# An Optimized Pilot-Assisted Channel Estimation Method For Low-dispersive Channels 

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#### Abstract

In this paper, we present a novel pilot-assisted channel estimation algorithm for orthogonal frequency-division multiplexing (OFDM) wireless communication systems. The proposed method uses fewer pilots, which helps to increase the bandwidth (BW) efficiency while maintaining the adequate bit-errorrate (BER). A systematical approach for optimization of the pilot pattern applied in the low-dispersive channels is proposed and the performances using both the proposed and conventional estimation methods are quantitatively compared the performance in terms of BER for a case study of DVB-T/T2 system in 2 K mode. MATLAB simulations are performed to evaluate the methods in the cases of the specific channel models (urban and rural models). From the simulation results, the novel estimation method gives out mostly the same BER while requiring only about $25 \%$ pilot number per symbol, compared with the conventional methods.


Keywords: Channel estimation, OFDM, comb-type, pilot, DVB-T

## 1 Introduction

The multipath effect in wireless/over-the-air communication can lead to intersymbol interference, attenuation in amplitude, and shift in phase of the signal at the receiver side. One of the main solutions to reduce the aforementioned effect and increase system efficiency is using the equalizer. In an OFDM system, the equalizer task requires preliminary information from the channel frequency response (CFR). The latter is often achieved by using pilot signal with two typical types of arrangements, block-type [1,2] and comb-type [3-5]. With the block-type arrangement, the pilot signal is assigned to a particular OFDM block, which is sent periodically in the time-domain. The frequency-domain interpolation is used to estimate the channel response and this method typically is adopted for slowly fading channels [1,2], [6]. In the fast-fading channels, comb-type pilot, where the pilot signals are uniformly distributed within each OFDM block, is adopted since and the channel information can be updated every OFDM symbol [3-5]. With the comb-type pilot arrangement, the channel response at data subcarriers can be interpolated from the CFR value at pilot subcarriers.

To reconstruct CFR from the scattered pilot information, several interpolation techniques such as linear, spline, cubic interpolation, can be used. The higher and more complex methods usually offer better accuracy and efficiency [6]. Other than
interpolation, in [7], the author presented a new approach for indirectly calculating CFR from the channel impulse response (CIR) estimation, which works well for the static channel. As a further extension from that approach, in [8], the authors proved that, with certain modifications, this method is promising for wide-band transmission in the low-dispersive channels. However, in [7], [8], the authors did not present the method for finding optimized pilot frequency spacing, and the evaluation was not quantitatively conducted for channel bit-error-rate (BER), especially with realistic channel models. In this paper, we proposed a systematical approach for optimization of the pilot pattern applied in the low-dispersive channel based on the original approach in [7], [8]. Also, we quantitatively compared the performance in terms of BER between the proposed method and the conventional interpolation ones for a case study of the DVB-T/T2 system in 2 K mode. The result shows that the proposed pattern achieves similar accuracy while having much better spectral efficiency.

The remaining of the paper is organized as follows. Section 2 describes conventional channel estimation methods based on interpolation techniques. Section 3 proposes a novel pilot pattern. Section 4 presents bandwidth extension and simulation results and section 5 concludes the paper.

## 2 Background of conventional estimation in the OFDM system

For comb-type pilot subcarrier arrangement, the $\mathrm{N}_{\mathrm{p}}$ pilot signals $x_{p}(m), m=$ $0 \div N_{p}-1$ are uniformly inserted in $X(k)$ according to the following equation [6]:

$$
X(k)=X(m L+l)=\left\{\begin{array}{lr}
x_{p}(m), & l=0  \tag{1}\\
\text { inf. data }
\end{array} \quad l=1, \ldots . L-1\right.
$$

where $\mathrm{L}=\left(\right.$ number of carriers $\left./ \mathrm{N}_{\mathrm{p}}\right)$ and $x_{p}(m)$ is the $\mathrm{m}^{\text {th }}$ pilot subcarrier value. Fig 1 illustrates the pilot arrangement in an OFDM system.

We define $\left\{H_{p}(k), \quad k=0,1, \ldots . N_{p}\right\}$ as the CFR at pilot subcarriers. The estimate of the channel at pilot subcarriers based on LS estimation is given by:

$$
\begin{equation*}
H_{e}=\frac{\hat{Y}_{p}}{X_{p}} \quad \mathrm{k}=0,1, \ldots . \mathrm{N}_{\mathrm{p}}-1 \tag{2}
\end{equation*}
$$

where $Y_{p}(k), X_{p}(k)$ are output and input at the $k^{t h}$ pilot subcarrier respectively.


Fig.1. The pilot arrangement in the OFDM system
A good example of the OFDM system that uses pilot-aided channel estimation is the DVB-T/T2 system, where the pilots will be used with interpolation techniques in the time domain and frequency domain. The channel estimation, which is performed independently at each symbol in the frequency domain, is called onedimensional (1D) estimation, while the estimation in both time and frequency domain is two-dimensional (2D) estimation.

In 1D channel estimation, the CFR will be estimated by using different interpolation methods such as: linear, cubic, spline interpolation... in the frequency domain. This method is easy to implement as it bases on the correlation function between the subcarriers in the frequency domain to estimate the CFR of the current symbol.

To increase the accuracy, the pilot information in both time and frequency domains will be used, which is called the 2D channel estimation. The 2D estimation method is illustrated in Fig. 2. The choice of 2D channel estimation gives a good tradeoff between performance and complexity [6]. First, the estimation is performed in the time domain using interpolation and pilot subcarriers of some OFDM symbols. The second step is to use these estimated values as pilots to estimate the channel response for all of the remaining subcarriers on each OFDM symbol.


PP2/DVB-T/T2
Fig. 2. Two-dimensional channel estimation in the DVB-T system

## 3 Optimized pilot pattern for low-dispersive channel

In [7] the multipath channel with N rays is considered, each of which has a delay of $\tau_{n}$, phase $-\varphi_{n}$, and intensity $-\mu_{n}$. In this case, the CFR of a multipath channel can be presented as:

$$
\begin{equation*}
H(\omega)=\sum_{n=1}^{N} \mu_{n} e^{-j\left(\varphi_{n}+\omega \tau_{n}\right)} \tag{6}
\end{equation*}
$$

Transmitting K of pilot signals through this channel on the known frequencies on the receiving side it is possible to define $\mathrm{V}_{\mathrm{k}}$ of the voltages generated by pilot signals.

$$
\begin{equation*}
V_{k}=\sqrt{\rho_{s}} H\left(\omega_{k}\right)+n_{k} \tag{7}
\end{equation*}
$$

where $\omega_{k}$ is the frequency of the $k$ pilot carrier, $n_{k}$ is a component of Gaussian noise power, and $\rho_{s}$ is the signal-to-noise ratio (SNR) for $k^{t h}$ component of the received pilot signal. Estimation of the CFR of a multipath channel (6) is described as a delay line with $K-1$ taps, with the same delay $\Delta \tau$ between them [7]

$$
\begin{equation*}
\widehat{H}_{k}=\sum_{m=1}^{K} z_{m} e^{-j \omega_{k}(m-1) \Delta \tau}, \Delta \tau=\frac{\tau_{\max }}{K}, \tag{8}
\end{equation*}
$$

where $z_{m}$ - complex parameters defining amplitude and signal phase for tap $m$ of a delay line, $\tau_{\max }$ - possible maximum time delay in the channel. In the estimation process, this value ( $\tau_{\max }$ ) will be substituted by $T_{\max }$ and the relation between these two values will be evaluated in section 4.
Determination of the multipath channel characteristics is reduced to an estimate of $z_{m}$ by the measured values of voltages $V_{k}$ generated by pilot signals on the receiver. The approaches of the theory of potential noise immunity [9] are based on the methods of mathematical statistics, which allows us to evaluate the unknown parameters of the
signal. Estimation of $z_{k}$ the parameter is performed by the maximum likelihood method. In this case, the logarithm of the likelihood function (9) for the $k^{\text {th }}$ component of the signal is expressed as

$$
\begin{equation*}
L_{k}\left(z_{1}, z_{2}, \ldots, z_{K}\right)=\left|V_{k}-\sqrt{\rho_{s}}\left[\sum_{m=1}^{K} z_{m} e^{-j \omega_{k}(m-1) \Delta \tau}\right]\right|^{2} \tag{9}
\end{equation*}
$$

The Maximum likelihood estimates of parameters $z_{1}, z_{2}, \ldots z_{K}$, are determined from K equations as follows

$$
\begin{equation*}
\frac{\partial L_{k}\left(z_{1}, z_{2}, \ldots, z_{K}\right)}{\partial \overline{z_{k}}}=\sqrt{\rho_{s}} e^{-j \omega_{k}(m-1) \Delta \tau} .\left(V_{k}-\sqrt{\rho_{s}}\left[\sum_{m=1}^{K} z_{m} e^{-j \omega_{k}(m-1) \Delta \tau}\right]\right) \tag{10}
\end{equation*}
$$

Using the vector form, we have the main equation (11), connecting the columns of measured values $\vec{V}$ and estimated parameters $\vec{Z}, \vec{Z}$ is the estimate of CIR

$$
\begin{equation*}
\vec{Z}=\left(\frac{1}{\sqrt{\rho_{s}}}\right)\left[\overline{A^{T}} \cdot A\right] \cdot \overline{A^{T}} \cdot \vec{V} \tag{11}
\end{equation*}
$$

where $\vec{V}=\left\|V_{k}\right\|, \vec{Z}=\left\|Z_{k}\right\|, A=\left\|a_{m k}\right\|$ - the phase shift matrix, with $a_{m k}=$ $e^{-j \omega_{k}(m-1) \Delta \tau}$.

The main difference between interpolation methods and the proposed method is the calculating procedure. The accuracy of the interpolation essentially depended on the number of the interpolated points, in other words, the greater number of pilots, the better the estimation results. With the proposed method, CFR is reconstructing via CIR by mathematical transformation, it is crucial to determine the right pilot spacing for accurately estimating CIR, the latter depends on the maximum delay $\tau_{\max }$ of the channel.
Fig 3 illustrates the result CIR and CFR using the proposed method with 24 pilot subcarriers.


Fig. 3. Channel estimation using the proposed method with $\mathrm{K}=24$

## 4 A case study: Channel estimation for DVB-T/T2 system

In the following, we present a case study for a practical commercial DVB-T/T2 system with major parameters listed in Table 1. We use MATLAB simulation with the Rayleigh channel model, the detailed power spectrum density and path delays of the channel are from [10]. Other simulation parameters (modulation type, bandwidth, subcarrier spacing, the central carrier frequency) are from the system DVB-T in 2 K mode. Our model simulates not only the CIR, CFR estimation but also evaluates the BER of the system, which is the major performance metric for the optimization process.

Table 1. Simulation setup using DVB-T system parameters.

| Parameters | Specifications |
| :---: | :---: |
| Guard interval | $1 / 8$ |
| The type of guard interval | Cyclic extension |
| Model of the channel | "Urban type channel", Rayleigh |
| Bandwidth | 6 MHz |
| Speed of the transmitter | $60 \mathrm{~km} / \mathrm{h}$ |
| Type of modulation | QPSK |

### 4.1 Optimizing the pilot spacing

In [7], the value $\tau_{\max }$ in (8) was substituted by the estimated value $T_{\max }$, in which the author proposed the ratio $\tau_{\max } / T_{\max }=(0.7-0.9)$. The work in [8] surveyed the method with the quasi-static channel [10], and this ratio was reduced to get better estimation results. As presented in [7], [8], when applied to a specific system, the ratio must be adjusted with the condition of the channel (maximum delay) and the system (bandwidth and subcarrier spacing).

In the following, we present an empirical approach to optimized pilot spacing in the case of the low-dispersive channel profile. A system model using the proposed method is simulated with the parameters taken from the DVB-T/T2 system, working in 2 K mode, with QPSK modulation as presented in Table 1. These MATLAB simulations are performed with different values of the ratio $\tau_{\max } / T_{\max }$ using some channel models from [10] to find out the optimal $\tau_{\max } / T_{\max }$ range based on BER.

Fig. 4 shows BER performance with the changes of the ratio $\tau_{\max } / T_{\max }$. In the specific case of a low-dispersive channel, the ratio $\tau_{\max } / T_{\max } \leq 0.7$ give the best BER performance, compared to other ranges. It is important to keep the first and the last pilot signals at the edge frequencies of the band, otherwise the estimated CFR might be incorrect at these locations. The number of pilots in use K depends on the spacing of two adjacent pilots and the width of the frequency band.


Fig. 4. BER performance with different $\tau_{\max } / T_{\max }$

### 4.2. Performance evaluations

This section presents the detailed simulation results and the comparison between the methods' efficiency using the BER parameter. With the analysis above, we simulate those estimation methods in MATLAB. The detailed parameters are presented in Table 1. Fig. 5 shows the results using 1D and 2D channel estimation. The simulation result in Fig. 5 shows that 1D channel estimation and 2D channel estimation both give adequately accurate results, but the 2D curve fits the actual CFR more closely than 1D with the tradeoff is the higher implementation complexity.


Fig. 5. Conventional 1D and 2D channel estimation

Applying the novel method in the case of the DVB-T system in 2 K mode with the above-mentioned ratio of $\tau_{\max } / T_{\max } \leq 0.7$, an optimized pilot pattern can be achieved as presented in Fig.6. As can be seen from the pilot location, the proposed pattern has only about one fourth of the conventional pattern pilot number. Besides, with these locations, the calculation procedure will be simpler, as the receiver does not need to recognize the order of the symbol for the pattern identification as in 1D and 2D methods.


Fig. 6. The location of pilot subcarriers in both methods
To evaluate and compare the efficiency of the estimation methods, we simulated the conventional methods and the proposed method on the same system configuration. Fig. 7 shows the BER performance in the cases of AWGN and the fading channel.


Fig. 7. BER performance of the channel with AWGN, 1D, 2D of the conventional estimation and the novel channel estimation.

In Fig. 7, the AWGN is the lowest line - as it is the ideal transmission condition. Other BER lines show that the performance of the novel pattern is better than 1D channel estimation but a little inferior to 2 D channel estimation. When comparing the conventional method and the novel method, with the same SNR (at 28dB), the BER performance of conventional 1D channel estimation is $1.146^{*} 10^{-3}$, while of novel method achieved $1.109 * 10^{-3}$ and BER of the 2D method is $9.9 * 10^{-4}$. From the results, it is clear that the proposed pattern gives similar accuracy while using a significantly less number of pilots (only one-fourth of the DVB-T/T2 pilot number - per symbol), which also means higher spectral efficiency. Besides, regarding the complexity of the estimation, with the constant location at every OFDM symbol, the proposed pattern using the novel method is less complex than those conventional ones as the symbol order identification is not required.

## 5 Conclusion

In this paper, we studied the channel estimation using the comb-type pilot arrangement and proposed an approach for optimization of the pilot pattern applied in the low-dispersive channel, where the CFR is indirectly calculated from the estimated CIR. The performance of the estimation method is quantitatively evaluated in terms of BER and compared with the conventional methods in DVB-T/T2 system in 2K mode. The simulation results show that the BER performances of the two methods are mostly equivalent while the proposed pilot pattern using significantly fewer pilots and a simpler calculation procedure, which essentially helps to save the system bandwidth and energy. With the inherent advantage in spectral efficiency, the proposed approach could be suited for other OFDM systems, especially for wide-band ones, though the broader study and analysis are still required for the solid validation.

## Reference

1. F. Yang, J. Wang, J. Wang, J. Song, and Z. Yang: Channel estimation for the Chinese DTTB system based on a novel iterative PN sequence reconstruction, IEEE International Conference on Communications, pp. 285-289, 2008.
2. Aida Zaier and Ridha Bouallègue: Channel estimation study for block- pilot insertion in OFFDM systems under slowly time-varying conditions, International Journal of Computer Networks \& Communications (IJCNC) Vol.3, No.6, November 2011.
3. Hala M. Mahmoud, Allam S. Mousa, Rashid Saleem: Channel Estimation Based in Comb-Type Pilots Arrangement for OFDM System over Time Varying Channel, Journal of the network, Vol. 5, No. 7, July 2010.
4. Fatima Abdeen Farah Edris, Dr. Ashraf Gasim Elsid, Dr. Mohamed. H. M. Nerma: A Study Of Channel Estimation In Fast Fading Environments, international journal of scientific \& technology research volume 4, Issue 08 , August 2015.
5. Spiridon Zettas, Pavlos I. Lazaridis, Zaharias D. Zaharis, Stylianos Kasampalis,
and John Cosmas: A Pilot Aided Averaging Channel Estimator for DVB-T2, IEEE, 2013.
6. Ahmad R.S. Bahai, Burton. R. Saltzberg, Mustafa Ergen: Multi-Carrier Digital Communications Theory and Application of OFDM, 2nd ed, 2004.
7. M. A. Bykhovskiy: Method opredeleniya parametrov mnogoluchevogo kanala svyazi [Method for determining the parameters of multipath communication channel], Electrosvyaz [Telecommunication], no. 3, pp. 46-49, 2010 [In Russian].
8. Tham Tran, Mikhail Prokopchuk and Alexander Dvorkovich: Study of channel response estimation method based on theory of optimum noise immunity, Engineering and Telecommunications - En\&T-2019, 2019.
9. V. A. Kotelnikov, The theory of optimum noise immunity. Mineola, 1960.
10. Digital Radio Mondiale (DRM), ETSI System Specification ES 201980 V4.1.2, April 2017.
