Caustic Analysis of Reflected Rays from a Spherical Reflector Antenna

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***Abstract* — Now, 5G mobile system is developing to increase user capacity and reduce connection latency. At radio wave technology, new challenges such as millimeter wave (28GHz), small cell size and multi beam base station are introduced. At millimeter wave, where antenna size becomes small size of almost 30cm, a reflector antenna becomes promising. In order to use a reflector antenna for multi beam application, a spherical reflector is suitable. However, a spherical reflector has a problem of distributed caustics. In designing antenna, to clarify caustic positions is important. In this paper, the caustic position equation is derived. Accuracy of the equation is ensured by comparing with MATLAB simulation results. Finally, useful data for dual spherical reflector designing is obtained.**

Index Terms — dual spherical reflector, caustic, ray tracing, multi beam, 5G mobile

I. Introduction

Now, 5G mobile system is developing to increase user capacity and reduce connection latency [1][2]. At radio wave technology, new challenges such as millimeter wave (28GHz), small cell size and multi beam base station are introduced [3]. At millimeter wave, where antenna size becomes small size of almost 30cm, a reflector antenna becomes promising because of obtaining high gain and fabrication ease.

Previously, for multi beam applications, a dual spherical reflector was developed [4]. The main and sub reflectors had the same center of circle. Even though good multi beam were obtained, spherical phase aberration on the aperture plane reduced antenna gain.

In order to reduce the spherical phase aberration, sub reflector shaping will be needed. Previously, some dual reflector shaping methods were developed. The method of designing the shaped sub-reflector [5] and employing an equivalent reflector [6] were typical shaping methods. The equivalent reflector method may be useful in the sub-reflector shaping. In designing sub-reflector shaping, to determine the sub-reflector size by taking into account the caustic of the spherical main-reflector.

In this paper, analysis of the caustic of the spherical main-reflector is performed. The equation for the caustic is derived. The accuracy of equation results are ensured through comparing with the results of MATLAB simulations. Finally, important data to determine the sub reflector size is obtained.

II. Derivation of Caustic Equation

Structural parameters of a spherical reflector antenna and related rays are shown in Fig.1. The radius of a spherical reflector is indicated as R. The additional circle shown by a dashed line is used to determine reflected rays from the spherical reflector. When the incident ray is parallel to the horizontal axis (z-axis), the angle of reflected ray (ϕ) is equally divided by the reflector radius. And α becomes same as the inclination angle (α) of the reflector radius. Therefore, the reflected ray point on the z-axis is given by the crossing point with a line that is tangential on the C_r circle. The distance of this crossing point and the center of the reflector circle is indicated as r. The distance of two reflected rays is shown by Δr. The caustics of reflected ray are shown by the ⊗ marks. In the case of a spherical reflector, caustics are distributed in a wide area. To understand the locus of caustics becomes very important.

From now, derivation of the locus is explained. Firstly, r is given by the next expression.

$$r\cos\alpha = \frac{R}{2} \tag{1}$$

As for the distance from the center of reflector circle, next equation is given.

$$(r+\Delta r)\cos(\alpha+\Delta\alpha) = \frac{R}{2} \tag{2}$$

The Eq. (2) is reformed as follows.

$$r\cos\alpha - r\sin\alpha\sin\Delta\alpha + \Delta r\cos\alpha - \Delta r\sin\alpha\sin\Delta\alpha = \frac{R}{2} \tag{3}$$

The Eq. (3) can be expressed in more simple form by neglecting the second order smalls.

$$r\sin\alpha\sin\Delta\alpha = \Delta r\cos\alpha \tag{4}$$

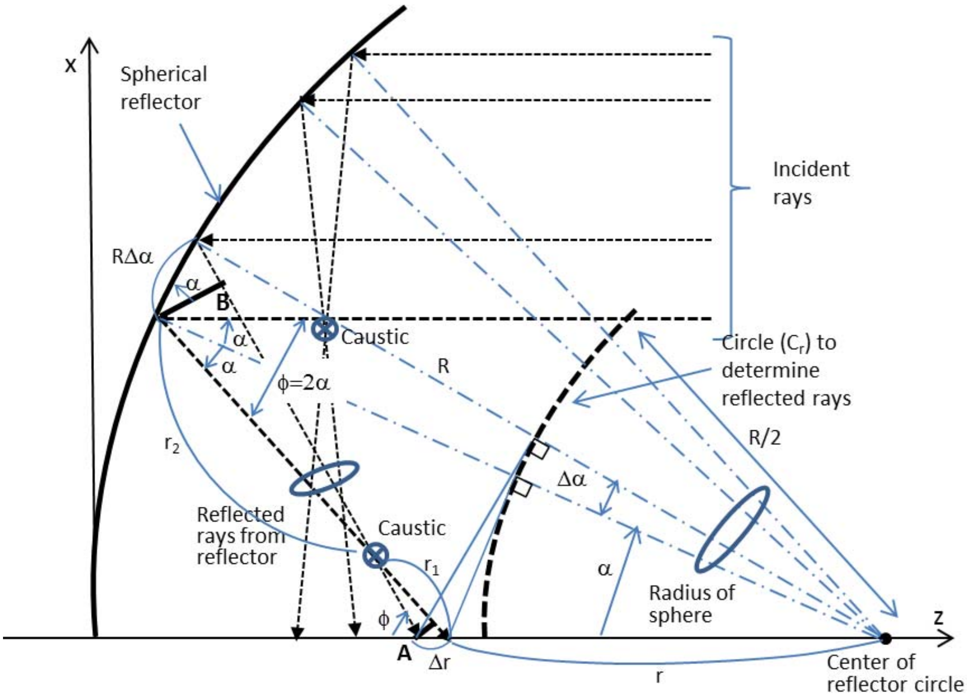

Fig.1 Parameters of reflected rays from spherical reflector

As a result, Δr is expressed by the next equation.

$$\Delta r = \frac{R \sin\alpha \sin\Delta\alpha}{2(\cos\alpha)^2} \tag{5}$$

III. Dual Spherical Reflector Design

In designing a dual spherical reflector, the sub-reflector size should be determined adequately for the size of a main-reflector. The situation is shown in Fig. 2. In the case of the reflection angle (ϕ_1) is large, the main-reflector radius (R_{M1}) and caustic (R_{C1}) are indicated in Fig. 2. For smaller reflection angle (ϕ_2) is large, the main-reflector radius (R_{M2}) and caustic (R_{C2}) are indicated in Fig. 2. The ratio of R_M/R_C can be calculated by using the r_2/r_1 ratio as shown by the next equation.

By using the Δr equation, the distance equation of A is obtained.

$$A = \Delta r \sin 2\alpha = \frac{R \sin\alpha \sin\Delta\alpha \sin 2\alpha}{2(\cos\alpha)^2} \tag{6}$$

The distance of B is expressed by the next equation.

$$B = R\Delta\alpha \cos\alpha \tag{7}$$

Finally the ratio r_2/r_1 that gives the caustic is expressed by the next equation.

$$\frac{r_2}{r_1} = \frac{B}{A} = \frac{2R\Delta\alpha(\cos\alpha)^3}{R\sin\alpha\sin\Delta\alpha\sin 2\alpha} = \frac{2(\cos\alpha)^3}{\sin\alpha\sin 2\alpha} = \left(\frac{\cos\alpha}{\sin\alpha}\right)^2 \tag{8}$$

$$\frac{R_M}{R_C} = \frac{r_1 + r_2}{r_1} = 1 + \frac{r_2}{r_1} = 1 + \left(\frac{\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{(\sin\alpha)^2} \tag{9}$$

When designing the sub-reflector, the radius should be larger than the caustic point so as to achieve ray tracing to a feed point successfully.

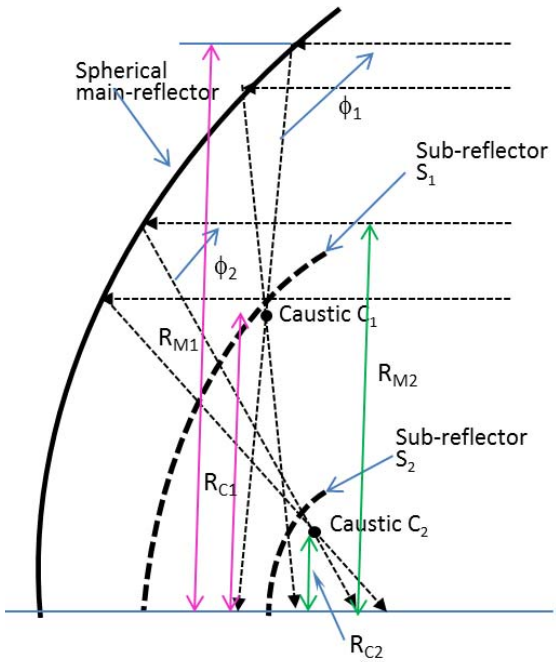

Fig.2 Configuration of dual spherical reflector

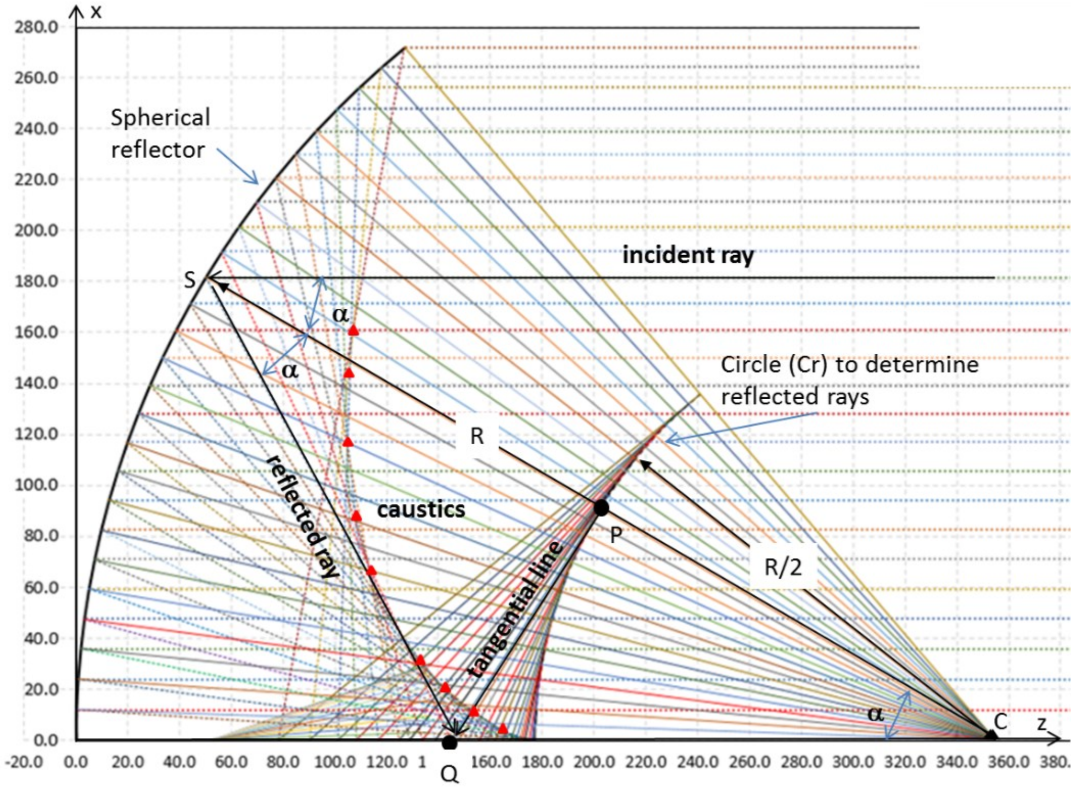

Fig.3 Ray tracing results by developed MATLAB program

IV. Ray tracing calculation by matlab program

The caustics are calculated by a developed MATLAB program. The content of the MATLAB program is shown in Fig. 3. The incident and reflected rays have the same angle (α) to the radial line (R) of the spherical reflector. So, the rectangles SQP and CQP become the same. Then, the point Q is given by the crossing point of tangential line at point P and the z-axis. And, the point Q becomes the crossing point of reflected ray and the z-axis. In this manner, all reflected rays are determined for given incident ray having angle (α). Finally, caustics of reflected rays shown by ▲ marks are determined. The x-axis values of incident ray and caustic correspond to R_M and R_C, respectively.

In this calculation, the radius of sphere of R = 354.8 mm and maximum angle of α = 50 deg. are given. Obtained relation of R_C/R_M and α is shown in Table1. At α = 50 deg., R_M = 270 mm and R_C = 160.8 mm. So, R_C/R_M = 0.6.

TABLE I
RC/RM vs α Values By MATLAB

α	20	25	30	35	40	45	50
R_M	115.3	148.7	166.9	200.2	227.5	245.7	270
R_C	15.2	27.3	42.5	72.8	100.1	130.4	160.8
R_C/R_M	0.13	0.18	0.26	0.36	0.44	0.53	0.6

V. MATLAB and Equation Results

Comparisons of Eq. (9) and MATLAB calculation results of Table1are shown in Fig.4. Good agreements between the equation and MATLAB results are obtained. Accuracy of the equation expression is ensured. The results of Fig. 4 is used to determine sub-reflector radius (R_S) for a given main-reflector radius (R_M). R_S should be selected larger than R_C in order to make the sub-reflector work effective. One more important result is that the reflection angle (ϕ = 2α) has a limit at the general antenna configuration of R_S/R_M < 0.25. The reflection angle (ϕ) should be selected ϕ < 60 degrees.

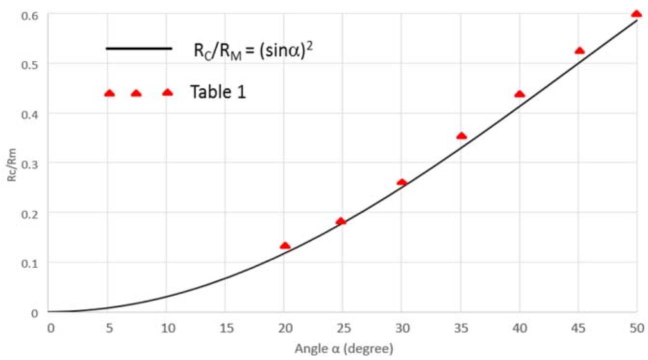

Fig.4 Comparisons of caustic results by equation and MATLAB calculation

VI. Conclusion

Analysis of caustic positions of a spherical reflector antenna is achieved. The equation expression for caustic positions is derived. The accuracy of the equation is ensured by comparing caustic positions of equation and MATLAB calculation results. An useful design chart that determine the possible sub-reflector size at a given main-reflector size is obtained. At the same time, the structural limitation in designing a spherical reflector is obtained.

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