Detection and Parameters Estimation of Binary Phase Shift Keying Signals in Low Signal to Noise Ratio

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Abstract: *In this paper, an algorithm is proposed to estimate the carrier frequency and symbol rate of binary phase shift keying modulation with Barker codes in a high noise environment. The proposed algorithm is based on cross-correlation functions between unknown and reference signals. This algorithm divides into two estimators. The first one is used to estimate the carrier frequency and second one is to estimate the symbol rate with different types of binary phase shift keying signals with Barker Codes (7, 11, and 13). The superiority of the proposed algorithm is demonstrated by the analysis and simulations. The simulation results show that the required value of signal-to-noise ratio is to obtain the perfect probability of correct estimations, where (*$\mathrm{P_{ce}} = 100\%$*) is* $\mathrm{SNR} \geq -26$ *dB for all test BPSK signals. It was confirmed that the algorithm is suitable for detecting and estimating the carrier frequency and symbol rate of the binary phase shift keying signals in a low signal to noise ratio.*

1. Introduction

Intra-pulse modulation have many applications in radar, sonar and communication systems. One of the most important class of such signals is the frequency modulation of the signals, as well as the phase modulation of the signals in modern radar systems. The most famous family of phase modulation signal is called BPSK. In radar and communication systems, BPSK signals are commonly used in the form of Barker codes (7, 11 and 13) to solve problems with resolution and anti-jamming.

On the other hand, carrier frequency and symbols play an important role in demodulation. In radar and communication applications, two parameters are needed to decode these signals. So, the problem of carrier frequency and symbol rate estimation of BPSK signals has received great attention in recent years. Early approaches were focused on the analysis of the single-component BPSK signal. The first method was known that a cepstral analysis and the required value of SNR to achieve 100 percent of correct estimation is $\mathrm{SNR} \geq 15\,\mathrm{dB}$ [1]. The later, new method used to estimate symbol rate is called the quadrature estimator. This method is suitable for estimating the symbol rate of the BPSK signal at $\mathrm{SNR} \geq 3\,\mathrm{dB}$ [2]. The third group of methods for estimating the carrier frequency and symbol rate of BPSK signals is based on Haar wavelet transform [3]. This method has the best performance (100 percent probability of correct estimation) for input BPSK signals with $\mathrm{SNR} \geq -2$ dB. The other method for estimating carrier frequency and symbol rate of BPSK signals is the blind cyclostationary estimator [4].

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The method approaches perfect probability of correct estimation for BPSK signals with $\mathrm{SNR} \geq -4$ dB. It can be seen that all the above-mentioned methods are suitable for estimating the carrier frequency and symbol rate of BPSK in noise. However, they have a problem with the parameter estimating BPSK signals in a lower SNR (where $\mathrm{SNR} = -5$ dB or lower). To deal with the problem, this paper is focused on examining the effect of the new algorithm on estimating the carrier frequency and symbol rate of BPSK signals with different types of Barker codes. The main idea of the method is employed in a cross-correlation function to find out the BPSK signals in a low SNR. The main task of this paper is focused on determining the minimum value of the SNR at which the proposed method still obtains a perfect probability of correct estimation. Firstly, it is used to estimate the carrier frequency and secondly for estimating the symbol rate of the BPSK signals. Next, the accuracy of these estimations is calculated against the SNR for each type of BPSK signals to find out the minimum value of the SNR at which the method still obtains 100 percent of correct estimation for all test BPSK signals.

A theoretical description of the cross-correlation function is presented in Section 2. In Section 3, algorithm steps are proposed for BPSK parameter estimation. In Section 4, the simulation results are presented and analyzed - test results with different SNR levels and a different types of BPSK waveform to determine the minimum value of SNR at which the method was able to detect signals and estimate their parameters. And finally, in Section 5, the main conclusions are drawn from the summarized simulation results.

2. Theoretical Description of Cross-Correlation Function

The cross-correlation function is a comparison between two waveforms and the measurement of the similarity. It is a time-domain analysis methodology this is particularly used in determining periodic signals hidden in the noise, determining coherence between random signals and determining signal sources and their transmission times.

In radar signal processing, the cross-correlation function is understood as the comparison between a signal and a known reference signal or more precisely: their degree of coincidence as a time-shift function between these two signals. It is a function of the propagation time between the signals whose mathematical equation is different for analog and digital signals. For analog signals, it is a function of the time of the sum of the matching areas of the two waveforms:

$$C_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt \tag{1}$$

For discrete or digital signals, it is a time function of the number of corresponding partial pulses:

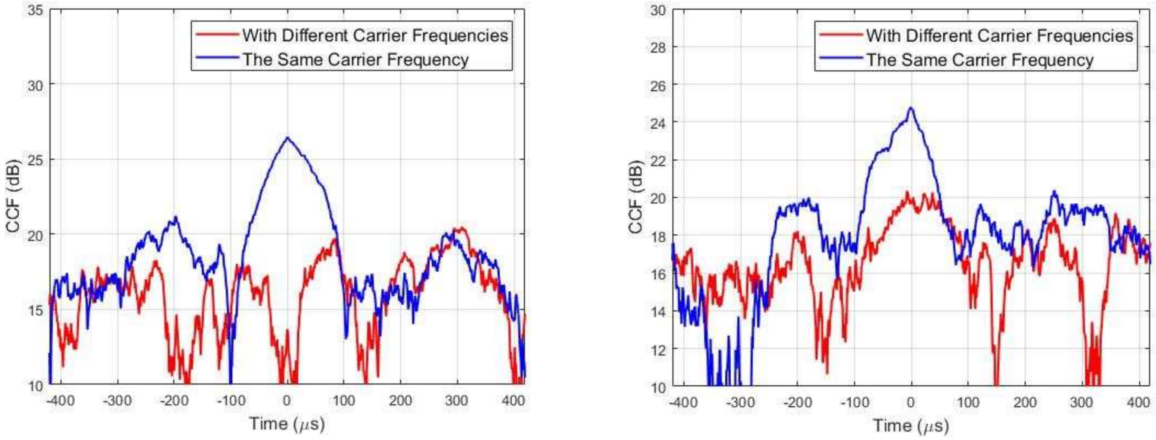
$$C_{xy}(m) = \sum_{n=-\infty}^{\infty} x_n\, y_{n-m} \tag{2}$$

The reference signal is usually “normalised”, i.e. the conditions of origin are standardized or even based on an ideal mathematical model. The amplitudes of the result, on the other hand, are not standardized, i.e. the maximum agreement is less than 1. If the shape of the signal to be compared is identical to the reference, this is called auto-correlation. A special case can be used to detect periodicities in a mixture of signals.

A practical application of the cross-correlation function is, for example, the matched filter as well as the method of distance measurement in a noise. Alternatively, the cross-correlation function is used as a method of identifying signals in noise [5].

3. The Proposed Method

As seen in the previous section, one of the common applications of the cross-correlation function is to detect an unknown signal in a mixture of signals and noise. Fig. 1 shows a plot of the cross-correlation function between two BPSK signals with Barker Codes 7 at different levels of SNR.

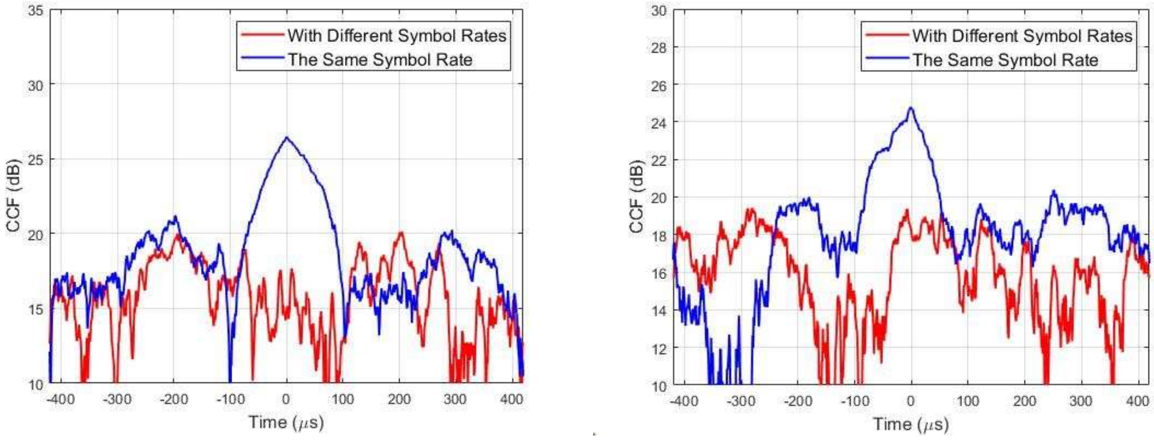


(a) SNR = -8 dB.

(b) SNR = -13 dB.

Figure 1. The CCF of two BPSK signals at different levels of SNR (a) SNR = -8 dB, (b) SNR = -13 dB.

The blue line is the cross-correlation function (CCF) of two signals which have the same carrier frequency, and the red line is the CCF between two signals that have different frequencies. Although the SNR is ultra-low ($\mathrm{SNR} = -13\,\mathrm{dB}$), the simulated results show that when two signals have the same carrier frequency, their CCF has the highest value when time $t = 0\ \mu\mathrm{s}$. The same case for BPSK signals with different symbol rates and the same carrier frequency, their CCF has a lower value than when they have the same symbol rate (see Figure. 2).



(a) SNR = -8 dB.

(b) SNR = -13 dB.

Figure 2. The CCF of two BPSK signals at different levels of SNR (a) SNR = -8 dB, (b) SNR = -13 dB.

From these results, we propose a new method for estimating the parameters of BPSK signals based on the theory of the cross-correlation function between the unknown and reference signals. The proposed method is divided into two parts. The first one is used to estimate carrier frequency f_c and the second one is for the symbol rate estimation f_b of BPSK signals. The basic idea behind the method procedure is written as below. The simulation parameters are given in Table I.

1. Generating reference BPSK signals $s(t)$ on wideband frequency B . The reference BPSK signal is expressed by the equation:

$$s(t) = [s_1(t), s_2(t), s_3(t), \cdots s_{n-1}(t), s_n(t)] \qquad (3)$$

   where n is the number of BPSK signals.
2. Calculating the spectrum $S_u(\omega)$ and $S(\omega)$ of the unknown $s_u(t)$ and reference BPSK signals $s(t)$ by using FFT.
3. Calculating the cross-correlation function between the unknown and reference signals via FFT by the equation:

$$R(t) = F^{-1}\{S(\omega) S_u^*(\omega)\} \qquad (4)$$

   where $S_u^*(\omega)$ is the complex conjugate of the $S_u(\omega)$, and F^{-1} is the inverse FFT.
4. Finding the peak value of the CCF to detect and estimate the carrier frequency of the unknown BPSK signal.
5. Generating new reference BPSK signals $s_n(t)$ with the known carrier frequency f_c which is estimated above and different symbol rates.
6. Calculating the FFT of the new reference BPSK signals. Then calculate the new CCF $R_n(t)$ between the unknown and new reference BPSK by using Equation (4).
7. Finding the peak value of the new CCR $R_n(t)$ to achieve the symbol rate of the BPSK signal.

Table I. Simulation parameters.

	Parameters	Value
Unknown BPSK signal	Carrier frequency (MHz)	1
	Barker Codes	7
	Bit duration (μs)	1680
	Initial phase (rad)	$\pi/3$
Frist reference BPSK signal	Carrier frequency (MHz)	0.1 ÷ 2
	Barker Codes	7
	Bit duration (μs)	1680
	Number of reference signals	100
Second reference BPSK signal	Carrier frequency (MHz)	1
	Barker Codes	7
	Bit duration (μs)	70 ÷ 6700
	Number of reference signals	100

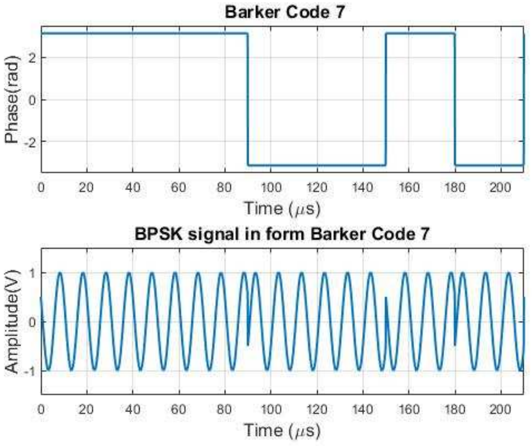
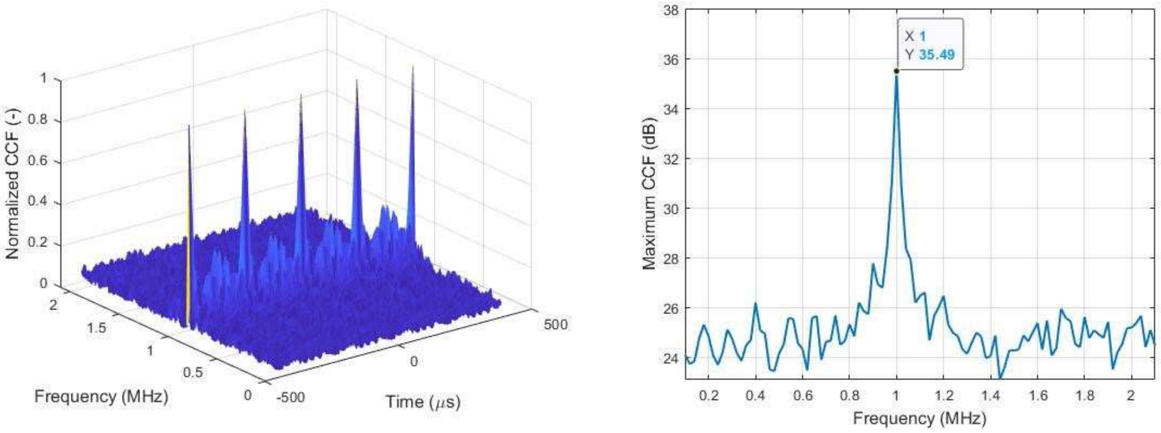


Figure 3. BPSK signal in form Barker Codes 7.

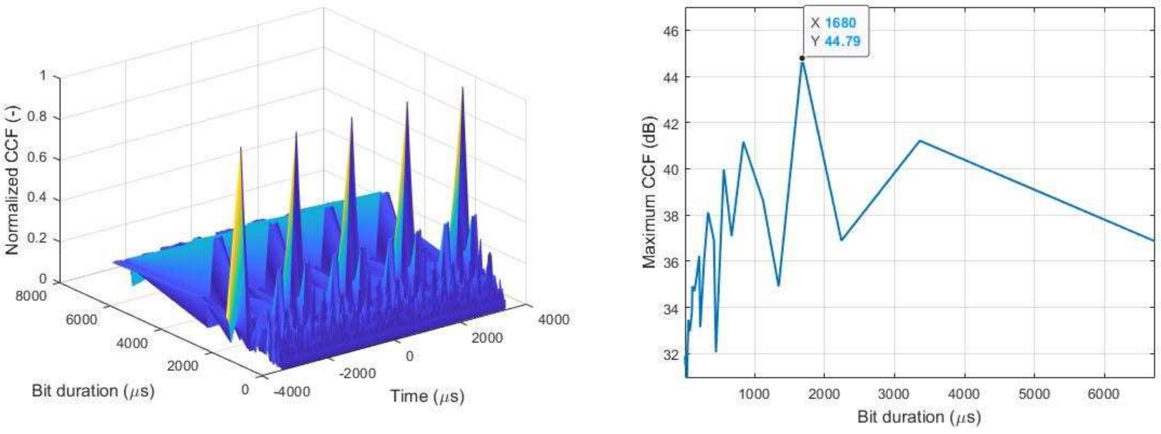
Fig. 3 shows plots of BPSK signals in the form of Barker Codes 7 against time. According to the above theoretical description, in order to estimate the carrier frequency of the unknown signal, the index of the reference signal should be determined with the highest value of CCF with the unknown BPSK signal (assume that, the frequency of the unknown signal is within the frequency range of the reference signal). The CCF between the unknown and first reference BPSK signal is shown in Figure 4(a). The maximum value of the CCF versus the index (i.e. the carrier frequencies of reference BPSK signals) are shown in Figure 4(b). It shows that the highest value of the CCF is $R(t) = 35.49\,\text{dB}$ at the frequency $f_c = 1$ MHz. It means that the unknown BPSK signal is close to a reference signal which has the carrier frequency $f_c = 1$ MHz. In other words, the carrier frequency of the unknown BPSK signal was estimated by using the first estimator ($f_c = 1$ MHz).



(a) 3D graph.

(b) Peak value of CCF versus carrier frequency.

Figure 4. The CCF between the unknown and first reference BPSK signal (a) 3D, (b) Peak value of CCF versus carrier frequency.



(a) 3D graph.

(b) Peak value of CCF versus bit durations.

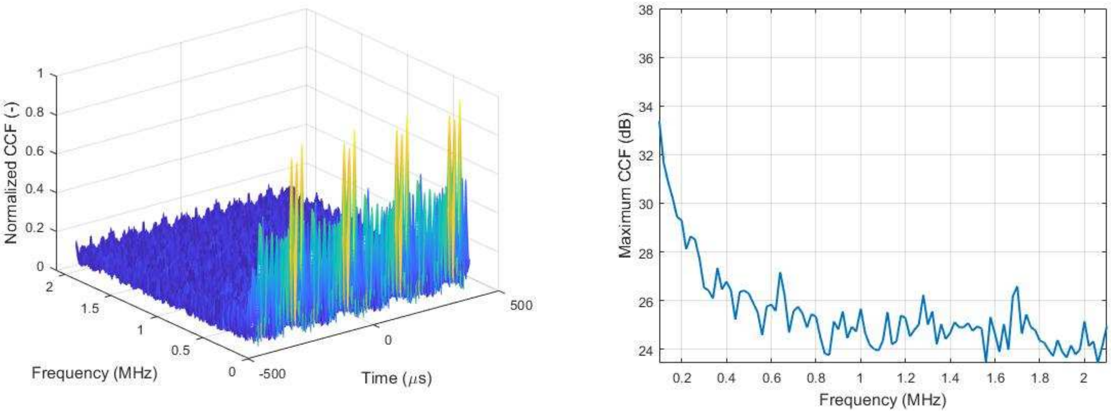
Figure 5. The CCF between the unknown and first reference BPSK signal (a) 3D, (b) Peak value of CCF versus bit durations.

The next step of this paper is to generate a new reference BPSK signal at the estimated carrier frequency with different symbol rates and then to calculate the new CCF between the unknown and these reference signals. As was the case above with estimating the symbol rate of the unknown BPSK signal, only the index of reference signal needs to be determined with the highest value of CCF with the unknown signal. The CCF between the unknown and second reference signals in 3D as shown in Figure 5(a). Fig. 5(b) shows the peak values of the CCF versus the index (i.e. bit durations of reference BPSK signals). It shows that the maximum value of CCF is $R(t) = 44.79$ dB at the bit duration $T_b = 1680\mu s$. The estimated parameters of BPSK at the level of SNR = -10 dB is given in Table II. According to the simulation results, a new method was proposed for the detection and parameter estimation of the BPSK signal in the form of Bark codes. In the next section of this paper, the effect of this method needs to be examined with different levels of SNR, different BPSK signals in the form of Barker codes 7, 11, 13. In addition, the minimum value of SNR should be determined at which this method is suitable to obtain the perfect probability of correct estimations ($P_{ce} = 100\%$) on signal analysis.

Table II. Estimated parameters of BPSK signal at SNR = -10 dB by running the system for 200 loops.

Parameters	Simulation values	Estimated values	Relative error (%)	RMSE (-)
Carrier frequency (kHz)	1000	1000	0	0
Bit duration (μs)	1680	1680	0	0

The last step in this section is provided simulate with a BPSK signal, whose carrier frequency or bit duration is outside the observed frequency range. The simulation results show that, the estimated frequency of the unknown BPSK signal is equal to the limit values of the observed frequency range (see. Figure. 5(b), upper or lower limit). In this situation, it is needed to extend the observed frequency range (resp. bit duration) and recalculate the frequency of the unknown signal.



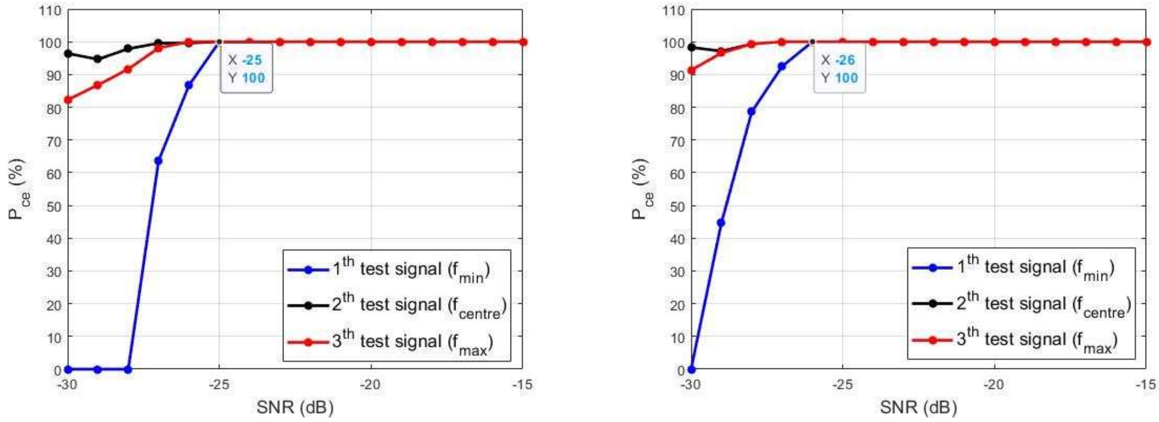
(a) 3D graph. (b) Peak value of CCF versus carrier frequency.

Figure 5. The CCF between the unknown and first reference BPSK signal (a) 3D, (b) Peak value of CCF versus carrier frequency.

4. Simulation Results

In the previous section, the theoretical description of the proposed method was explained. This section examines the effect of the proposed method for different simulation parameters of BPSK signals in the form of Barker codes 7, 11 and 13 by using the program MATAB. In order to verify the performance of this method, Monte Carlo simulation is performed for all tests of BPSK signals by running the system for 200 loops for an SNR range from -30 dB to -15 dB. The probability of correct estimation P_{ce} is calculated at the end of 200 loops. All tests are performed on the condition that the frequency and bit duration of the unknown BPSK signal is within the observed frequency and bit duration range.

The first simulation determines the required value of SNR at which the first estimator is still suitable for detecting the BPSK signal in the form of Barker Codes 7, i.e. for estimating its carrier frequency. The carrier frequency of the unknown BPSK signal was set at the minimum f_{min}, center f_c and maximum values f_{max} of the carrier frequencies of the reference BPSK signals.



(a) Barker Codes 7.

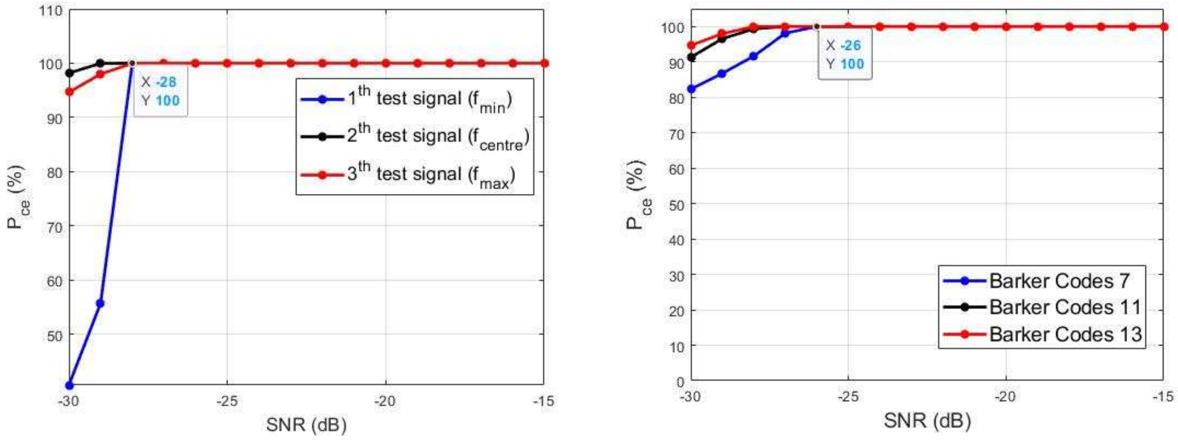
(b) Barker Codes 11.

Figure 6. The effect of the first estimator depends on SNR (a) Barker Codes 7, (b) Barker Codes 11.

Fig. 6(a) shows a plot of the probability of correct estimation versus the levels of SNR of the test BPSK signals in form of Barker codes 7 at special carrier frequencies. It shows that the effect of the method directly depends on the carrier frequency of test BPSK signals. At the same value of $SNR = -28$ dB, this method gives the best results ($P_{ce} = 97.87\%$) for the second test BPSK signal (its carrier frequency is equal to the centre of the carrier frequency of the reference signals) (black line), followed ($P_{ce} = 97.87\%$) by a third test BPSK signal (red line). The poorest result ($P_{ce} = 0\%$) is for the first test BPSK signal (its carrier frequency is equal to the minimum carrier frequency of the reference BPSK signal) (blue line). Altogether, the required value of SNR is $SNR \geq -25$ dB at all carrier frequencies of the unknown signals in order to obtain a perfect probability of correct estimation ($P_{ce} = 100\%$) for analysing BPSK signals in the form of Barker Codes 7. The same case is $SNR \geq -26$ dB for Barker codes 11 and $SNR \geq -28$ dB for Barker Codes 13 at all carrier frequencies of unknown signals when estimating the carrier frequency of the BPSK signal in the form of Barker codes 11 and 13 with only the difference being the value of SNR to achieve a 100 percent correct estimation. (see Figure 6(b), 7(a)).

Fig. 7(b) shows the effect of this method for estimating the carrier frequency of BPSK signals in the form of Barker codes 7, 11 and 13 against the SNR. The simulation results show that the proposed method performs best having a 100 percent correct estimation at SNR = -28 dB for Barker codes 13 (red line), followed by Barker codes 11 (black line) and 7 (blue line), requiring an additional SNR of 2 dB and 3 dB to achieve a perfect estimation.

Overall, the simulation results above show that the required value of SNR is $SNR \geq -26$ dB in order to obtain a 100 percent correct carrier frequency estimation for all test BPSK signals in the form of Barker Codes 7,11 and 13. It means that for an input BPSK signal with $SNR \geq -26$ dB the effect of the method doesn't depend on the carrier frequency and the types of Barker codes which were used for this BPSK signal.



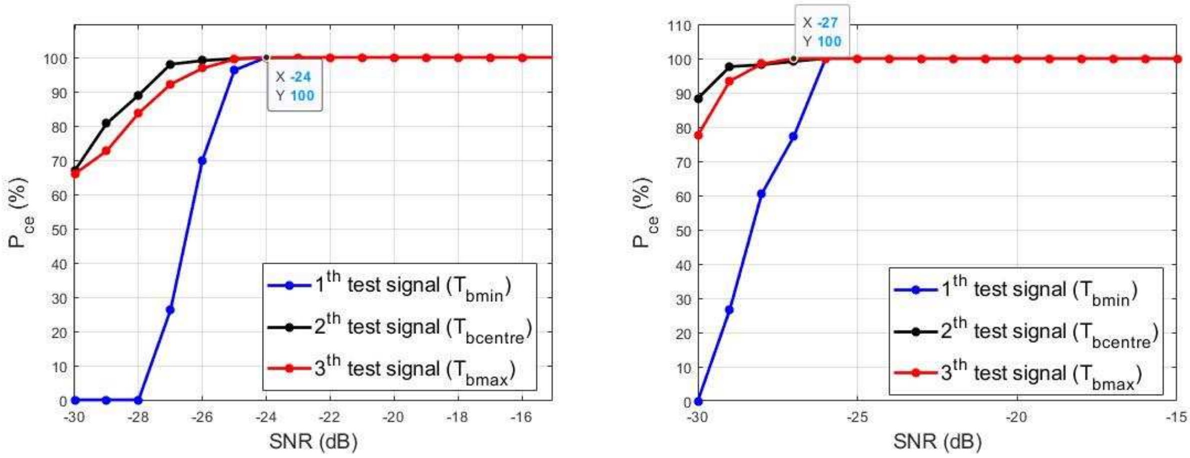
(a) Barker Codes 13.

(b) All types of Barker Codes.

Figure 7. The effect of the first estimator depends on SNR (a) Barker Codes 7, (b) All types of Barker Codes.

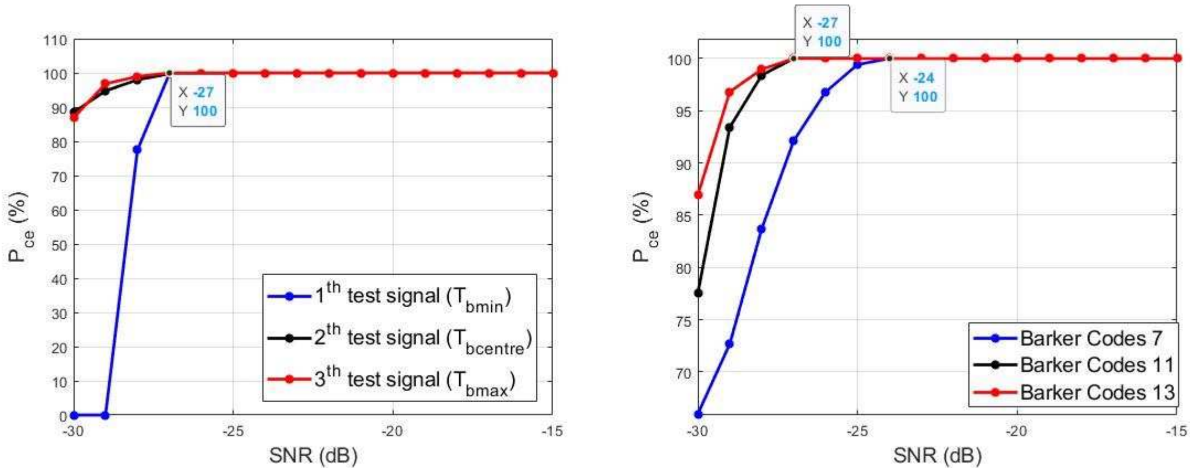
Similar to the first step, in this step the minimum value of SNR is determined where the method is still suitable for obtaining a perfect probability of correct estimation ($P_{ce} = 100\%$) for the symbol rate analysis for all test BPSK signals in the form of Barker codes. The bit durations of test BPSK signal were set at the minimum $T_{b\min}$, centre $T_{bcentre}$ and maximum values $T_{b\max}$ of the bit durations of reference BPSK signals. The effect of this estimator for each test BPSK signal is shown in Figure 8.

It is seen that irrespective of the time parameter being estimated, the effect of the second estimator depends on the bit duration of the test BPSK signals. This estimator performed the best result for the second test BPSK signal in each form of Barker codes 7, 11, 13 (black line) (their bit duration is equal to the bit duration of the centre reference signals), followed by the third test signal (red line). The poorest performance is for the first test BPSK signals (blue line) (their bit duration was set on the lowest bit duration of reference BPSK signals) (see Figure 8(a), (b) and (c)). At the same time, the simulation results show that this estimator has the highest efficiency for BPSK signals in the form of Barker codes 13 (red line), then for the BPSK signal in the form of Barker codes 11 (black line). The lowest effect is for estimating the BPSK signal in the form of Barker codes 7 (blue line). The required value of SNR is $SNR \geq -27$ dB to achieve a perfect symbol rate estimation for the BPSK signal in the form of Barker codes 11 and 13 and the required value is $SNR \geq -24$ dB for BPSK in form of Barker codes 7 (see Figure 8(d)). The minimum value of SNR is $SNR \geq -24$ dB in order to obtain 100 percent probability of correct symbol rate estimation for all types of BPSK signals with all types of Barker codes. It means that for an input BPSK signal with $SNR \geq -24$ dB the method performs a 100 percent correct estimation of its symbol rate analysis. The effect of this estimator doesn't depend on the input value of the bit duration and types of Barker codes.



(a) Barker codes 7.

(b) Barker codes 11.



(c) Barker codes 13.

(d) All types of Barker codes.

Figure 8. The effect of the second estimator depends on SNR (a) Barker Codes 7, (b) Barker Codes 11, (c) Barker Codes 13, (d) All types of Barker Codes.

The summarized simulation results in this section have confirmed that the proposed method was more effective than the above-mentioned methods in [2, 3, and 4] of BPSK signal analysis in low SNR. The required value of SNR is -26 dB in order to obtain perfect results on carrier frequency analysis and is -24 dB on symbol rate analysis by comparing with the methods [2] (requiring value of SNR is SNR = -5 dB) for all test BPSK signals. However, the effect of the proposed method depends on the number of reference signals. A larger number of reference signals gives better results but presents problems with the number of calculations and running time. The optimization of this method will be presented later.

5. Conclusion

In this paper, a new, fast and highly-accurate parameters estimation method was presented based on the cross-correlation function for estimating parameters of BPSK signals in form of Barker codes at a low SNR. Firstly, the cross-correlation between the unknown signal and reference signals was designed to determine the carrier frequency of the BPSK signal. This was then used to estimate the symbol rate (i.e. the bit duration of BPSK signals). This paper examined the effect of the proposed method for BPSK signals in the form of Barker Codes 7, 11 and 13 with different carrier frequencies and symbol rate for a SNR range from

-30 dB to -15 dB. The simulation results show that, the minimum value of SNR is $\mathrm{SNR} \geq -26\,\mathrm{dB}$ in order to obtain a perfectly correct estimation $(P_{ce} = 100\%)$ for all test BPSK signals. It confirmed that the proposed method is suitable for detecting and estimating the parameters of BPSK signal at an ultra-low SNR. On the other hand, the effect of this method directly depends on the number of reference BPSK signals. A larger number of reference signals gives the highest probability of correct estimation but it has problems with the runtime of all systems. Therefore, further studies are need to optimise this method in order to reduce the number of reference BPSK signals and the runtime of all systems. In addition, the method can be used as an off-line signal analysis and the estimated parameters can be used as an input to the classifier network.

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