gamma}_{opt}$ . The previous coherent region optimization algorithms often assumed that the  $\tilde{\gamma}_{opt}$  is a complex coherence without the contribution of any other scattering components. In practice, the polarization channels are always a mixed value of many different scattering components. Hence, this assumption often causes errors in the forest height estimation, especially in the complex structured forest areas. To overcome the above shortcomings, we propose an optimal iterative method to improve the accuracy of forest parameters estimation on complex structured forest areas. For each  $\tilde{\gamma}_v(h_v, \alpha)$  value on the coherence line, the complex coherent interference coefficients for each polarization channel can be estimated as follows:

$$\tilde{\gamma}_{est}(h_v, \alpha, L(\vec{\omega})) = e^{j\phi_0} (\tilde{\gamma}_v(h_v, \alpha) - L(\vec{\omega})(1 - \tilde{\gamma}_v(h_v, \alpha))) \quad (26)$$

where  $\tilde{\gamma}_v(h_v, \alpha)$  is the complex coherence coefficient of the direct scattering components from the canopy, which is represented in the Eq. (5). The new optimal iteration algorithm is implemented through four main steps as follows. Firstly, the forest height ( $h_v = h_{v-3stage}$ ) is used as a known parameter to find values for  $L(\vec{\omega})$  and  $\alpha$  coefficients based on an optimal standard  $\varsigma_{min} = |\tilde{\gamma}_{opt} - \tilde{\gamma}_{est}|$ . Where two coherences  $\tilde{\gamma}_{est}$  and  $\tilde{\gamma}_{opt}$  determined as in Eqs. (26) and (25), respectively. Secondly, by change the values of forest height  $h_v$  with hops  $\Delta h_v$ , ( $h_v \leq 2\pi/k_z$ ), we repeat the above loop and record the parameter pairs values  $L(\vec{\omega})$  and  $\alpha$  for each  $h_v$ . Then we can obtain the table of  $L(\vec{\omega})$  and  $\alpha$ . Thirdly, the  $\bar{L}(\vec{\omega})$  and  $\bar{\alpha}$  is the final pair of values selected by averaging all corresponding values in the above table. The complex interference coherent coefficient of the pure volume scattering component will be determined:

$$\tilde{\gamma}_v(h_v, \bar{\alpha}) = \frac{e^{-j\phi_0} \tilde{\gamma}_{opt} - \bar{L}(\vec{\omega})}{1 - \bar{L}(\vec{\omega})} \quad (27)$$

Finally, the forest height will be obtained by solving the Eq. (28):

$$\frac{e^{-j\phi_0} \tilde{\gamma}_{opt} - \bar{L}(\vec{\omega})}{1 - \bar{L}(\vec{\omega})} = e^{\frac{-\cos\theta k_z^2}{8z} + j k_z h_v} \cdot \frac{\text{erf}\left(\frac{4\bar{\alpha} h_v + j \cos\theta k_z}{2\sqrt{2\cos\theta}}\right) - \text{erf}\left(\frac{j k_z}{2} \sqrt{\frac{\cos\theta}{2\bar{\alpha}}}\right)}{\text{erf}\left(\sqrt{\frac{2\bar{\alpha}}{\cos\theta}} \cdot h_v\right)} \quad (28)$$

## 4. Experimental results and analysis

In this section, we use two different PolInSAR data types for purpose of demonstrating the effectiveness of proposed method. The proposed method is first applied to simulated PolInSAR data, which is generated from PolSARProSim simulation software (Williams, 2006), to evaluate the effectiveness, stability and reliability of its results. After that, based on the obtained results with simulated data, it is completely reliable to apply the proposed method to real data. Finally, we can make valid statements for the performance of proposed methods from obtained results.

### 4.1. Simulated PolInSAR data

In this paper, simulated data of tested forest scene is created from PolSARProSim 5.2 software (Williams, 2006). The simulated scenario in the paper is the coniferous forest zone, name as HEDGE in the PolSARProSim software at 1.3 GHz and at 40-degree angle of incidence. The stand height 20 m, and it is located on a 0.1% ground azimuth and 0.2% ground range slope. The parameters of the simulated forest area are detail represented in Table 1. To further evaluate the effectiveness of the proposed method, we apply the proposed method to the different simulation data types with different vertical wavenumber and terrain's slope. The effectiveness of the proposed method was assessed through the comparison with the Xiao method, Tayebe method and Khati method with the generated simulation forest data. Fig. 3 (a) depicts the simulated forest area in coding Pauli image of the surveyed forest area that includes with the size (191 × 231) pixels.

The forest height elevation estimated by the proposed method is shown in Fig. 3(b) and we can see that the forest height mean fluctuates around 20 m. Next, in order to increase the reliability and accuracy of the proposed method, we compare the results obtained from the proposed method with the Tayebe, Khati and Xiao methods. The results are shown above in Fig. 4.

Fig. 4 illustrates the forest height estimation of four methods above over the observed forest area. The Fig. 4 (d) shows that the average height from 18 m to 20 m is significantly higher than the other methods. Nonetheless, the forest heights that below 15 m and above 23 m of this method is lowest. This result shows that the forest height estimated by the proposed method is highly accurate and reliable.

Table 2 presents the results of the forest parameters estimated by four methods. In which the average forest height of the simulation data is 20 m. The results in Table 2 show that the average forest height estimated by the proposed method is 19,586 m. This result has the highest accuracy compared to four mentioned methods. At the same time, the average forest height estimated by Khati and Xiao method is 19,083 m and 19,265 m. These two methods also showed high accuracy and stability in determining the

Table 1  
Forest parameters of the simulation scenario.

Altitude	Vertical baseline	Central frequency	Horizontal baseline
3000 m	1.0 m	1.3 GHz	10 m
Incidence angle	Density	Tree species	Tree height
40°	900 stem/Ha	Hedge	20 m

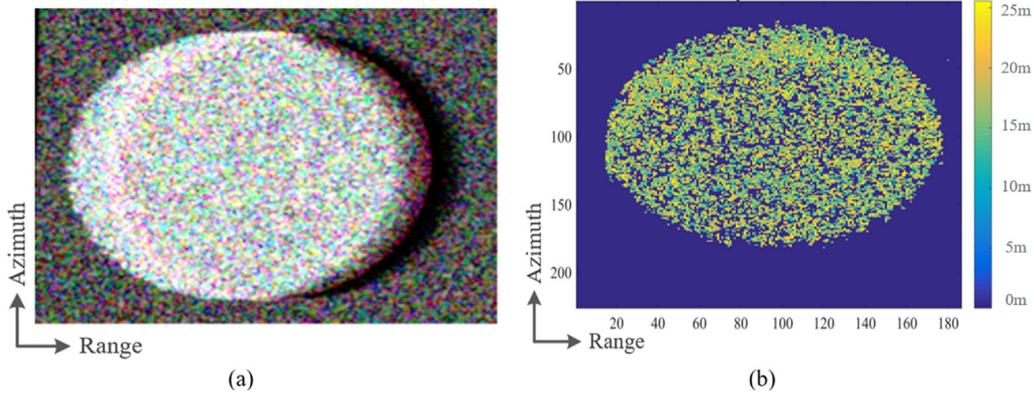


Fig. 3. (a) Pauli image of the observed area, (b) 2D image depicting forest height extraction.

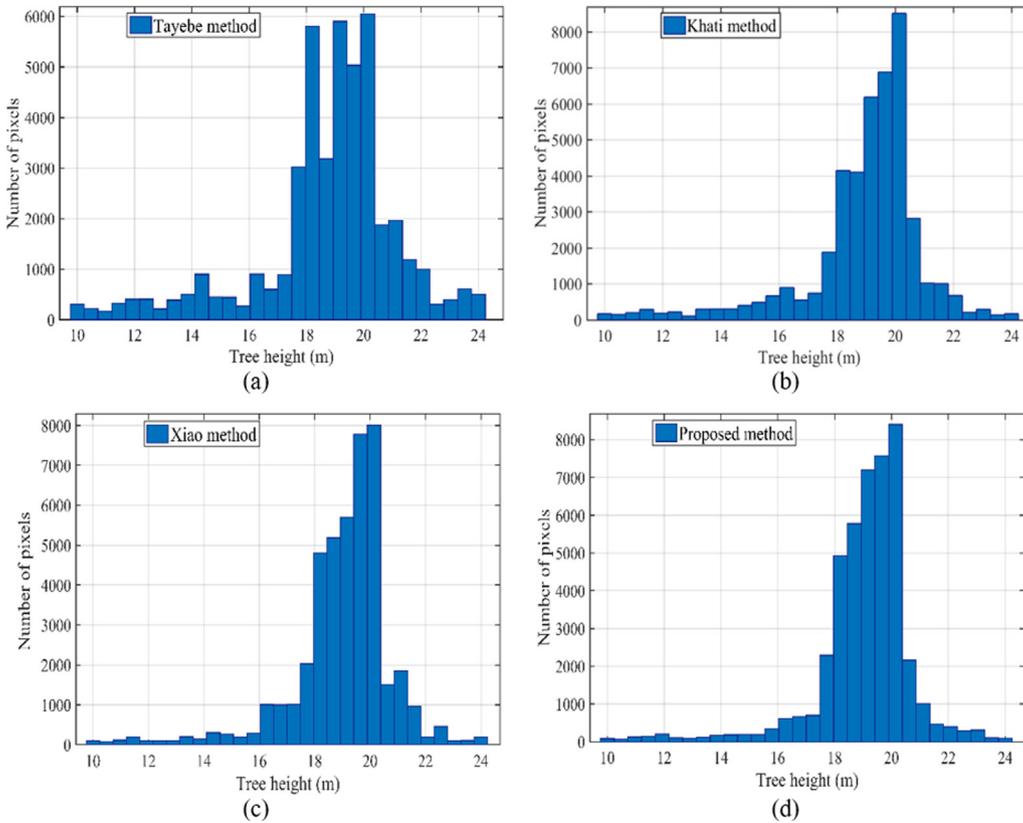


Fig. 4. Histograms of vegetation height. Results for the Tayebe method (a), Khati method (b), Xiao method (c) and Proposed method (d).

height of your forest. Meanwhile, the average forest height result estimated by Tayebe method is 18,531 m due to Tayebe's assumption that the optimal coefficients only have volume only scattering components without other scattering components (as ground, tree-trunk scattering

component) and use the line fit method to estimate the ground phase. Therefore, the estimation efficiency of the forest parameters of Tayebe's method is not high and could not accurately reflect the scattering process in the actual forest environment.

Table 2  
Results of forest parameters for the four methods.

Parameters	Real values	Tayebe method	Khati method	Xiao method	Proposed method
$h_v [m]$	20	18.531	19.083	19.265	19.586
$\phi_0 [rad]$	0.0982	0.1616	0.1218	0.1225	0.0918
$\sigma [dB/m]$	0.1262	0.2465	0.2006	0.1919	0.1860
$\bar{\alpha}$	—	—	—	—	0.0094
RMSE (m)	0	2.9566	2.4661	2.2622	1.9654
Accuracy (%)	100	92.655	95.415	96.325	97.929

In addition, regarding the mean squared error (RMSE) value and the accuracy of four methods, the proposed method has the highest efficiency. Table 2 also shows that the surface phase and the mean extinction coefficient estimated by the proposed method are more accurate and closer to the respective system values than that in other methods.

To further assess the impact of several scenario parameters on the estimation results, we applied the four methods to the scenario presented in Table 1 but with different vertical wavenumbers. The simulated forest data was generated by changing the satellite incident angle while the rest of parameters remain unchanged. The experiments were conducted with 12 data with different vertical wavenumbers, the incident angle varies from 15 degrees to 70 degrees (the incidence angle of each simulation data is 5 degrees), corresponding to the values of the vertical wavenumber coefficient from 0.489 rad/m to 0.0217 rad/m. In the reference (Kugler et al., 2015), Kugler has been shown that the complex interferometry coherence coefficients of HH, HV and VV polarization channels are significant influenced by vertical wavenumber. Therefore, changes of the vertical wavenumber may cause an error of several meters in the estimated forest height.

Fig. 5 shows the estimated average forest height of four methods with different vertical wavenumbers. In which, the forest height was estimated by the proposed method (red line), Xiao method (green line), Khati method (black line) and Tayebe method (blue line). The results show that the estimated average forest height of four methods is highly

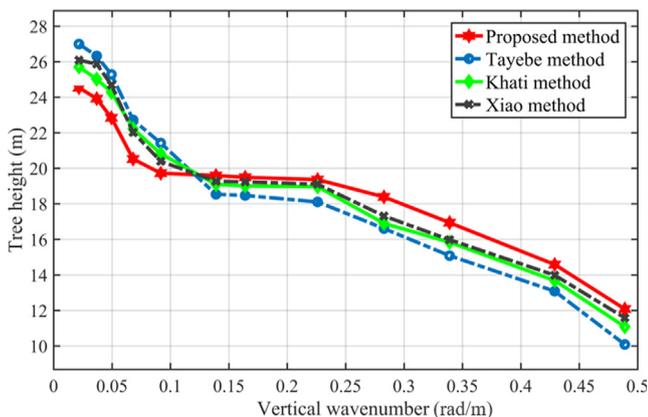


Fig. 5. Mean forest height results with different  $k_z$ .

stable and accurate with vertical wavenumbers in the range  $(0.0917 \div 0.226)$  rad/m. With the vertical wavenumbers smaller than 0.0917 rad/m, the forest height is usually overestimate due to residual decorrelation, which will be lead the unfavorable coherence to height scaling. On the contrary, when the vertical wavenumber increased significantly (greater than 0.2828 rad/m), the contribution of residual nonvolume decorrelation  $\tilde{\gamma}_{Res}$  (see Kugler et al., 2015) into complex coherence coefficient may increase, so the forest height is underestimated. Therefore, when vertical wavenumber too small or too large, the estimation results of the average forest height of all four methods not accuracy and reliable. It can be concluded that the vertical wavenumber is an important parameter for the effectiveness of the proposed method and forest height retrieval.

Next, to analyze the effect of slope on the estimated parameters, we applied the proposed method to 8 different slope terrains. These data were generated by changing the slope in the range direction of the terrain, which change from 0 degrees (0%) to 38.6 degrees (80%). We note that, the complex interferometric coherence coefficient is affected remarkably by ground ranges slopes. So, the forest height inversion methods related with complex interferometric coherence coefficient will hard to obtain the right value. In proposed method, the accuracy of forest height estimation is significantly improved by choosing the best fit parameter for finding the complex volume only coherence based on the constrained optimizations. From Table 3, it can be seen that the phase and magnitude of optimal complex coherence as well as estimated forest height by proposed method all increase with slope of terrains. Especially, when the slope of terrain is larger than 30.9 degrees, the estimating forest parameter results lead to significantly error. However, the estimation error of the proposed method is relatively small and the forest parameters estimated by this method ensure the accuracy and close to the actual value of the system. The estimated forest parameters in Table 3 demonstrate that, determining correctly the optimal interference coherent and the center phase of the volume scattering component will affect the accuracy of the forest height of the applied algorithm.

#### 4.2. Spaceborne PolInSAR data

In this section, we perform applying the four methods to space-borne PolInSAR data, which acquired from SIR-C

Table 3  
Results of proposed method with different slope terrains.

Slope of terrain (degree)	Center phase of volume scattering $\phi_{opt}$ [rad]	Magnitude of the optimal coherent factor $ \tilde{\gamma}_{opt} $	Average tree height (m)
$0^{\circ}$	4.965	0.769	19.586
$5.7^{\circ}$	4.982	0.782	19.599
$11.3^{\circ}$	5.298	0.806	19.623
$16.7^{\circ}$	5.299	0.819	19.685
$21.8^{\circ}$	5.566	0.846	19.821
$26.6^{\circ}$	5.681	0.859	20.386
$30.9^{\circ}$	5.788	0.861	20.571
$34.9^{\circ}$	6.004	0.882	20.892
$38.6^{\circ}$	6.186	0.922	21.519

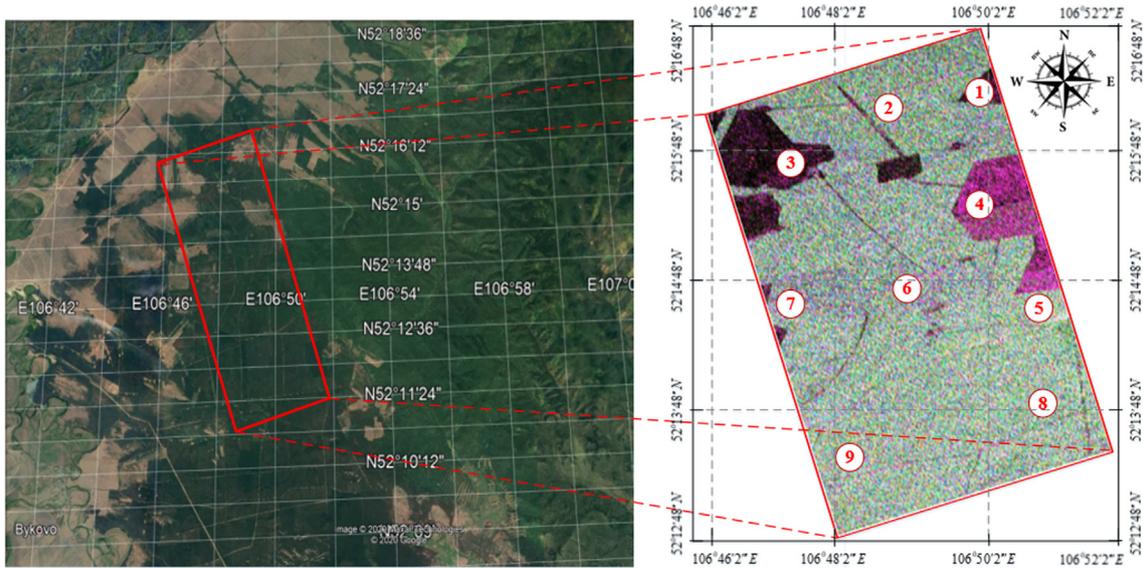


Fig. 6. (a) The optical image of test site from Google Earth; (b) RGB Pauli coding image.

system. The data set used in this paper was taken from an images pair of the forest area at KUDARA, LAKE BAIKAL, RUSSIA. This region has a mixed forestry, road and agricultural area. They consist of quad-pol interferometric data at L band with a 24.569 - degree angle of incidence and 413.8 m baseline. Fig. 6(a) depict the Google image for whole observation area, in which the test area shown by red rectangle. Fig. 6(b) is the composite image of the evaluation patch on Pauli basis.

The surveyed region includes some objects such as forest area, road and agricultural area (violet area). One of cause induces the error in forest height estimating that is the presence of temporal decorrelation in the PolInSAR data. However, the magnitude of the interference correlation coefficient of the PolInSAR data are almost greater than 0.7 and the forest tree do not much change in a month. Based on the analyzing of Papathanassiou (Papathanassiou and Cloude, 2003), we can ignore the effect of the temporal decorrelation when estimating forest height. After co-registering PolInSAR images, we selected a survey area with  $495 \times 495$  pixels. The observation region is a temperate forest with a variety of tree species

placed on small rough terrain. According to the survey, the vegetation is mainly Taiga Forest and forest height is between 15 and 30 m. The average forest height of the experimental data is approximate to 20 m.

The Fig. 7 illustrate the estimated vegetation in the test site area. Due to lack of real forest height information, we adopted performance of proposed method with simulated data and gave a qualitative analysis. In bare land such as roads or agriculture regions, the average vegetation height is less than 3 m. In forest areas, the tree heights range from 15 m to 32 m and the average forest heights is 20.169 m. The inversion result may be slightly overestimated due to lack of terrain slope information. However, the results obtained are completely consistent with the distribution of the vegetation in the test site area. By applying the proposed method and above mentioned methods to the spaceborne data, the results of these methods are shown in Table 4. From this table, we shown that the tree height estimated by Tayebe, Khati and Xiao methods are approximately 18.81 m, 19.68 m and 19.85 m, respectively.

This result is completely logical with the results of forest height estimation which are implemented in the Section 4.1.

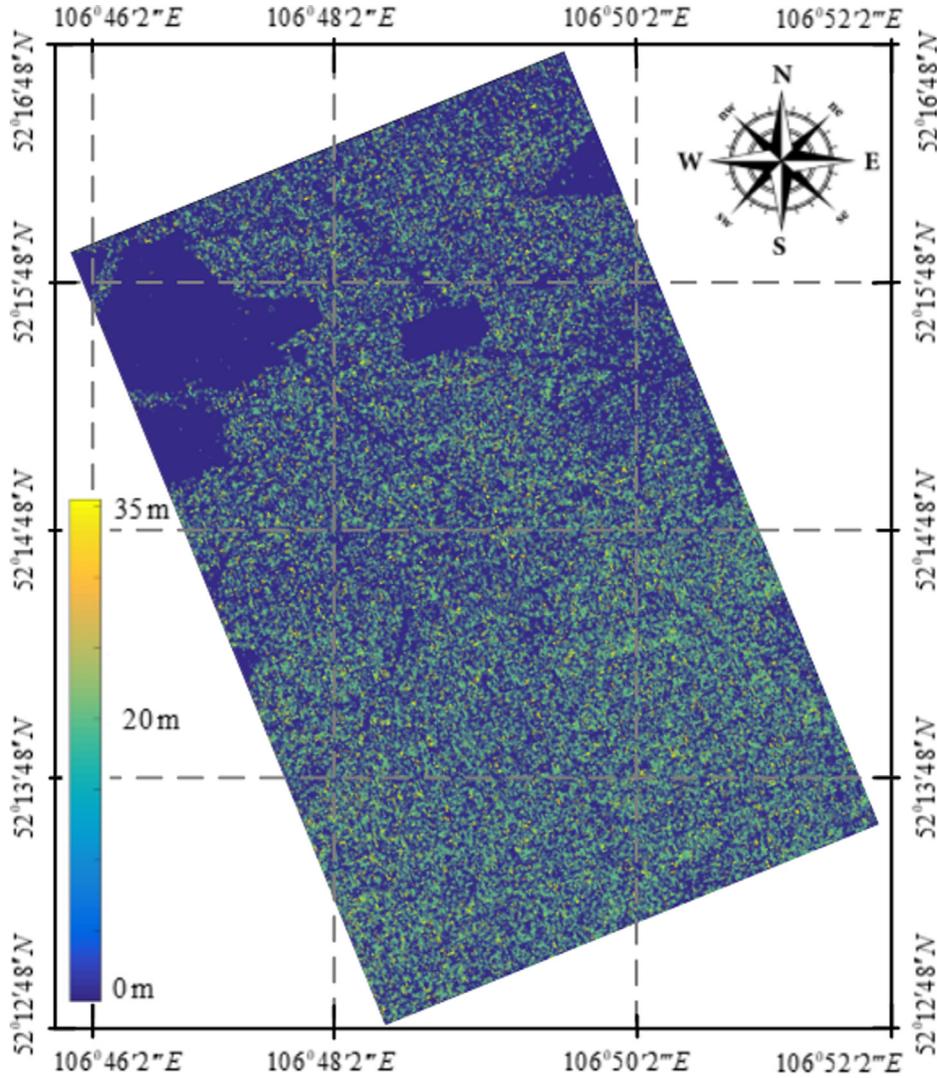


Fig. 7. The estimated forest height of the test site region.

Table 4  
Results of forest parameters of the methods with spaceborne data.

Parameters	Tayebe's method	Khati's method	Xiao's method	Proposed method
$ \tilde{\gamma}_{opt} $	0.683	0.772	0.779	0.792
$\phi_{opt} [rad]$	4.362	4.589	4.657	4.682
$h_v [m]$	18.814	19.683	19.852	20.169
$\phi_0 [rad]$	-0.1265	-0.0971	-0.0892	-0.0682
$\alpha$	-	-	-	0.0096
$\sigma [dB/m]$	0.2739	0.2469	0.2549	0.2095
$RMSE (m)$	2.9983	2.4627	2.2592	2.0885

In addition, the remain parameters also reflect the estimation efficiency of forest parameters of the proposed method as more accurate and reliable than the other methods.

The proposed method provides two additional physical parameter maps: the mean extinction coefficient and the topographic phase in Fig. 8(a) and 8(b), respectively. The penetration depth depends on the mean extinction coefficient. In other words, the extinction coefficient,  $\sigma$ , corresponds to a mean extinction value for the vegetation

layer. In forest areas, where the canopy covers most of the surface, and volume scattering dominates, so the extinction coefficient in volume is very large. Small scattering surfaces result in very little (the minimal wave scattering from the ground) scattering waves from the ground. Therefore, the ground phase is underestimated in these areas. In contrast, in road and farm areas, extinction coefficients are low, and the ground phase is estimated to be relatively large as detailed in Fig. 8.

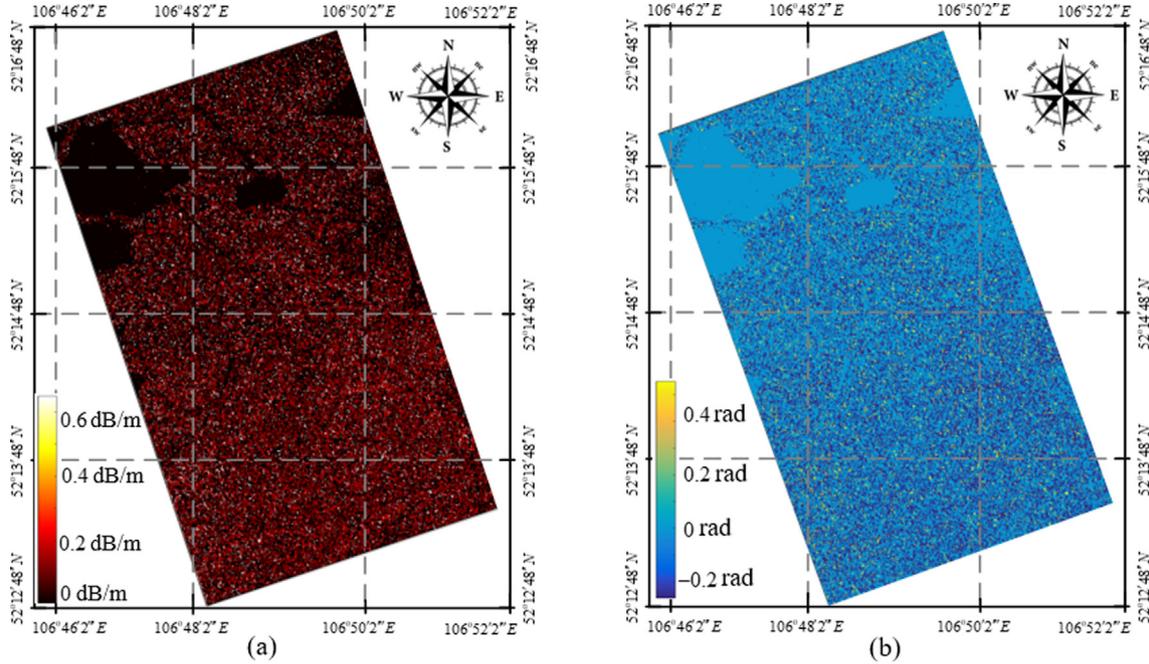


Fig. 8. (a) The estimated extinction coefficient and (b) The estimated ground phase.

Based on the results shown in the Figs. 7 and 8(a), we can state that the wave penetration coefficient and the estimated forest height are correlated with each other. Two figure also show that the wave penetration coefficient over the entire observed forest area is proportional to the estimated forest height. In Fig. 8(a) the wave penetration coefficient in agricultural land and road areas is very small, in the range (0–0.1) dB/m. The mean values of the forest height and wave attenuation coefficients of the proposed method are about 20 m and 0.2095 dB/m, respectively. These values are relatively accurate and completely consistent with the real value of the observed area.

Finally, in order to fully evaluate the effectiveness of the proposed method, we examined the forest parameters on the 9 areas (size 10x10) highlighted in Fig. 6. We applied the proposed approach to 9 areas and gave two parameters: the mean forest height and the mean wave attenuation coefficient. Then evaluating the accuracy of the proposed method as well as the relationship between the wave

attenuation coefficient and the results of the estimation of forest height. Fig. 9(a) shows that the height of the forest is usually less than 3 m for agricultural or farm areas such as zone 1 or zone 3. In addition, the rest of the area shows a height of about 20 m. The areas of 1 and 3 have sparse density and very low height, so the wave propagation and reception of scattering is quite favorable. As a result, the wave loss coefficient is very small. Conversely, for dense forested areas, canopy and branches will cause multiple scattering, so the coefficient of wave attenuation in these areas is greater and usually ranges from (0.164–0.269) dB/m.

From the results obtained with the simulation data and the space-borne data, we see that the proposed method is more effective than previous forest height inversion methods, such as three-stage, Tayebe, Khati and Xiao. The ground phase estimation’s accuracy has a significant role in improving the efficiency of the forest parameter conversion algorithms. In previous studies (Xiao and

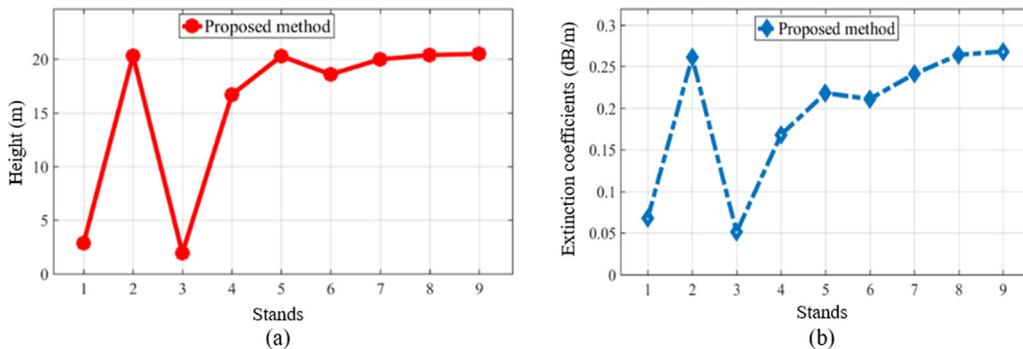


Fig. 9. (a) Estimated tree height, (b) extinction coefficient at 9 stands.

Feng, 2019; Khati et al., 2017; Tayebe et al., 2018; Cloude and Papathanassiou, 2003), mostly use the TLS algorithm to estimate the topography phase. However, this algorithm often leads low efficiency because of difficulty to choose the best fitting coefficient, the accuracy of this method depends on the number of polarizing channels used. The ground phase accuracy of the proposed method is significantly improved by using the ATLS method based on the coherence set theory. The ATLS method not only overcomes the disadvantages of the TLS method but also combines to some optimal binding conditions to improve the accuracy of ground phase estimation. The presented results in Tables 2, 4, and Fig. 8 (b) show that the terrain phase estimated by the proposed method always has higher accuracy and is closer to the real value than the remaining methods.

The previous forest height inversion methods (Xiao and Feng, 2019; Khati et al., 2017; Tayebe et al., 2018; Cloude and Papathanassiou, 2003) often used some assumptions to find the volume-only interferometric coherence coefficient and based on this parameter to extract the forest height. However, the influence of ambiguity of the non-volume decorrelation and ground contribution caused a significant error in the forest height inversion. This disadvantage can be handle by modifying the middle point of the ambiguity to be the volume-only interferometric coherence value. In the proposed method, we adjust this middle point by the determination process of the optimal volume coherent coefficient  $\tilde{\gamma}_{opt}$ , which reduced the coherence ambiguity region from nonvolume decorrelation and ground contribution. In addition, the proposed method has also developed an optimal loop algorithm to improve the accuracy of the forest height estimation with the wave extinction coefficient that varies with the respective tree height. Therefore, the efficiency of the proposed method has been significantly improved compared to other inversion methods. The results show that the proposed method always leads stability and accuracy when applying calculation parameters to many different forest types (tropical forests, temperate forests, mountainous forests).

#### 4.3. Unmanned aerial vehicle synthetic aperture radar data

In order to provide further verification of the of this new forest height estimation technical, the proposed method is also applied to other fully polarimetric InSAR images. The data set was collected by a synthetic aperture radar located on the unmanned aerial vehicle (Unmanned Aerial Vehicle Synthetic Aperture Radar - UAVSAR) of NASA/JPL by AfriSAR project. The device operates in L-band, of which incidence angles are from  $21^{\circ}$  to  $65^{\circ}$  and baseline is from 0 to 160 m. Accordingly, this data set provides PolInSAR images for estimating canopy heights in Gabon Lope National Park. Both data sets were measured on 25th February 2016 by NASA collaboration with ESA and the Babon Space Agency.

The study area is hilly terrain with different slopes, rich forest resources of various species, and tree heights. To accurately estimate the forest height of the study area, the project published LiDAR measured data corresponding to each forestry part. LiDAR measured data will be used as a reference to evaluate the accuracy of the proposed method. LiDAR data was downloaded from the Distributed Active Archive Center for Biogeochemical Dynamics (ORNL DACC).

Fig. 10 (a) shows the optical image of Lope National Park, Gabon from Google Earth, providing the positions of the 12 plots within the selected study area. The survey areas were placed in optical images from google earth (white rectangles), and the Plot ID of these survey positions are denoted as FA1, FA2, and FA3. All of the ground survey positions were conducted by the working group at Lope National Park. From Fig. 10(a), it can be seen that the study regions include different forests in term tree height, tree density, species. The dimension of each region approximates 575x685 pixels, which are divided into four equal parts. For simplicity in representation as well as performance evaluation of proposed method, these sub-areas are denoted by sub-IDs of FA1s01, FA1s02, etc. as shown in Table 5.

Fig. 11 (a) shows the estimated forest heights of the proposed method for the FA1, FA2, and FA3 study areas, respectively. The FA1 study area shows that the height of the forest is highly concentrated at approximately 40 m. However, there are rivers and bare lands included in this area, so the average tree height of FA1 is lower than that of the rest of the study areas. The FA2 and FA3 study areas show more uniform tree heights and densities than FA1, and most estimated tree height is usually higher than 40 m. In addition, the 2D results obtained in Fig. 11 (a) show topographic similarity with selected study areas of optical image on google earth (Fig. 10 (a)).

Fig. 11(b) represents the results of the forest height valuation of the 30 selected stands for each tested area. Compared with the tree height value obtained from LIDAR, the root-mean-square error (RMSE) of the proposed method for three selected regions are 2.63, 2.91 and 3.26 m, respectively. Also, they get  $R^2$  about 0.815, 0.909 and 0.915 that correspond FA1, FA2 and FA3. The estimated tree height of FA1 largest diffused from fitting line, which denote the influence of more sparsely tree density. The inverted tree height of FA3 obtained best result with RMSE of 3.26 m and  $R^2$  of 0.915. However, the RMSE value of result in this region is higher than two remain areas. The main reason that the tree density and tree height in this area are largest, which may increase the ambiguity of volume only coherence. This is the main cause of the increase in RMSE value. However, these values are completely consistent with the actual conditions and it reflects the effectiveness of the proposed method.

Next, to give a better assessment of the results about forest estimating parameters of the proposed method, we

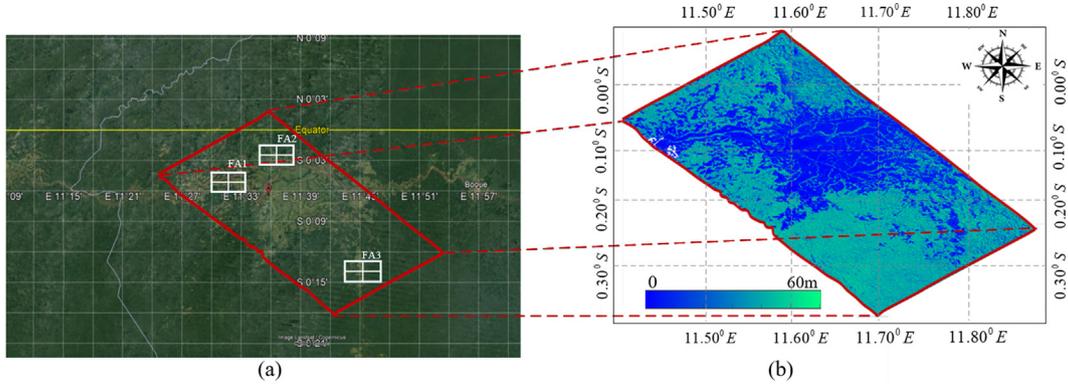


Fig. 10. (a) The optical image of Lope National Park, Gabon from Google Earth; (b) LiDAR image of Lope National Park, Gabon.

Table 5  
Results of forest parameters by the proposed method and LiDAR data set.

Forest Area	Sub ID	LiDAR height (m)	The forest parameters were estimated by the proposed method		
			Forest height (m)	Vertical wavenumber (rad/m)	Extinction value (dB/m)
FA1	FA1s01	24.36	21.17	0.089	0.182
	FA1s02	36.15	37.32	0.065	0.219
	FA1s03	19.24	17.62	0.137	0.166
FA2	FA1s04	34.68	33.16	0.071	0.301
	FA2s01	25.76	26.07	0.066	0.211
	FA2s02	37.05	33.69	0.091	0.317
	FA2s03	34.41	31.27	0.084	0.196
FA3	FA2s04	30.18	29.41	0.109	0.115
	FA3s01	35.65	32.24	0.078	0.261
	FA3s02	42.94	41.02	0.052	0.352
	FA3s03	36.62	33.15	0.075	0.252
	FA3s04	41.27	39.65	0.069	0.259

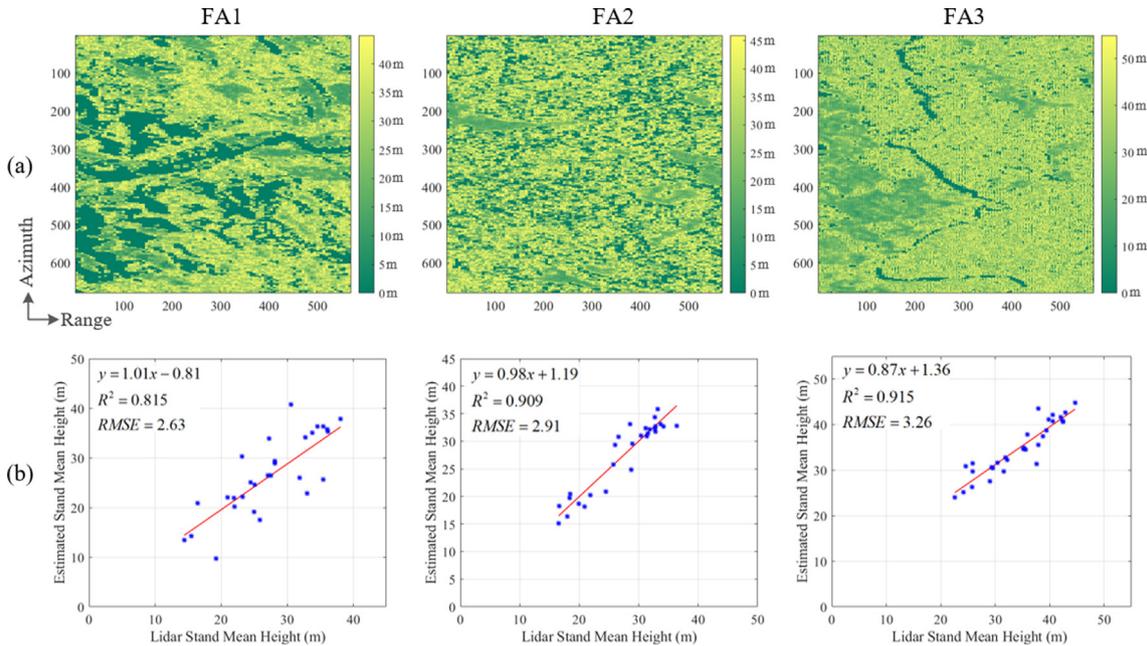


Fig. 11. (a) 2D Forest height mapping of proposed method; (b) Tree height estimation accuracy of proposed method.

subdivide the study areas into four equal parts (Fig. 10 (a)). Table 5 shows forest heights in the forest area sub of the LiDAR data and the proposed method. The tree height

results extracted by the proposed method are usually lower than the tree heights of the LiDAR data in the respective forest area subs (except FA1s02 and FA2s01 obtained the

better results). In addition, the parameters such as vertical wavenumber and extinction coefficient estimated by the proposed method are also presented in detail in Table 5. The forest parameters results estimated in Table 5 show high accuracy and reliability of the proposed method compared with the results of the LiDAR data. As a result, a more accurate inversion forest height can be obtained by proposed method.

## 5. Conclusion

A new method to retrieve the forest parameters by mean coherence set over heterogeneous vertical forest structure area is developed. In the proposed method, the terrain ground phase is extracted by combining the numerical range theory and adaptive total least square line fit method, while comprehensive search method with two constrain condition is used to extracting the forest parameters. Additionally, the vertical varying extinction coefficient is used to determine the volume only coherence, which makes proposed method more appropriate for estimating forest parameters in the dense forest areas. The proposed method achieved a higher performance than the previous methods when using the both simulated data and real data. Experimental results are shown that the proposed method not only enhance the reliable and stability in forest parameter inversion but also truly reflects the scattering process of radar waves in natural forest environment.

Three potential issues were found with the forest parameters estimation. First, the proposed method is still reasonable to retrieve the forest parameters for dense and homogeneous forest area because the wave extinction coefficient varies with the tree height. Second, it is stability to estimate forest height over flat as well as slope terrain. Third, it reduces some undesirable errors (phase disturbance, error model. . .), which lead to no overlapped ambiguity and bringing errors to forest height inversion. Beside, the complexity and computation time of the proposed method increases significantly due to the use of many loops and the addition of many binding conditions during the forest height estimation stage. It is also a disadvantage of the proposed method in forest parameters estimation, especially when calculating with large forest areas. In the future, we will try to improve the processing speed and reduce the computational complexity of the proposed method so that the measurements can be applied to large forest areas.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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