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TECHNICAL PAPER

Determine the dynamic parameters in mechanical system of the crab-shaped MEMS vibratory gyroscope

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Abstract

The inertial element, damping element and elastic element are the main components of a mechanical oscillation system. The values of these elements affect the resonant frequency of the mechanical system in a MEMS vibratory gyroscope. Besides, it is a necessary factor to determine the operating frequency of the mechanical system. In the mechanical system of the MEMS vibratory gyroscope, the elastic elements such as the elastic beams are directly connected with the proof-mass. The typical dynamic parameters of the oscillator system need to be determined to study the vibration characteristics of the MEMS vibratory gyroscope with some initial conditions. The construction of modeling and calculation of the dynamic parameters of a crab-shaped MEMS vibratory gyroscope which consists of a proof-mass with 2 in-plane degree-of-freedoms realized in this paper.

1 Introduction

The MEMS (Micro-Electro-Mechanical System) vibratory gyroscope (MVG) is a kind of complex micro-sensor. Its operation is based on the Coriolis principle to transfer the vibration energy of a rotation body from the primary mode into the secondary one (Acar and Shkel 2008; Shkel et al. 1999; Apostolyuk and Tay 2004). The amplitude of the secondary vibrational mode is proportional to the angular velocity of the rotation body which the MVG is integrated into. Thence, this sensor is used to detect and determine the angular velocity or rotational angle in the rotational motion. The applications of this sensor are common and known in the global position system to locate the body or combining with the acceleration sensor to apply on the game program in the smartphones (iPad, iPhone, etc.). The MVGs are known for some advantages over traditional gyroscopes by their small size, low power consumption, low cost, batch fabrication, and high performance (Acar and Shkel 2008; Shkel et al. 1999; Apostolyuk and Tay 2004; Alper and Akin 2004; Sayed et al. 2017).

Nowadays, MVGs are still attractive research objects interested in scientists. There are many mechanical structures with the different shaped beams of the MVG introduced in the previous researches (Acar and Shkel 2008; Shkel et al. 1999; Apostolyuk and Tay 2004; Alper and Akin 2004; Sayed et al. 2017; Preethi et al. 2012). The problem in this field focuses on constructing a new structure with some advantages of the performance, increasing the quality factor, or decreasing energy consumption. Besides, the analysis of the dynamic response of the mechanical system in the MVG is interested to optimize the structure. In order to perform this analysis by a numerical method, it is necessary to determine the dynamic parameters of the vibration system in a MVG. In some research about the MEMS actuators, the dynamic parameters of the mechanical system are determined by using an analytical method to evaluate the influence of the design parameters on the working stability of the electro-thermal V-shaped actuator (Hoang et al. 2020). These parameters are determined by the different techniques such as using the equivalent expressions, or simulation software. Each object is studied in the individual method to find out the resolution while the result is by not only the calculating method but also the geometric parameters of the model. There are a lot of configurations for the elastic beam of the single MVG for example the single beam (Alper and Akin 2004), T-shaped (Sayed et al. 2017; Preethi et al. 2012). In this paper, the single MVG model is proposed with the

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"Crab"-shaped of the elastic beams. Each beam consists of two perpendicular bars with suitable dimensions to allow the proof-mass to oscillate in the corresponding directions. Besides, the dynamic parameters of the vibration-mechanical system are calculated basing on both the analytical and simulation method. These parameters are used to study on the dynamic response of the proposed MVG by the analytical–numerical method in the next researches.

2 Construction the configuration

The oscillation pattern of the mechanical system in MVGs is depicted in Fig. 1a. A single MVG includes a proof-mass m suspended on the substrate by the bent elastic beams. These beams allow the proof-mass to freely vibrate in two perpendicular directions to create two major modes called primary and secondary mode (or driving and sensing mode) of the single MVG. The 3D model of the single MVG is shown in Fig. 1b. There are a lot of models for the elastic beam. In Fig. 1b, the proposed MVG includes four beams with one fixed ending and the guided other. Each beam is created from two perpendicular bars created the "L" shaped (or "Crab" shaped). This structure of the single MVG is called the "Crab" model. The dimension of the bar is defined as $L \times b \times h$, where L, b, h is the length, wide, and thickness of each bar respectively. The thickness of the bar is defined as the subtraction between the thickness of the SOI (silicon on insulator) wafer and the gap from the bottom face of the proof-mass to the substrate. The rest parameters influent to the mass and the stiffness of the vibration-mechanical system.

The differential motion equations of the vibration-mechanical system are described in some of the references (Acar and Shkel 2008) by the following:

$$M\ddot{X} + C\dot{X} + KX = F(t) \tag{1}$$

where, X, $\dot{X}$, and $\ddot{X}$ is the motion, velocity, and acceleration of the proof-mass in the vibrational directions respectively; M, C, and K is the mass matrix, damping matrix, and stiffness matrix respectively; $F(t)$ is the driving force applying on the proof-mass in the driving direction.

The solutions of the Eq. (1) show the dynamic response of the system to evaluate the performance of the mechanical system or optimize the mechanical structure for increasing the quality factor of the sensor. In order to solve these equations, it is necessary to determine the dynamic parameters of the mechanical introduced above. Thence, in the following section, the vibrational mass, the damping coefficient and the stiffness coefficient of the proposed mechanical system will be calculated by using both the analytical method and simulating method.

The dimensions of the proposed MVG are listed in Table 1.

3 Dynamic parameter of the mechanical system

3.1 Effective mass

Considering a mass m put on a beam which is fixed at one of the endings and free at the other one as shown in Fig. 2 (this beam is called a fixed-free beam). The dimensions of the beam are defined as $L \times b \times h$ with the rectangular section. In order to increase the accuracy of studying the vibration response of the system, it is necessary to not only determine the stiffness and the damping coefficient of the system but also calculate the vibrational mass taking into account the individual mass of the beam. The mass of the vibrational beam is converted at the free end of the beam in vibrational status and called as the "effective mass".

In the model shown in Fig. 2, the effective mass of the beam is calculated basing on the guaranty the total kinetic

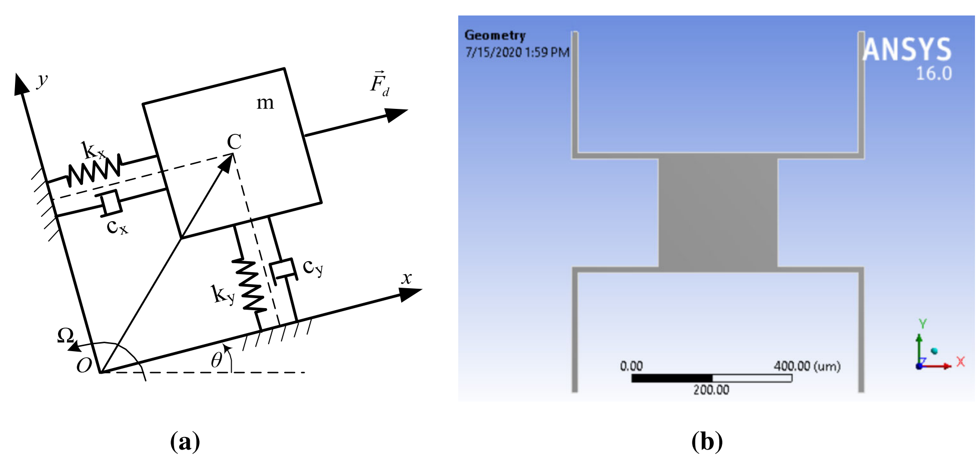


Fig. 1 Mechanical diagram **a** and 3D model **b** of the single MVG

Table 1 The parameters of the crab structure MVG

Parameter	Value
Density of silicon	2330 kg/m^3
Total dimension of the MVG ($x \times y$)	732 × 900 μm
Thickness of the MVG	30 μm
Wide of the beams	16 μm
Length of the driving beams	300 μm
Length of the sensing beams	200 μm
Dimension of the proof-mass ($a \times a$)	300 × 300 μm
Mass of the proof-mass	62.9104×10^{-10} kg
Mass of a crab beam	5.7709×10^{-10} kg

energy remains constant (the total kinetic energy of the whole vibrational system equal to the kinetic energy of the effective mass). Thence, the effective mass is determined as the followed expression:

$$m_{eff} = \rho_b \int_0^L \left[\frac{y(x)}{y(a)}\right]^2 dx = \frac{m_b}{L} \int_0^L \left[\frac{y(x)}{y(a)}\right]^2 dx \tag{2}$$

where ρ_b is mass density in the length of the beam; m_b is the mass of the whole beam; $y(x)$ and $y(a)$ is the displacement of the beam in the y-direction (see Fig. 3) at the considered point and the ending point, respectively.

Thence, the effective mass of the beam in the 3D model of the single crab MVG is defined as:

$$m_{eff} = \frac{33}{140} m_b. \tag{3}$$

And the total vibrational mass of the whole single crab MVG system is:

$$M = m + 4m_{eff} = m + \frac{33}{35} m_b. \tag{4}$$

Refer to the dimensions in Table 1, the vibrational mass of the system with the above expression is:

$$M = m_p + 4m_{eff} = m_p + \frac{33}{35} m_b = 68.35 \times 10^{-10}(\mathrm{kg}).$$

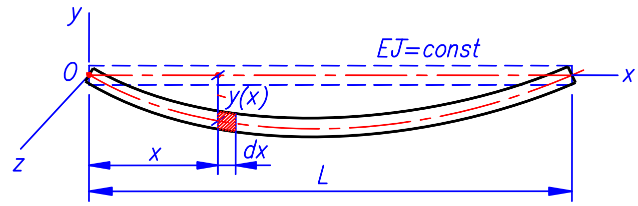


Fig. 3 The displacement of the bending beam

3.2 Equivalent stiffness

Considering the fixed-free beam in Fig. 2b, in case of the assumption that the behavior of the material is linear. The equivalent stiffness of the beam is calculated basing on the known analytical expressions in the lectures as following (Acar and Shkel 2008; Allen 2005):

$$k = \frac{3EJ}{L^3} \tag{5}$$

E—elastic module (Young's module) of the material, J—cross-section area moment inertia of the beam.

Figure 4 shows some different structures of the beam used in the MEMS devices.

The equivalent stiffness of the crab beams in Fig. 4a is determined corresponding as (Allen 2005):

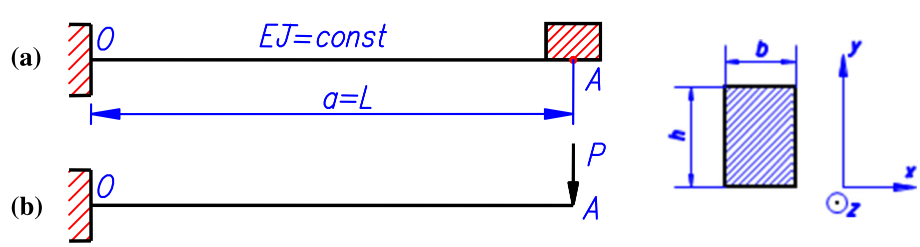
$$k_x = \frac{Eh}{4}\left(\frac{b_2}{L_2}\right)^3 \left(\frac{4L_2 + \alpha L_1}{L_2 + \alpha L_1}\right);\ k_y = \frac{Eh}{4}\left(\frac{b_2}{L_2}\right)^3 \left(\frac{4\alpha L_1 + L_2}{\alpha L_1 + L_2}\right) \quad \text{with } \alpha = \left(\frac{b_2}{b_1}\right)^3. \tag{6}$$

And the equivalent stiffness of the folder beam in Fig. 4b is determined as followed:

$$k_x = \frac{Eh}{4}\left(\frac{b}{L}\right)^3 \left(\frac{\alpha L' + 2L}{2\alpha L' + L}\right) \quad \text{with } \alpha = \left(\frac{b}{b'}\right)^3. \tag{7}$$

Besides, the equivalent stiffness can be determined by using the simulated method based on the assumption of the linear behavior of the material (see Fig. 5). The stiffness coefficient is defined as the ratio of acting force F and the displacement δ of the structure at the point where the force applies:

Fig. 2 Single beam model with a vibrational mass



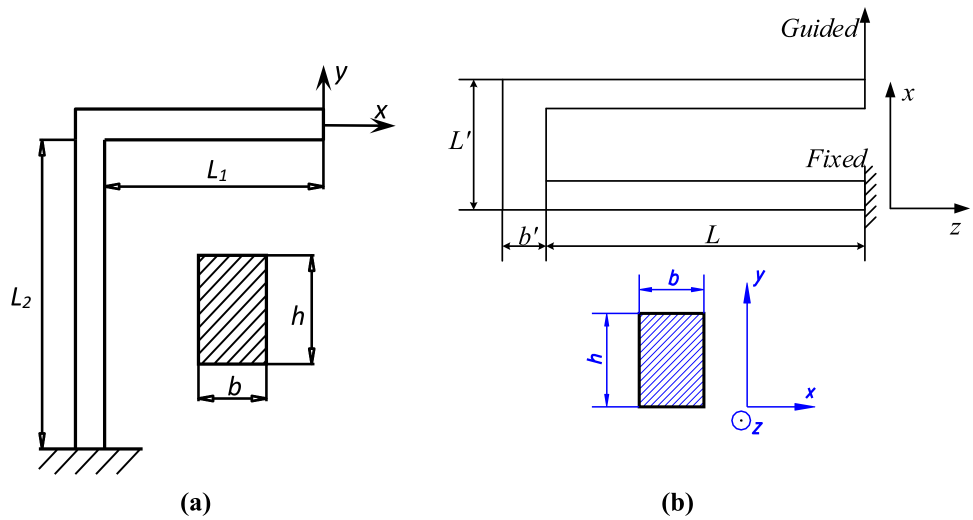


Fig. 4 The crab (**a**) and folder (**b**) beam

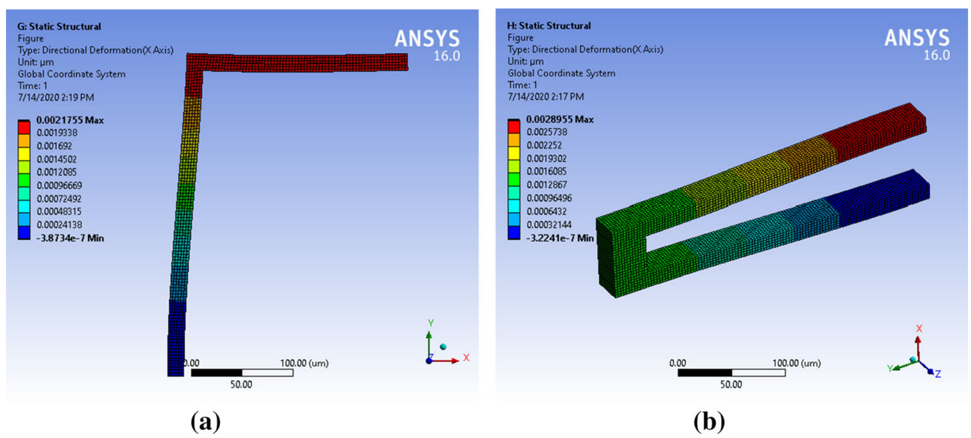


Fig. 5 Simulating the deformation of the crab (**a**) and folder (**b**) beam

$$k = \frac{F}{\delta}. \tag{8}$$

The displacement δ is determined taking into account the individual mass of the beam simulating the acting force and gravity force applied to the structure. Thence, the operating conditions of the structure are more real and the result is found more correctly. For example, consider the model of the single fixed-free beam in Fig. 2 with the same dimension and material (silicon—the common material in MEMS) but in two different calculating methods, the obtained results calculating equivalent stiffness depending on the length beam are listed in Table 2 where K_A and K_S is the equivalent stiffness coefficient of the single fixed-free beam using analytical and simulation method, respectively.

Table 2 The dependence of the equivalent stiffness on the length of the single beam

Length beam (µm)	300	350	400	450	500	600
K_A (N/m)	192.28	121.09	81.12	56.97	41.53	24.04
K_S (N/m)	192.8	121.42	81.34	57.12	41.64	24.09
Error (%)	0.27	0.27	0.27	0.26	0.25	0.22

The result shows that the stiffness depends clearly on the length of the beam. The values obtained using two different methods are the nearly same with 0.27% higher in the simulation method. This characteristic allows using the simulation method to determine the stiffness coefficient for the mechanical structure, especially for complex structures to guarantee both the accuracy and reducing time calculating. Thence, the equivalent stiffness of the crab beam and folder beam in Fig. 4 can also be determined using the simulation method through the expression (8). The initial and boundary condition is established as similar to the single beam above but one of the bars is guided to create the fixed-guided beam model.

In the same boundary condition, the dependence of the equivalent stiffness k on the length of the beam is shown in Fig. 6. The equivalent coefficient of the single beam is higher than the remaining two models with the same length

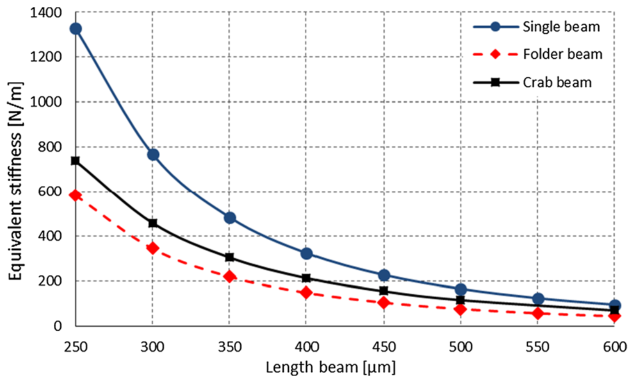


Fig. 6 Dependence of the stiffness on the length beam

and cross-section. The longer the length of beams is, the more nearly the same value of the stiffness is. The variation of the stiffness coefficient of the single beam is more intense than the two other types of the beam. The length of the single beam increases two times (from 250 to 500 µm), the stiffness reduces about eight times. In the case of the dimension as the above structure of the proposed crab MVG, the stiffness coefficient in x and y-direction is k_{x-} = 459.65 N/m, k_y = 1158.96 N/m, respectively.

3.3 Conversion damping

Considering the single beam model in Fig. 2, the conversion damping coefficient (C) is determined based on guaranteeing the damping force caused by the conversion damping coefficient equal to the total damping force of the air F_c to the whole movement structure as the followed expression:

$$C\dot{y} = F_c. \tag{9}$$

The air damping force acting on the structure consists of two components as: the air damping force on the front surface of the proof-mass (F_{c-p}), and the air damping force on the front surfaces of the beams in moving (F_{c-b}).

Consider an element beam dx moving in the y-direction, the air damping force acting on the moving beam is defined as:

$$dF_c = \Delta P h dx \tag{10}$$

where ΔP is the differential pressure between the front and the behind surface of the considered element beam. This parameter is defined as following (Christian 1966):

$$\Delta P = 4\sqrt{\frac{2}{\pi}}\dot{y}_x P_0\sqrt{\frac{M_a}{R_0 T_0}} \tag{11}$$

with M_a = 28.97 (kg/kmol) is mass of the air molecular; R_0 = 8.314 (J mol^{-1} K^{-1}) is ideal gas constant; T_0 = 25 °C is room temperature; P_0 = 101325 (Pa) is air pressure at room temperature.

Thence, the air damping force acting on the front surface of the moving beam becomes as follows:

$$F_{c-b} = 4\sqrt{\frac{2}{\pi}}P_0\sqrt{\frac{M_a}{R_0 T_0}}h\int_0^L \dot{y}_x dx. \tag{12}$$

The air damping force acting on the front surface of the proof-mass is described as the express:

$$F_{c-p} = 4\sqrt{\frac{2}{\pi}}P_0\sqrt{\frac{M_a}{R_0 T_0}}ah\dot{y}. \tag{13}$$

The conversion damping in the y-direction is calculated from the expression (9) becomes:

$$C = 4\sqrt{\frac{2}{\pi}}P_0\sqrt{\frac{M}{R_0 T_0}}h\left(a + \int_0^L \frac{y_x}{y_L}dx\right). \tag{14}$$

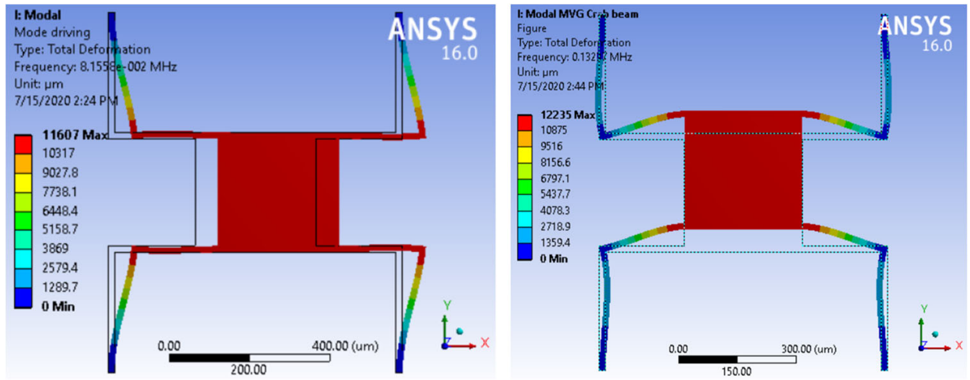
By the same analysis, the conversion damping in the x-direction is also determined while the system vibrates in the x-direction.

4 Dynamic parameter of the mechanical system

The dynamic parameters determined above are used to study on the mechanical response of the MVG system. One of the responses is the resonator frequency of the mechanical system. This frequency is defined as the operating frequency, where the amplitude of the driving and sensing vibration reaches the maximum value. There are two resonator frequencies for the 2-degree-of-freedom mechanical system in two major directions. They are determined by solving the free vibration equations with the analytical analysis or the modal analysis in the simulation software. By using the modal analysis modulus in ANSYS software, the two major vibration modes and the corresponding resonant frequencies are specified as showed in Fig. 7.

The resonant frequencies are determined in software as 81.55 and 132.17 kHz for driving and sensing mode, respectively. They depend on the stiffness and the moving mass of the mechanical system (includes the mass of the proof-mass and the effective mass of the beams). In the analytical method, the equivalent stiffness of the system included four crab beams with the dimensions in Table 1 is $K_x = 4k_x$ = 1838.6 N/m and $K_y = 4k_y$ = 4635.85 N/m in the driving and sensing, respectively. The total vibration mass of the proposed MVG is 68.35 × 10^{-10} (kg).

The amplitude-frequency response of the system is shown in Fig. 8 by specifying the harmonic response in ANSYS software. Thence, the driving and sensing



(a) Driving mode **(b)** Sensing mode

Fig. 7 Two modes of the proposed MVG

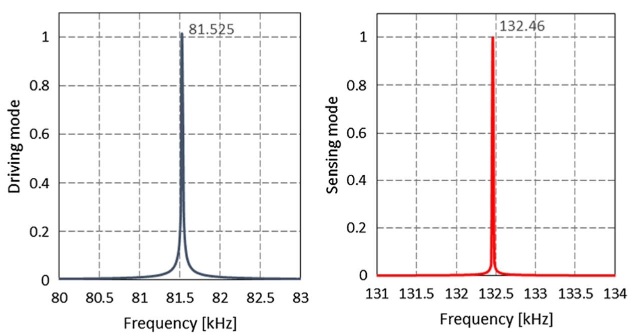


Fig. 8 Two major frequency of the proposed MVG

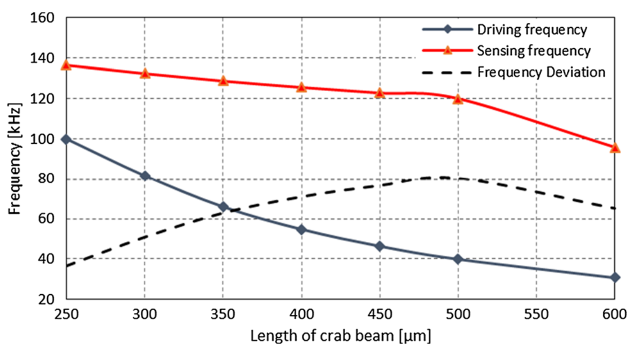


Fig. 9 Frequency deviation of the proposed MVG

frequency is confirmed again as 81.525 and 132.46 kHz for driving and sensing mode, respectively.

The length of the beam affects the resonant frequency in the directions. The longer the length beam is, the lower the equivalent stiffness so the lower frequency is. The frequency deviation between two major modes needs to be considered with a suitable value to ensure the bandwidth of the MVG [about less than 100 Hz (Weinberg and Kourepenis 2006)]. However, the result shows that the value of the frequency deviation reaches the value much greater than expected while the length of the crab beam is variable from 250 to 600 µm (see Fig. 9). This value larger 360–800 times than the expected value. So, it is necessary to optimize the dimensions of the crab beam to reduce the deviation frequency to the expected value. This issue will be carried out in the next researches.

5 Conclusion

Study on the mechanical systems of MEMS devices is a necessary issue and needs to be solved carefully. The dynamic parameters in these systems are the basis to study on the dynamic responses, performance, bandwidth, accuracy, quality factor, … of the system. In this paper, a crab single MVG model with 3D configuration was proposed and the manner to determine the dynamic parameters of this mechanical system of the proposed MVG was given. The vibration mass was calculated taken into about the mass of the elastic beams in the configuration to take out the exact asymptotic value. The stiffness and damping coefficient also was determined by using the analytical expressions and the simulation software to obtain similar values with a 0.27% error. The major frequencies were determined in both the analytical and simulation method and reach 81.5 and 132.4 kHz for the driving and sensing direction, respectively. These values of the frequencies are still far from the expected one, so the optimizing issue needs to be carried out to ensure the matching frequency in both the driving and sensing direction of the proposed MVG.

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