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Determined of Equivalent Fixed Depth in the 3D Frame

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Abstract. The paper presents the results of solving the problem of identifying the equivalent depth of the linkage of the frame - pile structural system. The computational model of the problem is a frame - pile structural system in the form of three-dimensional frame, linear elastic deformation, pile - soil link is replaced by a hard restraint (fixed) with equivalent restraint depth. The problem is solved by the method of penalty function to combine with the finite element method. Examples of numerical calculations show that models, algorithms and calculation programs can be trusted and acceptable.

1. Introduction

Pile foundation is a type of structure that used a lot in construction, transportation, irrigation, and offshore constructions... In the calculation, it is possible to replace the pile-soil link with the hard restraint (fixed) at the equivalent fixed depth (equivalent fixed). This equivalent fixed depth can be determined either by standard or experience, but does not completely accurately reflect the actual working of the pile. On the other hand, during the using process, piles can be reduced the link to the ground over time, so the equivalent fixed depth is changed. Determine the equivalent fixed depth to the actual working state of the pile (based on the specific vibration frequencies measured in the field) to determine the technical state of the structure is very important.

The equivalent restraint depth can be determined according to the design standard [1], [2]:

$$l_e = 2 \cdot \left(\frac{k \cdot b_c}{E \cdot I} \right)^{-1/5} \quad (1)$$

where:

k is Scale factor, kN/m⁴, according to table B.1 [2];

E is Elastic strain modulus of pile materials, kN/m²;

I is Inertial module cross - section of the pile, m⁴;

b_c is conventional width of piles (m); with pile pipes as well as column piles and bored piles with a diameter of 0.8m and larger than b_c = d + 1m, and other types of piles and cross sections b_c = 1.5d + 0.5m;

d is outside diameter of round section piles, side of square or rectangular pile section perpendicular to the applied load, m;

According to [3], for offshore constructions, the equivalent restrain depth l_e , is determined by geology, such as:



- + $l_e = (3.5 \div 4.5)d$: in the case of clay.
- + $l_e = (7 \div 8.5)d$: in the case of alluvial soil.
- + $l_e = 6d$: when the geological conditions of the construction site have not yet been determined.

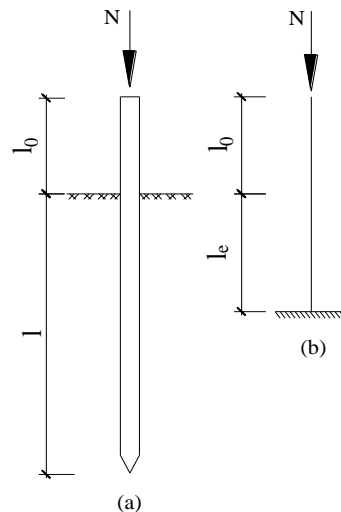


Figure 1. Model of the equivalent restraint depth.

However, the above determination methods are only suitable for a certain range, with (1) applied to piles with cross section with width less than 250 mm, constructed by close or pressed method, with (2) for large-diameter pipe piles for petroleum works. On the other hand during use and extraction, the piles may be reduced in connection with the ground over time, so the equivalent fixed depth is changed. Determining the equivalent fixed depth corresponding to the actual working state of the pile (based on the specific vibration frequencies measured in the field) to determine the technical state of the structure is very importance.

The problem of structural identification has been mentioned by scientists around the world since the late 1970s and early 80s of the 20th century. In particular in the aerospace and oil - shore there are many authors interested in research such as Farrar and Doebling [4], Ghoshal et al. [5], Friswell and Penny [6], Lee and Shin [7], ... Chan Ghee Koh, Lin Ming See, Thambirajah, Balendra [8] evaluated the hardness index of each floor to diagnose damage of frame structure. Narkis Y. [9] locates cracks in the beam structure. Hassiotis S. and Jeong G.D. [10] use the global planning method with finite element method to solve the problem of structural identification. J.K.Sinha, M.I. Friswell, S. Edwards [11], J.K.Sinha, P.M.Mujumdar, R.I.K.Moorthy [12], M.I.Friswell, J.E.T.Penny, S.D.Garvey [13], M.I. Friswell [14], J.K.Sinha, M.I. Friswell [15] studied the problem of diagnosing structural damage, stiffness link by the penalty function method.

2. Methods

2.1. The equations of motion of the frame - pile structural system in three-dimensional frame

Investigation of frame-pile structural system in the form of three-dimensional frame under dynamic load effect (Figure 2) in the coordinates Oxyz.

Recognize the following assumptions:

- Pile-soil link is replaced by hard equivalent fixed link.
- The strain of the frame - pile structural system is linear and small.

The analysis model of the structure is shown in Figure 3.

To build the equation of the motion of the frame - pile structural system, the finite element method (FEM) will be used.

The equations of motion of the frame - pile - soil structural system according to FEM method [4], after applying boundary conditions to the system, can be formulated as follows:

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = P(t) \tag{3}$$

Where $U(t), \dot{U}(t), \ddot{U}(t)$ respectively are the displacement vector, velocity and node acceleration of the bar - pile structural system.

M, K, C respectively are mass, stiffness, and damping matrices of the structural system.

$P(t)$ is the nodal load vector of the structural system.

The damping matrix of structural system can be calculated according to the mass matrix and stiffness matrix as:

$$C = \alpha M + \beta K \tag{4}$$

Where α, β are the factors depend on the specific vibration frequencies of the system and the viscous damping factors of the material.

α_1, α_2 are Rayleigh damping coefficient, determined by the lowest individual frequencies of the structure ω_1, ω_2 ; and the corresponding damping ratios ζ_1, ζ_2 according to the formula:

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} = \begin{Bmatrix} \frac{2(\zeta_1\omega_2 - \zeta_2\omega_1)\omega_1\omega_2}{\omega_2^2 - \omega_1^2} \\ \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2} \end{Bmatrix} \tag{5}$$

Where ω_1, ω_2 are 1st and 2nd individual frequencies of the structural system.

ζ_1, ζ_2 are damping ratios depend on the structural material also as nature of work of the system.

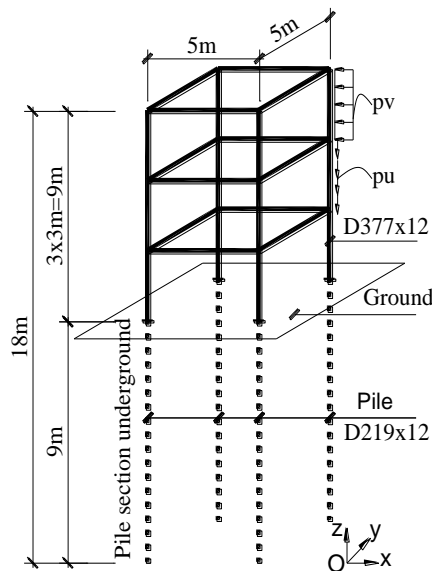


Figure 2. Real model of Frame - pile space structural system.

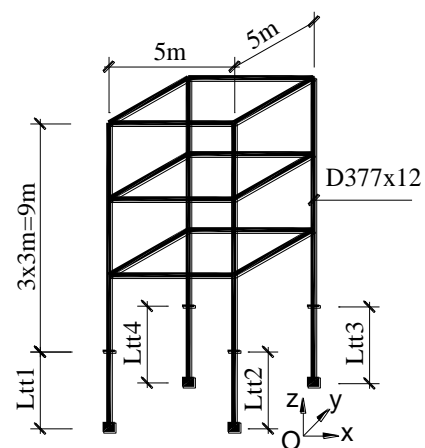


Figure 3. Structural analysis model Frame - pile of space.

The matrices of the whole system in equation (3) can be built from the matrices of FEM in the system by the "direct stiffness" method [4]. The following are the matrices of FEM for the three- dimensional Frame - pile structural system.

In order to establish the overall matrix of structural system M, K and nodal load vector P , it is necessary to define mass matrices m , stiffness matrices k , and nodal load vector p of element in local coordinate system. Mass matrices m , stiffness matrices k , and nodal load vectors p of elements in the local coordinate system have the following form:

$$m = \frac{ma}{420} \begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22a & 0 & 54 & 0 & 0 & 0 & -13a \\ 0 & 0 & 156 & 0 & 22a & 0 & 0 & 0 & 54 & 0 & -13a & 0 \\ 0 & 0 & 0 & \frac{140I_p}{F} & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 \\ 0 & 0 & 22a & 0 & 4a^2 & 0 & 0 & 0 & 13a & 0 & -3a^2 & 0 \\ 0 & 22a & 0 & 0 & 0 & 4a^2 & 0 & 13a & 0 & 0 & 0 & -3a^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13a & 0 & 156 & 0 & 0 & 0 & -22a \\ 0 & 0 & 54 & 0 & 13a & 0 & 0 & 0 & 156 & 0 & -22a & 0 \\ 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 & \frac{140I_p}{F} & 0 & 0 \\ 0 & 0 & -13a & 0 & -3a^2 & 0 & 0 & 0 & -22a & 0 & 4a^2 & 0 \\ 0 & -13a & 0 & 0 & 0 & -3a^2 & 0 & -22a & 0 & 0 & 0 & 4a^2 \end{bmatrix}$$

$$k = \begin{bmatrix} \frac{EF}{a} & 0 & 0 & 0 & 0 & 0 & -\frac{EF}{a} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{a^3} & 0 & 0 & 0 & \frac{6EI_z}{a^2} & 0 & -\frac{12EI_z}{a^3} & 0 & 0 & 0 & \frac{6EI_z}{a^2} \\ 0 & 0 & \frac{12EI_y}{a^3} & 0 & \frac{6EI_y}{a^2} & 0 & 0 & 0 & -\frac{12EI_y}{a^3} & 0 & \frac{6EI_y}{a^2} & 0 \\ 0 & 0 & 0 & \frac{GI_p}{a} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_p}{a} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{a^2} & 0 & \frac{4EI_y}{a} & 0 & 0 & 0 & \frac{6EI_y}{a^2} & 0 & \frac{2EI_y}{a} & 0 \\ 0 & \frac{6EI_z}{a^2} & 0 & 0 & 0 & \frac{4EI_z}{a} & 0 & -\frac{6EI_z}{a^2} & 0 & 0 & 0 & \frac{4EI_z}{a} \\ -\frac{EF}{a} & 0 & 0 & 0 & 0 & 0 & \frac{EF}{a} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{a^3} & 0 & 0 & 0 & -\frac{6EI_z}{a^2} & 0 & \frac{12EI_z}{a^3} & 0 & 0 & 0 & -\frac{6EI_z}{a^2} \\ 0 & 0 & -\frac{12EI_y}{a^3} & 0 & \frac{6EI_y}{a^2} & 0 & 0 & 0 & \frac{12EI_y}{a^3} & 0 & -\frac{6EI_y}{a^2} & 0 \\ 0 & 0 & 0 & -\frac{GI_p}{a} & 0 & 0 & 0 & 0 & 0 & \frac{GI_p}{a} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{a^2} & 0 & \frac{2EI_y}{a} & 0 & 0 & 0 & -\frac{6EI_y}{a^2} & 0 & \frac{4EI_y}{a} & 0 \\ 0 & \frac{6EI_z}{a^2} & 0 & 0 & 0 & \frac{4EI_z}{a} & 0 & -\frac{6EI_z}{a^2} & 0 & 0 & 0 & \frac{4EI_z}{a} \end{bmatrix}$$

$$p = \left[\frac{p_u a}{2} \quad \frac{p_v a}{2} \quad \frac{p_w a}{2} \quad \frac{p_\varphi a}{2} \quad -\frac{p_w a^2}{12} \quad \frac{p_v a^2}{12} \quad \frac{p_u a}{2} \quad \frac{p_v a}{2} \quad \frac{p_w a}{2} \quad \frac{p_\varphi a}{2} \quad \frac{p_w a^2}{12} \quad -\frac{p_v a^2}{12} \right]^T \quad (6)$$

Where I_y, I_z are Moment of inertia for the axis y, z ; I_p is Polar inertia moment; F is Area section of the element; a is length of the element; m is Mass distributed on the long unit of element; p_u, p_v, p_w are Vertical load and distributed horizontal; p_ϕ is Distributed torsional load; G is Elastic shear modulus of materials, kN/m^2 .

2.2. The problem of identification of the equivalent fixed depth and the solution method

Investigate the Frame - pile structural system in the form of existing space frame at the site. The problem here is to determine (or identify) the equivalent fixed depth of each pile on the basis of the specific vibration frequencies measured by dynamic testing of structures at the site.

To solve the problem, we will apply the penalty function method of the FEM update model in structural dynamics [16], [17], whereby the identification parameters of the problem are determined on the basis of minimizing the penalty function - is the sum of squares of errors between measured individual values and calculated values.

Symbols: $\theta = [\theta_1, \theta_2, \dots, \theta_j, \dots, \theta_p]^T = [l_1, l_2, \dots, l_j, \dots, l_p]^T$ is the vector of the identification parameters has an unknown value; $z_e = [\lambda_{e1}, \lambda_{e2}, \dots, \lambda_{ei}, \dots, \lambda_{eN}]^T$ is the vector of the first N values obtains from measurement when dynamically testing the structure at the site; $z_c = [\lambda_{c1}, \lambda_{c2}, \dots, \lambda_{ci}, \dots, \lambda_{cN}]^T$ is the vector of the first N values receives from the analysis, depending on the identification parameters, $z_c = z_c(\theta)$.

$\varepsilon = (z_e - z_c(\theta)) = \varepsilon(\theta)$ is errors vector between measured individual values and calculated values.

The penalty function $J(\theta)$ has the form

$$J(\theta) = \|\varepsilon(\theta)\|^2 = (z_e - z_c(\theta))^T W_\varepsilon (z_e - z_c(\theta)) = \sum_{i=1}^N W_{\varepsilon i} (\lambda_{ei} - \lambda_{ci}(\theta))^2 \tag{7}$$

where $W_\varepsilon = \text{diag}(W_{\varepsilon 1}, W_{\varepsilon 2}, \dots, W_{\varepsilon i}, \dots, W_{\varepsilon N})$ is diagonal matrix is positive and is usually the inverse matrix of the variance of the eigenvalues measurement data.

The functions $\varepsilon(\theta)$ and $J(\theta)$ usually high-level nonlinear functions of updated parameters θ . Therefore, the solution θ of the minimization problem of the aforementioned penalty function is hard to get in closed form by the precise analysis method. In this paper, instead of using the correct method, we use the iterative method, called the penalty function method. Below are set up the equations and algorithms according to this iterative method.

Develop Taylor error vector $\varepsilon(\theta)$ according to certain identification parameters in a given vector $\theta = \theta_k$, obtaining

$$\varepsilon_k(\theta) = (z_e - z_c(\theta_k)) + \left[\frac{\partial(z_e - z_c(\theta))}{\partial \theta} \right]_{\theta=\theta_k} \delta \theta_k + \text{higher order components} \tag{8}$$

If only the first two elements of the string (8) are retained, resulting in

$$\varepsilon_k(\theta) \approx (z_e - z_c(\theta_k)) + \left[\frac{\partial(z_e - z_c(\theta))}{\partial \theta} \right]_{\theta=\theta_k} \delta \theta_k = \delta z_k - S_k \delta \theta_k = \varepsilon_k(\delta \theta_k) \tag{9}$$

Where $\delta \theta_k$ is the increment of the identification parameter; δz_k is errors vector between eigenvalues measured and eigenvalues calculated when $\theta = \theta_k$.

$$\delta z_k = z_e - z_{c,k} \tag{10}$$

$$z_{c,k} = z_c(\theta_k) = [\lambda_{c1,k}, \lambda_{c2,k}, \dots, \lambda_{ci,k}, \dots, \lambda_{cN,k}]^T \tag{11}$$

S_k is sensitivity matrix - the first derivative of eigenvalues calculated according to the identification parameters at $\theta = \theta_k$.

$$S_k = \left[\frac{\partial z_c(\theta)}{\partial \theta} \right]_{\theta=\theta_k} = \begin{bmatrix} S_{11,k} & S_{12,k} & \dots & S_{1j,k} & \dots & S_{1p,k} \\ S_{21,k} & S_{22,k} & \dots & S_{2j,k} & \dots & S_{2p,k} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{i1,k} & S_{i2,k} & \cdot & S_{ij,k} & \cdot & S_{ip,k} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{N1,k} & S_{N2,k} & \cdot & S_{Nj,k} & \cdot & S_{Np,k} \end{bmatrix}_k \quad (12)$$

with:

$$S_{ij,k} = \frac{\partial \lambda_{ci,k}}{\partial \theta_j} \quad (13)$$

The penalty function $J(\theta)$ is in this form

$$J_k(\theta) = \varepsilon_k^T (\delta \theta_k) W_\varepsilon \varepsilon_k (\delta \theta_k) = (\delta z_k - S_k \delta \theta_k)^T W_\varepsilon (\delta z_k - S_k \delta \theta_k) = J_k(\delta \theta_k) \quad (14)$$

The solution of equation (14) is obtained by minimizing the function $J_k(\delta \theta_k)$ follow $\delta \theta_k$, whereby

$$\frac{\partial J_k(\delta \theta_k)}{\partial \delta \theta_k} = 0 \quad (15)$$

Replace $J_k(\delta \theta_k)$ from (14) to (15), we get

As of (9), minimize (7) according to $\delta \theta_k$ have the result:

$$\delta \theta_k = \left[S_k^T W_\varepsilon S_k \right]^{-1} S_k^T W_\varepsilon \delta z_k \quad (16)$$

Because the function (9) is a linear approximation function θ , to get as close to the exact value of the problem as iterative. If performed:

$$\delta \theta_k = \theta_{k+1} - \theta_k \quad (17)$$

from (16) may write:

$$\theta_{k+1} = \theta_k + \left[S_k^T W_\varepsilon S_k \right]^{-1} S_k^T W_\varepsilon \delta z_k \quad (18)$$

or:

$$\theta_k = \theta_{k-1} + \left[S_{k-1}^T W_\varepsilon S_{k-1} \right]^{-1} S_{k-1}^T W_\varepsilon \delta z_{k-1} \quad (19)$$

Here (k-1), k, (k+1) index indicate the iteration steps.

The looping process ends when the solution of the problem converges with the required accuracy.

The elements of a sensitive matrix S can be obtained from partial vibration differential equation of the structure:

$$[K - \lambda M] \varphi = 0 \quad (20)$$

where: λ and φ are the normalized eigenvalues and eigenvectors of the structure.

The result we have:

$$S_{ij} = \frac{\partial \lambda_{ci}}{\partial \theta_j} = \varphi_i^T \left[\frac{\partial K}{\partial \theta_j} - \lambda_i \frac{\partial M}{\partial \theta_j} \right] \varphi_i \quad (21)$$

With: λ_i, φ_i are Normalized eigenvalues and eigenvectors i of the structure.

At the iteration k the above quantity has the form:

$$S_{ij,k} = \frac{\partial \lambda_{ci,k}}{\partial \theta_j} = \varphi_{i,k}^T \left[\frac{\partial K}{\partial \theta_j} - \lambda_i \frac{\partial M}{\partial \theta_j} \right]_{\theta=\theta_k} \varphi_{i,k} \quad (22)$$

Where: $\varphi_{i,k}$ is Normalized vector of the structure corresponding to the i value at the k iteration (or at $\theta = \theta_k$).

According to the iterative algorithms established above, the content of calculating the updated parameters is conducted in the following order:

* *The original analysis*

Select vector of the parameters need to be identified θ of the structure model (here is equivalent restrain depth vector).

$$\theta = [\theta_1, \theta_2, \dots, \theta_j, \dots, \theta_p]^T = [l_1, l_2, \dots, l_j, \dots, l_p]^T$$

- Set up the stiffness matrix K and $\frac{\partial K}{\partial \theta_j}$ ($j = 1, 2, \dots, p$), mass matrix M and $\frac{\partial M}{\partial \theta_j}$ ($j = 1, 2, \dots, p$)

according to the identification parameters θ .

- Select a vector of initial identification parameters θ_0 .

$$\theta_0 = [\theta_{1,0}, \theta_{2,0}, \dots, \theta_{j,0}, \dots, \theta_{p,0}]^T = [l_{1,0}, l_{2,0}, \dots, l_{j,0}, \dots, l_{p,0}]^T$$

- Calculate the matrix M , K when $\theta = \theta_0$.

- Calculate eigenvalues $\lambda_{ci,0}$ ($i=1, 2, \dots, N$), eigenvectors $\varphi_{i,0}$ ($i=1, 2, \dots, N$) and $\left[\frac{\partial K}{\partial \theta_j} \right], \left[\frac{\partial M}{\partial \theta_j} \right]$ of the structural system when $\theta = \theta_0$.

- Calculate the sensitivity $S_{ij,0}$ according to formula (22) (with index $k=0; i=1, 2, \dots, N; j=1, 2, \dots, p$) and form the sensitivity S_0 by the formula (12) with $k=0$.

- Forming eigenvalue measurement vector z_e

$$z_e = [\lambda_{e1}, \lambda_{e2}, \dots, \lambda_{ei}, \dots, \lambda_{eN}]^T$$

- Calculate the vector of eigenvalue error δz_0

$$\delta z_0 = z_e - z_{c,0}$$

$$\text{With: } z_{c,0} = [\lambda_{c1,0}, \lambda_{c2,0}, \dots, \lambda_{ci,0}, \dots, \lambda_{cN,0}]^T$$

* *Repeatability*

At the iteration k ($k=1, 2, 3, \dots$), calculate identification parameter vector θ_k , eigenvalues $\lambda_{ci,k}$, eigenvector $\varphi_{i,k}$, error matrix $z_{c,k}$, eigenvalue error vectors $\delta z_k, \left[\frac{\partial K}{\partial \theta_j} \right]_{\theta=\theta_k}, \left[\frac{\partial M}{\partial \theta_j} \right]_{\theta=\theta_k}$, sensitivity

$S_{ij,k}$, sensitivity matrix S_k , parameter vector θ_{k+1} according to the formulas set above.

- Compare errors between θ_{k+1} and θ_k with allowed errors ε .

- Check $\frac{\theta_{k+1} - \theta_k}{\theta_{k+1}} < \varepsilon$? If not satisfied: repeat calculation; If satisfied: stop calculating.

Based on the received algorithms, the author has built the **UFEM** program to solve the problem of identifying the equivalent fixed depth of the frame - pile structure working according to three-dimensional model in MATLAB language [18], [19]. **UFEM** has been tested for reliability [17].

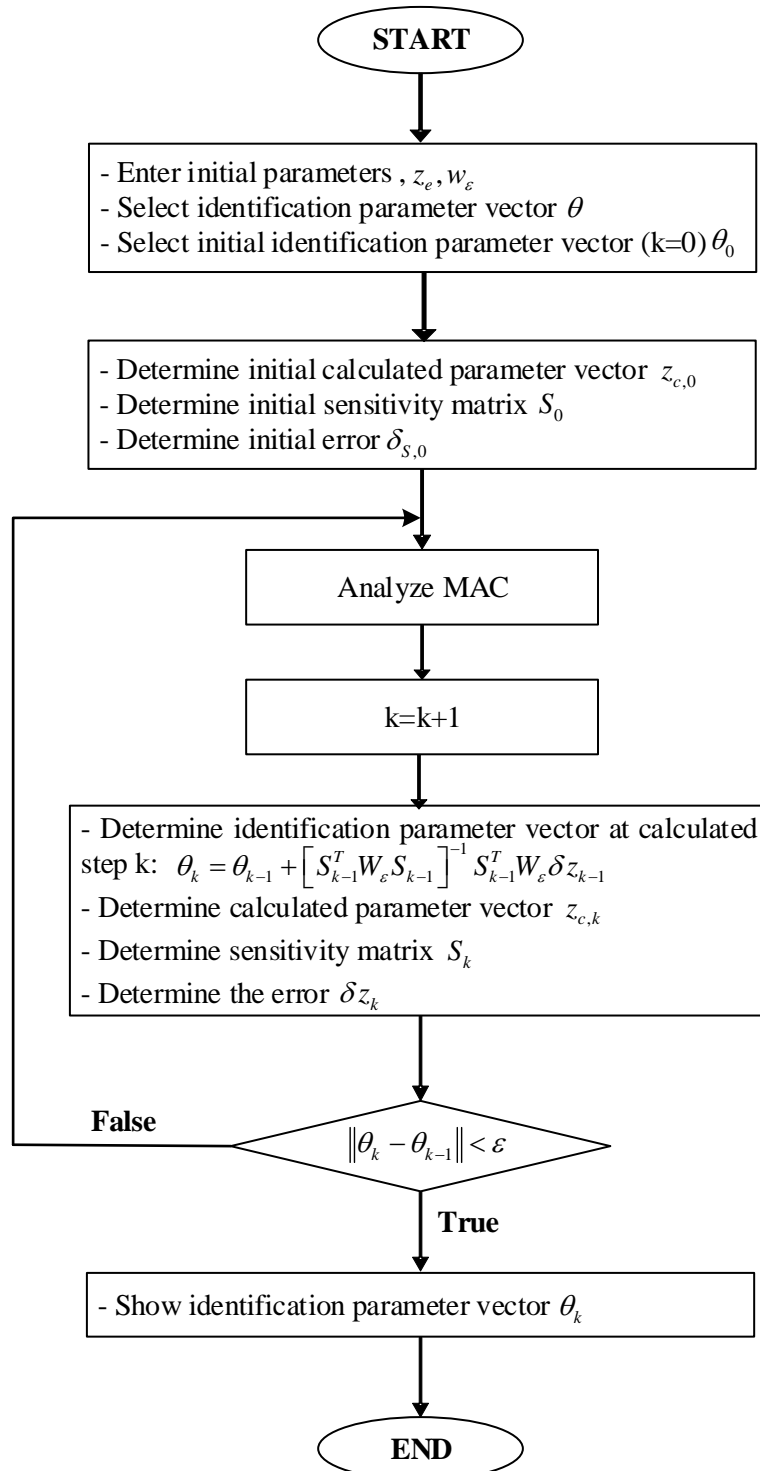


Figure 4. UFEM program block diagram.

3. Results (Calculate by number)

The numerical calculations below are performed to check the reliability of established algorithms and programs. Identify equivalent fixed depth of the structure (Figure 2, Figure 3), with 04 equivalent fixed depths of 04 different piles (therefore, there are 04 identification parameters).

* Starting data:

- Structure of frame - pile made of steel pipes, the size shown in figure 1, horizontal-section of bars on the ground type $\phi 377 \times 12 \text{mm}$; pile section in soil 9m long, horizontal-section type $\phi 219 \times 12 \text{mm}$; elastic modulus of steel: $E = 21000000 \text{ T/m}^2$; Specific gravity of steel: $\gamma = \rho / g = 7.8 \text{ T/m}^3$.

- Measurement vector (assumed):

$$\lambda_e = [\lambda_{e1} \ \lambda_{e2} \ \lambda_{e3} \ \lambda_{e4} \ \lambda_{e5} \ \lambda_{e6} \ \lambda_{e7} \ \lambda_{e8}] = [37.258 \ 4032.649 \ 4814.176 \ 5277.911 \ 5560.007 \ 9242.009 \ 10904.999 \ 20061.865] \quad (23)$$

- The assumed eigenvalues here are not actually true measurements, it is eigenvalues calculate for the bar-pile structure with a given equivalent restrain depth

$$l_u = [l_1 \ l_2 \ l_3 \ l_4]^T = [1.00 \ 1.00 \ 1.00 \ 1.00]^T \quad (24)$$

- Choose the allowable error: $\varepsilon = 0.5\%$.

* Calculation results:

- Select the identification parameters as the equivalent restrain depth: $l_u = [l_1 \ l_2 \ l_3 \ l_4]^T$.

- Discrete FEM for structure.

- Select initial parameters: $\theta_0 = l_{u,0} = [l_{1,0} \ l_{2,0} \ l_{3,0} \ l_{4,0}]^T = [0.80 \ 1.20 \ 1.20 \ 0.80]^T \text{ (m)}$.

- Solution of the problem: The solution of the problem when calculating iteration needs to focus on the values of vector (24) with allowed errors $\varepsilon = 0.5\%$. The results of calculating the value of the identification parameters according to the iterative calculation steps are shown in table 1 and figure 5.

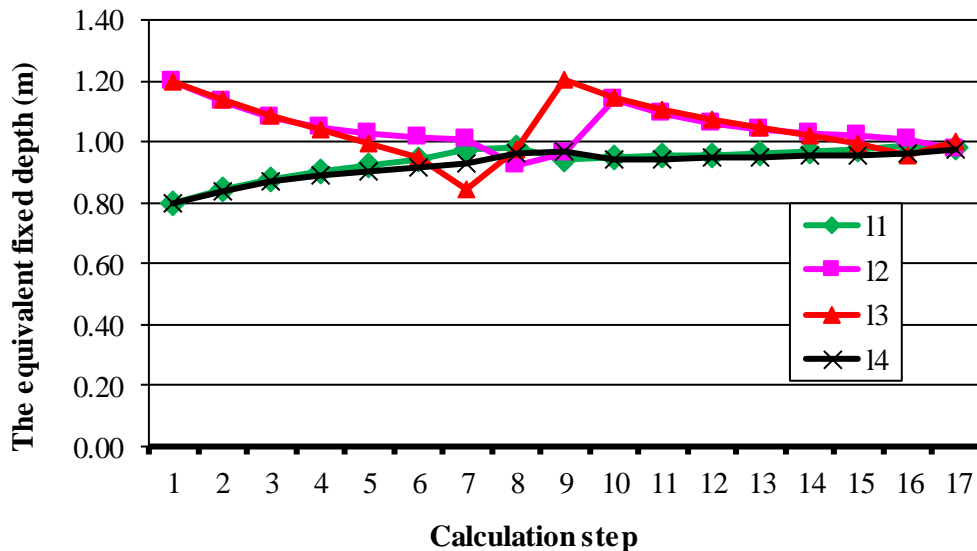


Figure 5. Identification parameter graph by calculation step.

Solution of convergence problems after 17 steps calculated with the results:

$$\theta = l_u = [l_1 \ l_2 \ l_3 \ l_4]^T = [0.984 \ 0.974 \ 1.003 \ 0.976]^T \quad (25)$$

Table 1. Identification parameters by calculation step.

Calculation steps	Identification parameters			
	$\theta_1(l_1)$	$\theta_2(l_2)$	$\theta_3(l_3)$	$\theta_4(l_4)$
1	0.800	1.200	1.200	0.800
2	0.846	1.136	1.142	0.840
3	0.878	1.083	1.089	0.869
4	0.902	1.047	1.041	0.890
5	0.924	1.027	0.999	0.906
6	0.946	1.018	0.950	0.919
7	0.978	1.009	0.843	0.933
8	0.981	0.925	0.977	0.963
9	0.941	0.965	1.207	0.970
10	0.953	1.138	1.147	0.940
11	0.956	1.094	1.109	0.944
12	0.959	1.062	1.075	0.948
13	0.964	1.043	1.046	0.952
14	0.970	1.031	1.019	0.955
15	0.977	1.022	0.994	0.959
16	0.987	1.011	0.959	0.964
17	0.984	0.974	1.003	0.976

4. Conclusions

The solutions of the above problem converge on the values to be sought, the numerical analytical data demonstrate reability of studied results, proving that the established algorithms and programs (UFEM) can be used to identify the equivalent fixed depth of the structure in three-dimensional model.

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