Focal Region Ray Tracing of Dual Spherical Reflector Antenna

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***Abstract* – 5G mobile communications require multibeam base station antenna to operate at millimeter frequency band. Therefore, dual spherical antenna is promising for 5G application. As for achieving high gain, optimum antenna configuration should be found out. In this paper, the focal region ray tracing method is developed. Firstly, analytical equations for the focal region ray tracing are clarified. The key equation of determining the caustic position is then solved by MATLAB program. In addition, some ray tracing results are shown to ensure the usefulness of the developed ray tracing method.**

***Keywords* — Dual spherical reflector, ray tracing, focal region, feed position, 5G mobile base station**

I. INTRODUCTION

The future radio wave technologies of 5G mobile base station are millimeter wave, multibeam and small cell size [1]. Dual spherical reflector antenna is an attractive candidate for multibeam performance at small geographical cell area covering millimeter waves transmission [2]. The antenna size is designed with diameter of 30 cm.

Previous dual spherical reflector antenna for multibeam is introduced by Ishimaru, et al. in [3]. In his configuration, spherical aberration could not be eliminated which could reduce the antenna gain. Therefore, optimum configuration of dual spherical antenna should be found out to overcome this drawback. In this paper, focal region ray tracing is developed to found out the optimal configuration.

First and foremost, ray tracing equations for dual spherical reflector are derived. Following it, reflection ray angle is then solved by MATLAB program. This focal region study is analyzed in the receiving mode as in [4]. It is a trial and error procedure to obtain the ideal feed region close to each other.

II. GENERAL GEOMETRY OF DUAL REFLECTOR

Antenna design parameters and its configuration are shown in Fig.1. Initial incidence ray onto main reflector, $M(r_2, \varphi_o)$ gives the angle to its reflection ray, $\varphi_o = 40°$. Few equations are acquired from ray tracing technique to find the reflection angle, γ on the convex sub-reflector plane expressed by polar coordinate $S(r_1, \beta)$ Sub-reflector height is set based on the examined caustic curve due to main reflector spherical aberration, with angle from its radius at $\beta_o = 7°$.

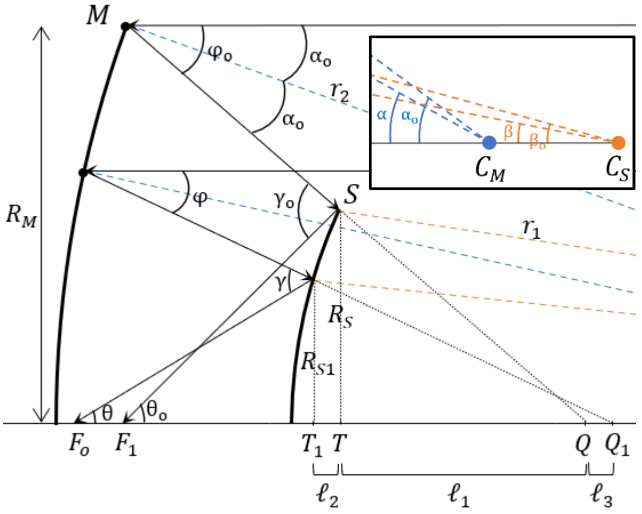

Fig. 1. Configuration of dual spherical reflector antenna

TABLE 1 PARAMETERS GIVEN AND SOLVED

	Main reflector	Sub-reflector	Focal
Initial ray condition	$R_M = 150mm,\ r_2$ $\varphi_o = 2\alpha_o$	$R_{S,}\ r_1$ β_o, γ_o	θ_o
To be solved	$\varphi = 2\alpha$	β, γ (MATLAB)	θ (result)

III. FOCAL REGION ANALYSIS EQUATION

Preliminary parameter is initialized for main reflector and have radius as follows:

$$r_2 = \frac{R_M}{\sin \alpha_o} \tag{1}$$

Reflected ray onto the sub-reflector geometry based on caustic ratio derivation in [5] provides us limit ratio of $R_S/R_M < 0.25$. It emphasizes on the position of sub-reflector vertex where second reflection occurs at R_S should be larger than the caustic point with radius

$$r_1 = \frac{R_S}{\sin \beta_o} \tag{2}$$

Triangles formed from sub-reflector profile due to the caustic lines from main reflector give us below derivation:

$$\ell_1 = \mathrm{QT} = \frac{R_S}{\tan \varphi_o} = \frac{r_1 \sin \beta_o}{\tan \varphi_o} \tag{3}$$

Incoming middle ray tracing onto $\mathrm{M}(r_2, \varphi)$ is computed to solve β for the next corresponding φ.

$$\ell_2 = TT_1 = r_1(\cos\beta - \cos\beta_o) \tag{4}$$

$$\ell_3 = QQ_1 = \frac{r_2}{2}\left(\frac{1}{\cos\alpha} - \frac{1}{\cos\frac{\varphi}{2}}\right) \tag{5}$$

Equation (3) to (5) have relation as (6) and solved in MATLAB to get angle β and γ. The rest is mathematical expression to find the coordinate geometry of sub-reflector by using line equation; $y = mx + c$.

$$R_{S1} = r_1 \sin\beta = (\ell_1 + \ell_2 + \ell_3)\tan\varphi \tag{6}$$

Reflection ray angle onto z-axis is as follows:

$$\gamma = 2(\theta + \beta) \tag{7}$$

IV. RESULTS AND DISCUSSION

Here, the computed dual spherical reflectors and the focal region by manual drawing is presented together with MATLAB result. Table 1 shows result for choosing ratio of 0.15, hence $R_S = 22.5$ mm at $\beta_o = 7°$. β and γ values solved in MATLAB are relatively agreed with the calculated values from manual drawing on a graph paper.

TABLE 2 SOLVED ANGLE AND MANUALLY OBTAINED ANGLE

	Equation solved by MATLAB		**Manual drawing**	
φ_o	**β_o**	**γ_o**	**β_o**	**γ_o**
40°	7°	66°	7°	66°
φ	**β**	**γ**	**β**	**γ**
35°	6.6188°	56.7623°	6.5°	58°
25°	5.2413°	39.5175°	5°	40°
15°	3.3330°	23.3340°	3°	23°
5°	1.1409°	7.7182°	1°	8°

$\mathrm{S}(r_1, \beta_o)$ is positioned once initial reflection ray having angle $\varphi_o = 40°$ is at the height of R_S. The next sub-reflector coordinates are calculated based on its corresponding β to find the reflection point onto it. Second reflection rays onto z-axis which determines the focal point are evaluated at equivalent angle respective to the radius line, r_1 .

In later part of ray tracing in MATLAB, two cases with different R_S are computed to illustrate different focal regions.

V. RAY TRACING IN MATLAB

Focal region ray tracing for sub-reflector at $R_S = 22.5$ mm is shown in Fig. 3 whereas Fig.4 at $R_S = 50$ mm. In this analysis, the focal region is examined at eight different incident angles from main reflector, within range 5° to 40°.

From the figures, it can be observed that as R_S is increased, the focal point moves closer to the main reflector dish. As the radius of sub-reflector is also enlarging along with its height, the focal region distribution between ray at $\varphi_o = 40°$ to 5° became closer to each other.

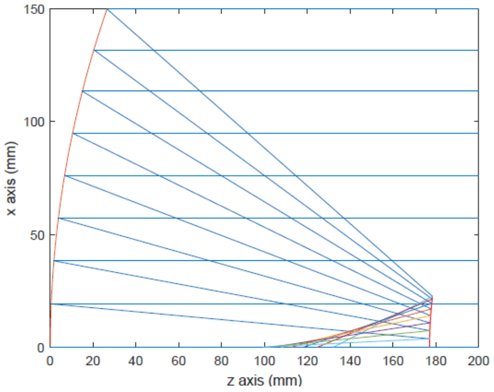

Fig. 2. 2D ray tracing for $R_S = 22.5$ mm

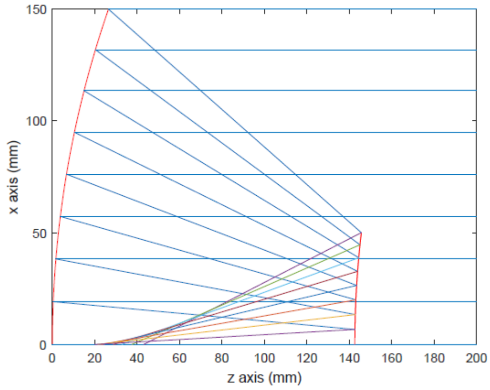

Fig. 3. 2D ray tracing for $R_S = 50$ mm

TABLE 3 PARAMETERS FOR SIMULATION

	Case 1	**Case 2**
R_S	22.50 mm	50.00 mm
r_1	184.62 mm	410.28 mm

VI. CONCLUSION

As for usefulness of this ray tracing, illustration results of focal region ray tracing are shown. The equation expression to get β and γ is derived and solved successfully to extrapolate into ray tracing diagram for the whole dual spherical reflector system.

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