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The design of a quasi-time optimal cascade controller for ball and beam system

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Abstract. In this paper, a methodology for designing quasi-time optimal cascade controller for ball and beam system is presented and the result is compared with LQR method as well as implemented on real system. Ball and beam system is a highly non-linear system, its parameters are difficult to estimate accurately and easily affect by disturbance. In designed method, a mathematical model describing the system is built, including a motor that creates a rotation of beam and is divided into two subsystems when synthesizing the cascade controller for the system. The first floor is the beam subsystem with the output is the angle as the set value for the second floor, which is ball subsystem. The controller is synthesized for each subsystem based on the quasi-time optimal control. The advantage of this method is the synthesizing control law with non-linear system. The simulation results show the effectiveness of the proposed design.

1. Introduction

Ball and beam system is a typical system used in study of modern and intelligent control theory. It consists of a rigid beam rotating freely in a vertical plane around the axis while a Ball rolls along the beam. It can be categorized into two configurations. The first configuration is often called as “Ball and Beam Balancer”, in which the beam is supported in the middle, and it rotates against its central axis. The second configuration is built with the beam supported by two level arms on both sides. One of the level arms acted as the pivot, and the other is coupled to motor output gear. The purpose of ball and beam system is to hold the ball in a desired position on the beam, the ball position cannot be controlled directly but only through changing the angle of beam.

Among the studies on the first model, the study [10,11] uses the classic PID controller but the results are not good because the model is highly nonlinear, so the classical controller is not suitable for nonlinear control applications. The study [2] presents the approach based on Lyapunov method combined with the nonlinear observer showed positive results, but when designing the controller, the control quality is not really good. In the works [12] presenting LQR method for shading system and bar with good results. But this method is limited when using a linear model, it does not show all the dynamic nature of the system. Some studies using intelligent control methods such as [3-5] present the method of designing controllers using Neuron Network and Fuzzy Logic Type-2 that have been successful and have good simulated results, but not yet applied on real system. In the study [1,6], the authors have used quasi-time optimal control laws for very good simulation results, but the control laws are too complicated.

The rest of the paper is organized as follows: Part II is for physical modelling of ball and beam system; part III is an overview of control structure diagram and the method to synthesis the quasi-time optimal cascade control for the ball and beam system; the simulation results are evaluated and compared with the LQR. Finally, the enforcement of quasi-time optimal cascade controller on actual ball and beam system is indicated in part IV.

2. Dynamical model of the ball and beam system

The first configuration is shown in Figure 1, the beam is supported in the middle, and it rotates against its central axis. Most ball and beam systems use this type of configuration. This type has the advantage of easiness of building and the simplicity of the mathematical model, but the motor need the high torque.

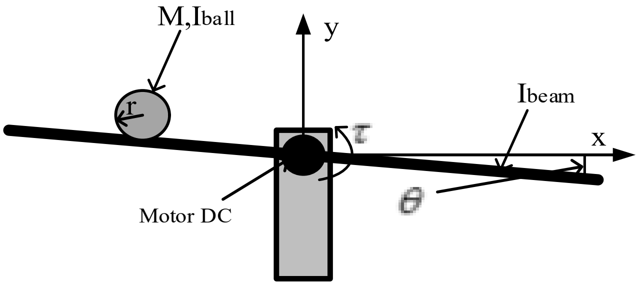

Figure 1. Ball and beam system

The parameters of system as follows: ball mass M = 0.05(kg); ball radius r = 0.002(m); moment of inertia of the ball Iball = 2.10-6(kg.m2); moment of inertia the beam Ibeam = 0.02(kg.m2); gravity g= 9.81(m/s2); moment of inertia of the motor Jm = 0.049.10−4(kg.m2); R=2.7 (Om)- armature resistance; motor's Torque constant Kt = 5.3.10-3(Nm/A); back EMF constant Ke = 5.10-3 (Vs/rad); gear ratio Kc = 27.

Assuming that friction at all connected points is zero, applying Lagrange method, the mathematical model of the ball and beam system can be described as follows [1,9]:

$$\begin{cases} \left(M+\frac{I_{ball}}{r^2}\right)\ddot{x}+Mg\sin\left(\theta\right)-Mx\dot{\theta}^2=0 \\ R\left(B_m+mx^2\right)\ddot{\theta}+2mx\dot{x}\dot{\theta}+\left(K_c^2K_tK_e+RK_c^2b\right)\dot{\theta}+Rmg\cos\left(\theta\right)x=K_cK_tU \end{cases} \tag{1}$$

From equations (1), the state equation with the state variable $(x_1,x_2,x_3,x_4)^T=(x,\dot{x},\theta,\dot{\theta})^T$ as follows.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix} x_2 \\ \frac{M}{A}x_1x_4^2-\frac{Mg}{A}\sin\left(x_3\right) \\ x_4 \\ \frac{K_cK_t}{R\left(B+Mx_1^2\right)}U-\left(\frac{K_c^2K_eK_t}{R\left(B+Mx_1^2\right)}+\frac{K_c^2b}{\left(B+Mx_1^2\right)}\right)x_4-\frac{2M}{\left(B+Mx_1^2\right)}x_1x_2x_4-\frac{Mg}{\left(B+Mx_1^2\right)}x_1\cos\left(x_3\right)\end{bmatrix} \tag{2}$$

Where $A = M + \frac{I_{ball}}{r^2}$; $B = I_{ball} + I_{beam} + K_c^2 J_m$;

3. The design of cascade controller with quasi-time optimization for ball and beam system

3.1 Control cascade structure diagram of ball and beam system

In the study [1,6-9], it is proposed the control law with quasi-time optimization, but the control law is too complicated to implement on embedded systems. In the ball and beam system, it is necessary to control the position of the ball and also to bring the beam to the horizontal position. Therefore, the authors propose the structure diagram of the ball and beam system with two controllers: the position of ball and the angle of beam. The control structure diagram of the ball and beam system is shown in Figure 2. The position controller based on the desired position value and current state to give the set value to the angular control of the ball and beam system. The angle controller based on the error between the set value and the state of the system gives the torque impacting on the ball and beam system.

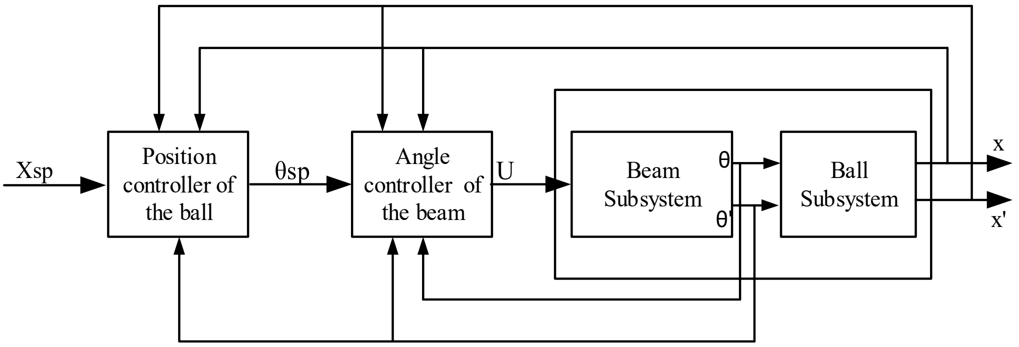

Figure 2. The control structure diagram of ball and beam system

3.2 The synthesis of quasi-time optimal cascade controller for ball and beam system

In order to synthesize the quasi-time optimal cascade control law on the embedded system and simplifies the control law, the authors propose to separate the original system into the two subsystems with the mathematical model of each subsystem after linearizing the equations (2) as follows.

The model of ball subsystem with state variable $(x_1, x_2)^T = (\theta, \dot{\theta})^T$:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{Mg}{A} U_1 \end{cases} \tag{3}$$

The model of beam subsystem with state variable $(z_1, z_2)^T = (\theta, \dot{\theta})^T$:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\left(\frac{K_c^2 K_e K_t}{RB} + \frac{K_c^2 b}{B} \right) z_2 - \frac{Mg}{B} x + \frac{K_c K_t}{RB} U_2 \end{cases} \tag{4}$$

The direction of quasi-time optimal control is presented in the studies [1,6-9]. For subsystems (3), (4), the selected virtual equations has the form (5):

$$\begin{cases} \dot{y}_1 = -v_1 \dfrac{y_1}{\sqrt{y_1^2 + \varepsilon_1^2}} + y_2 \\ \dot{y}_2 = -\dfrac{y_2}{\varepsilon_2} \end{cases} \tag{5}$$

System (3) and (4) has a controlled Jordan form. The virtual system is selected as (5) then the ball position will be guaranteed to return to the preset position with quasi-time optimization. The quasi-time optimal control laws are found when solving equations (4) with $y_1 = x_1 - x_{sp}$ in equations (4) and $y_1 = z_1$ in equations (5). The formula of U_1 and U_2 have the form (6) and (7).

$$U_1 = \frac{A}{Mg}\left(v_1\varepsilon_{11}^2 \frac{\dot{x} - \dot{x}_{sp}}{\left(\left(x - x_{sp}\right)^2 + \varepsilon_{11}^2\right)^{1.5}} + \frac{1}{\varepsilon_{12}}\left(\dot{x} + v_1 \frac{x - x_{sp}}{\left(\left(x - x_{sp}\right)^2 + \varepsilon_{11}^2\right)^{0.5}} \right)\right) \tag{6}$$

$$U_2 = \left(K_c K_e + \frac{R K_c b}{K_t} \right)\dot{\theta} + \frac{MgR}{K_c K_t} x - v_2 \varepsilon_{21}^2 \frac{\dot{\theta}}{\left(\theta^2 + \varepsilon_{21}^2\right)^{1.5}} + \frac{1}{\varepsilon_{22}}\left(\dot{\theta} + v_1 \frac{\theta}{\left(\theta^2 + \varepsilon_{21}^2\right)^{0.5}} \right) \tag{7}$$

4. Simulation and experiment results

4.1 Simulation results

By the method described above, the synthesized control laws U_1, U_2 provides the stable control over the ball and beam system. The simulation results with initial values are as follows: $x_{sp} = 0$, $x(0) = 0.5$, $\dot{x}(0) = 0$, $\theta(0) = 0.4$, $\dot{\theta}(0) = 0$. The parameters of quasi-time optimal controller with U_1, U_2: $v_1 = v_2 = 1$, $\varepsilon_{11} = 0.4; \varepsilon_{12} = 0.3; \varepsilon_{21} = 0.4; \varepsilon_{22} = 0.5$. Figure 3 shows the response of the ball position with the proposed method (CAS-KOB) and LQR is presented in the study [1]. It is clear that the position response of ball to proposed method is more stability than the LQR controller. Figure 4 shows the angle response of beam according to the proposed method for faster and more stable response than LQR, although the higher overshoot at the first time.

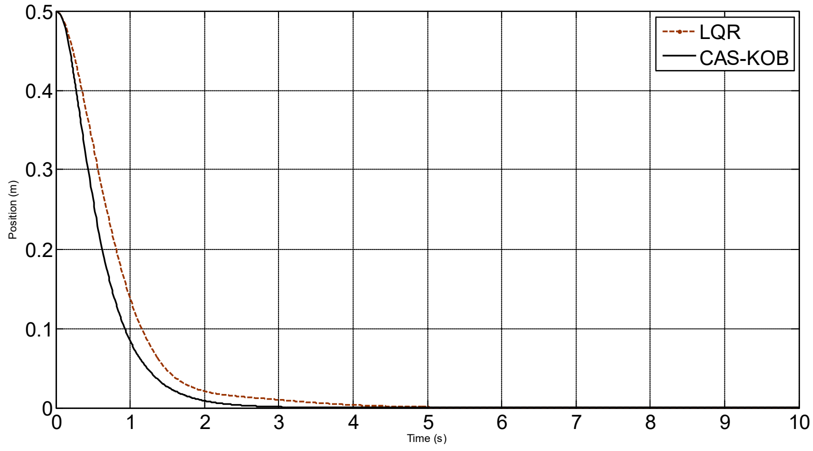

Figure 3. The position response of ball

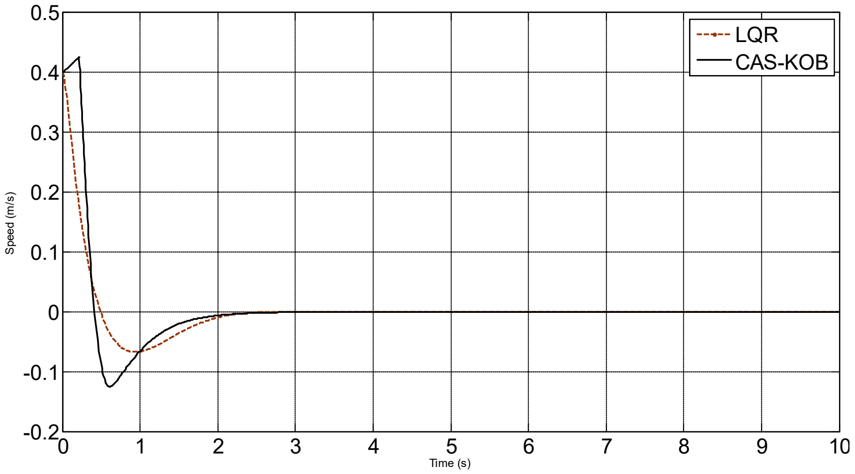

Figure 4. The angle response of beam

4.2 *Experiment results*

The quasi-time optimal cascade control law is implemented on real-time embedded system and the model made by the authors (Figure 5). The parameters are as follows: the travel distance of ball is 0.5 (m), the other parameters are the same as the simulation.

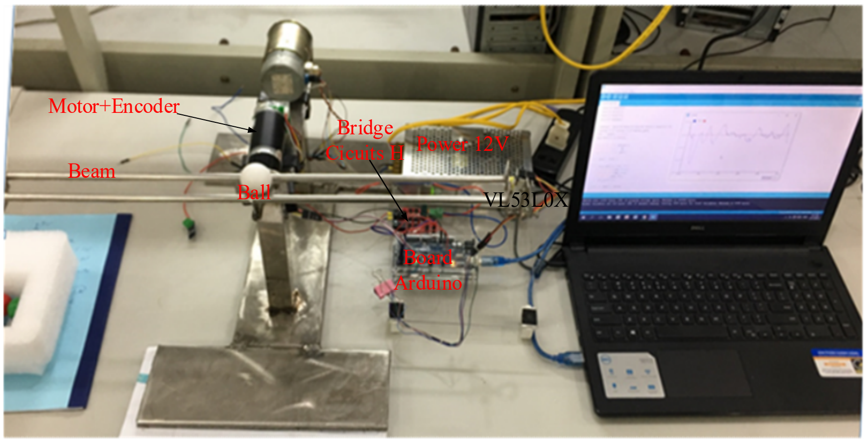

Figure 5. Experimental model of ball and stick systems.

Figures 6 and 7 show the angular and position response of the ball and beam system using the proposed controller, and we see that the optimal controller has worked with the designed ball and beam model. The ball is always guaranteed at the desired position and only fluctuates slightly around the equilibrium position. The fluctuations of the ball position around the set point tend to fade. At the same time, we also see that the response of the beam angle and the ball position sometimes fluctuate due to the mechanical errors of the gearbox, the measurement error when using the radar VL53L0X distance sensor. Another reason is that the control law is not optimal with the real model because υ, ε in the simulation are only approximate.

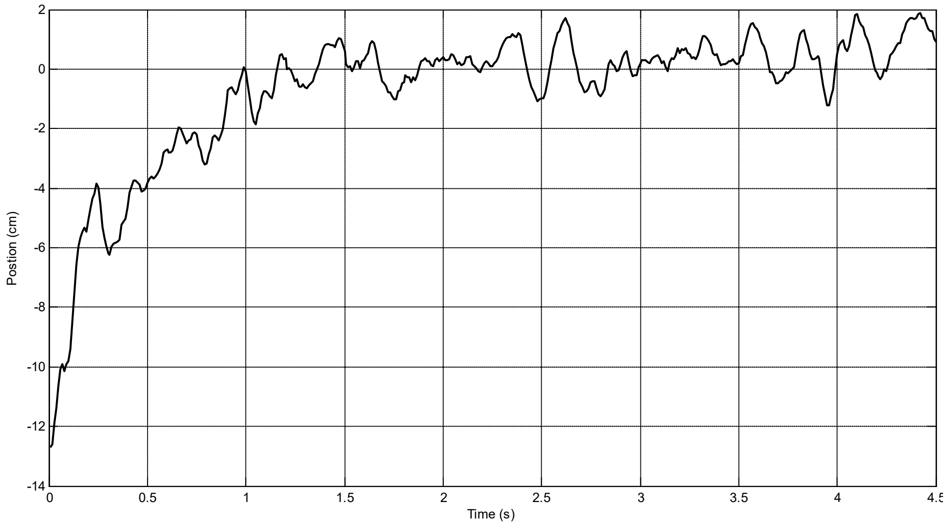

Figure 6. The position response of ball on embedded system.

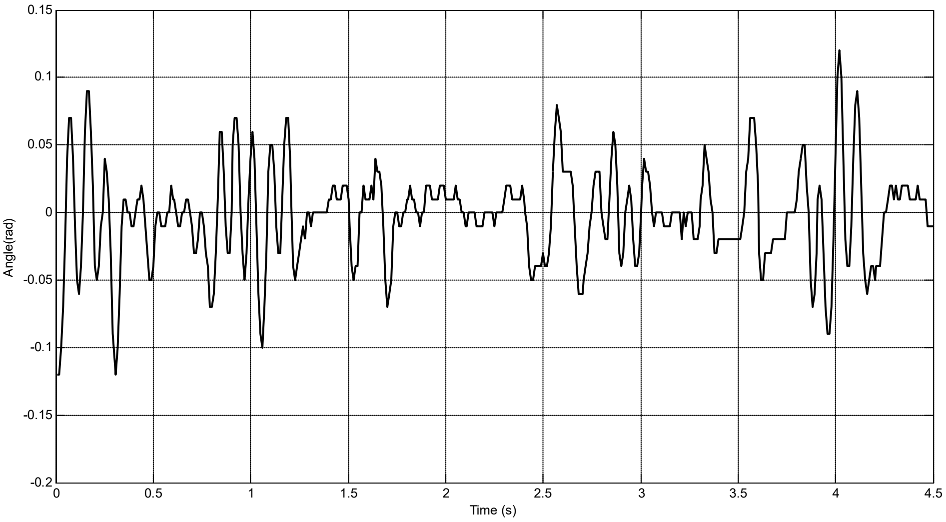

Figure 7. The angle response of beam

on embedded system.

5. Conclusion

The results obtained when synthesizing control laws of the ball and beam system according to the proposed method show that the control law still remains the advantages of the quasitime optimal control for complex technical systems with high nonlinearity. In addition, the proposed method is much simpler than the quasi-time optimal control law when not separated into two subsystems. Therefore, the proposed method of this method are easily implemented on embedded systems. Comparison with the traditional LQR controller shows that the setting time when using the synthesized controller proposed in the paper is better. Future studies will supplement the research on sustainable controller and adaptive parameters of the control law when the object model is uncertain.

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