Image Reconstruction Utilizing Algebraic Helmoltz Inversion and Passband Filtering Applied to Viscoelasticity

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***Abstract*— Shear wave viscoelasticity imaging is a quick and convenient imaging model for collecting images of tissues. The mechanical properties of soft tissues (elasticity, viscosity) are expressed through the complex shear modulus (CSM) parameter, a useful information for tissue pathological diagnosis. In this paper, we proposed a new solution in which we use FDTD method for modeling the wave propagation in 2D environment using FDTD method; then apply the a band-pass filter (BPF) to reduce the noise from the measured particle velocity of the shear wave; finally, we use the Algebraic Helmholtz Inversion (AHI) algorithm to directly estimate CSM. The results of numerical simulation have proven the effectiveness of the proposed algorithm.**

Keywords*— *Shear wave viscoelasticity imaging, complex shear modulus (CSM), viscosity, elasticity, finite-difference time-domain method (FDTD), Algebraic Helmholtz Inversion (AHI).

I. INTRODUCTION

In 1998, Sarvazyan et al. introduced the technique of shear wave elastic imaging (SWEI) for medical diagnosis [1]. In 2004, Chen and colleagues formulated an expression which shows that shear wave velocity is related to the elasticity and viscosity of the medium [2]. Accordingly, they proposed a quantitative method of estimating tissue elasticity and viscosity by measuring shear wave velocity. In 2010, Orescanin Marko et al. applied the Maximum Likelihood Ensemble Filter (MLEF) to estimate CSM parameters for homogeneous environments based on the Kelvin-Voigt model [3]. In 2013, an estimate of CSM using MLEF for heterogeneous environments was proposed in [4]. The downside of the MLEF is that the computational volume is large and therefore not suitable for orientations to real-time CSM imaging devices. So far, researches on CSM estimation and shear wave imaging has been of great interest to various research groups [5], [6], [7], [8], [9].

The shear wave propagation in biological soft tissue is modeled by the basic wave equation, which shows that the particle velocity value of the shear wave is a function that is related to the past particle velocity value. However, this model only suits a homogeneous environment. In heterogeneous environments (such as in biological soft tissue), the velocity value of a shear wave depends on its historical value and is also influenced by the value immediately preceding the investigated location.

Therefore, in this paper, we have applied the FDTD to model the shear wave propagation in biological soft tissue (a complex heterogeneous environment). This method shows that the particle velocity value of the shear wave is a function that is influenced by both space and time. Particle velocity data of the measured shear wave include the measurement noise. Therefore, a proposed band-pass filter is used to reduce frequency fluctuations and measurement noise. Finally, we apply the AHI algorithm to directly estimate the CSM (elasticity and viscosity). From there, we can represent a 1D or 2D images of tissue elasticity and viscosity. It is known that the activated frequency of an elastic ultrasonic imaging system is a single frequency. Previously, in order to reduce frequency fluctuations and measurement noise, a low-pass filter (LPF) has been proposed [11], however, measurement noise is not completely eliminated. In order to receive signals around the operating frequency, a band-pass filter has been proposed. The numerical simulation results have shown that the normalized error is significantly reduced in comparison with the previous study applying low-pass filter. Moreover, the proposed algorithm does not need to use optimal filtering, or adaptive filtering as previous studies [11]. Therefore, the complexity of the imaging system will be lower and facilitate the application for real-time imaging.

II. METHODOLOGY

A. Shear wave propagation

In the shear wave viscoelasticity imaging system, a needle vibrates at a specified frequency along the Z axis, shear waves, then propagate on the X-Y plane (see Fig. 1). We can collect particle velocities using Doppler ultrasonic devices [12].

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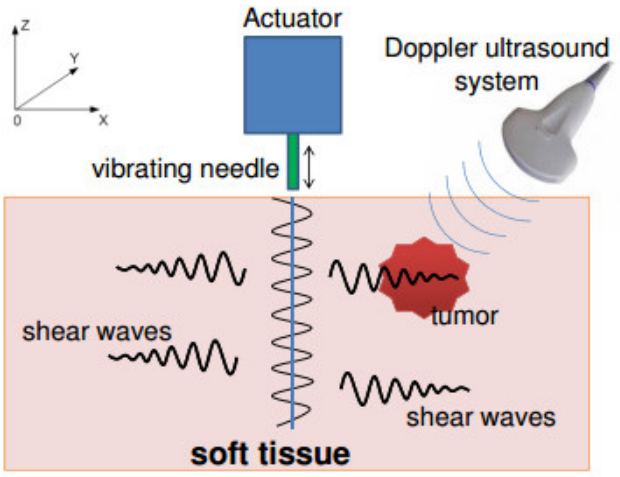


Figure 1. Excitation and measurement of shear wave.

Equation (1) is used to calculate the particle velocity $v(r,t)$ at spatial position r and time t [12]

$$v(r,t) = \frac{1}{\sqrt{r-r_0}} A e^{-\alpha(r-r_0)} cos[\omega t - k_s(r-r_0) - \phi] \quad (1)$$

where A is the vibration amplitude of the needle, r_0 is the spatial position of the needle, ϕ is the initial phase, α and k_s are the attenuation coefficient and the wavenumber in the spatial position r.

The advantage of formula (1) is its simplicity. However, this formula does not reflect shear wave propagation in real tissue, especially in heterogeneous environments. Therefore, the FDTD method is used with the assumption that the shear wave propagation shifts along the radial axis and ignores the absorption of the medium. The relationship between the particle velocity vector v_z in the direction of x-wave propagation in the Cartesian coordinate system and the compression tensor σ_{zx} can be described by equations (2) and (3) [13]:

$$\rho \partial_t v_z = \partial_x \sigma_{zx}, \quad (2)$$

$$\partial_t \sigma_{zx} = (\mu + \eta \partial_t) \partial_x v_z, \quad (3)$$

where ∂_t is the partial derivative operator ∂/∂_t applied to the right value of the symbol, ∂_x is the partial derivative operator ∂/∂_x applicable to the right value of the symbol, ρ is tissue density, μ and η are the tissue elasticity and viscosity, respectively.

CSM, $G(x,\omega)$, is the complex function of the angular frequency ω as follows:

$$G(x,\omega) = \mu(x) - i\omega\eta(x) \quad (4)$$

where μ is the elasticity and η is the viscosity to be estimated.

To discrete equations (2) and (3), the following symbols are used:

$$v_z(x,t) = v_z(i\Delta x, n\Delta t) = v_z^n|_i \quad (5)$$

$$\sigma_{zx}(x,t) = \sigma_{zx}(i\Delta x, n\Delta t) = \sigma_{zx}^n|_i \quad (6)$$

where Δx is the distance between consecutive spatial positions, Δt is the sampling period, index i is the spatial step, and index n is the time step.

FDTD method is applied to (2) and (3) as follows:`

$$v_z^{n+1}|_i = v_z^n|_i + \frac{\Delta t}{\rho \Delta x}\left(\sigma_{zx}^{n+\frac{1}{2}}\Big|_{i+\frac{1}{2}} - \sigma_{zx}^{n+\frac{1}{2}}\Big|_{i-\frac{1}{2}} \right) \quad (7)$$

$$\sigma_{zx}^{n+\frac{1}{2}}\Big|_{i+\frac{1}{2}} = \sigma_{zx}^{n-\frac{1}{2}}\Big|_{i+\frac{1}{2}} + \frac{\mu \Delta t}{\Delta x}(v_z^{n+1}|_{i+1} - v_z^{n+1}|_i) + \frac{\eta}{\Delta x}(v_z^{n+1}|_{i+1} - v_z^{n+1}|_i) - \frac{\eta}{\Delta x}(v_z^n|_{i+1} - v_z^n|_i) \quad (8)$$

B. Enhanced signal by using band-pass filter

The particle velocity may be disturbed from the noises such as the measured noise, reflection, etc. An efficient filter is needed to enhance the signal before applying AHI to directly estimate CSM. In this paper, we designed a band-pass filter to reduce the noise from the particle velocity that is collected from the Doppler ultrasound system. In [14], the noise impact was reduced by applying a low-pass filter. This filter only can remove the high frequency noise. In this paper, we only keep the frequency band that consist the exciting frequency.

The particle velocity signal, expressed as $v_z(n)$, is transmitted into the tissue and is affected by noise, represented as $z(n)$. Together, they form the $v_z(n)$ noise signal described by:

$$v(n) = v_z(n) + z(n) \quad (9)$$

This noisy signal $v(n)$ is applied as the input of the band-pass filter to extract an estimate of the desired signal $v_z(n)$. The output of band-pass filter or filtered signal is $\hat{v}_z(n)$. The noisy signal is sampled and it forms a vector containing N samples.

$$v(n) = [v_z(0)\ v_z(1)\ \dots\ v_z(N-1)], \quad (10)$$

and the coefficients of the filter are expressed as follows:

$$\omega(n) = [\omega(0)\ \omega(1)\ \dots\ \omega(L)] \quad (11)$$

where L is the order of the filter. The filter coefficients, also called the weights $\omega(n)$. Magnitude frequency response of the designed filter is shown in Fig. 2.

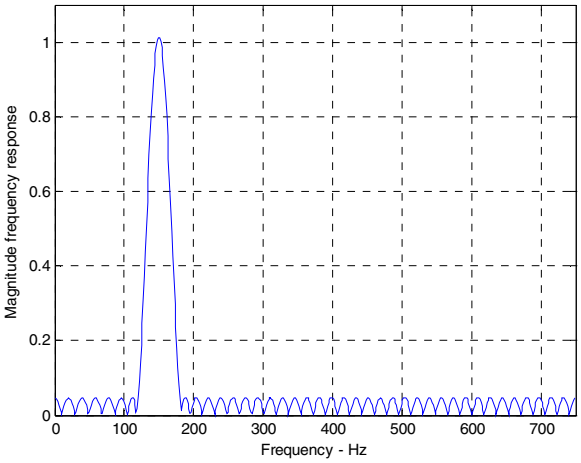


Figure 2. Magnitude frequency response of the designed filter

C. Direct inverse transformation using AHI to estimate CSM

In this step, AHI algorithm [14] is used to calculate the CSM. For small volumes, suppose that the viscous elasticity of tissue is isotropic. We integrate (2) and (3) to obtain:

$$\rho \frac{\partial^2 v_z}{\partial t^2} = G'(x,t)\nabla^2 v_z \tag{12}$$

where $G'(x,t)$ is the CSM in the time domain and $\nabla^2 v_z$ is the Laplace operator of v_z defined as $\nabla^2 v_z = \frac{\partial^2 v_z}{\partial x^2}$.

The AHI algorithm is used to solve (12), then becomes the Helmholtz equation:

$$\left(\frac{G(x,\omega)}{\rho}\nabla^2 + \omega^2\right) V_z(x,\omega)|_{\omega=\omega_0} = 0 \tag{13}$$

where $G(x,\omega)$ is CSM in the frequency domain and is defined in (4), $V_z(x,\omega)$ is a Fourier transform over time of particle velocity $v_z(x,t), V_z(x,\omega) = F_t\{v_z(x,t)\}$, and ω_0 is the angular frequency $\omega_0 = 2\pi f_0$. From (13), we can see that CSM can be directly estimated as follows:

$$\mu(x) = \Re\left\{\frac{-\rho\omega_0^2 V_z(x,\omega_0)}{\nabla^2 V_z(x,\omega_0)}\right\}$$
$$\eta(x) = \Im\left\{\frac{-\rho\omega_0^2 V_z(x,\omega_0)}{\nabla^2 V_z(x,\omega_0)}\right\} \tag{14}$$

where $V_z(x,\omega_0)$ is calculated using the Fourier transform at the specified angular frequency ω_0; $\nabla^2 V_z(x,\omega_0)$ is calculated using the discrete Laplace function (The MathWorks) $del2\left(V_z(x,\omega_0)\right)$ returns the discrete approximation of the Laplace differential operator applied to $V_z(x,\omega_0)$.

The proposed algorithm for estimating CSM is summarized in Algorithm 1.

Algorithm 1. *The proposed algorithm of estimating CSM*

Step 1. Initiate the simulation.
Step 2. Choose excitation frequency $f_0 = 150Hz$.
Step 3. Emitting shear wave by needle oscillation.
Step 4. Collect velocity particles with noise at 120x120 spatial positions.
Step 5. Filter the noise using the band-pass filter.
Step 6. Remove the transient parts of the filtered signal.
Step 7. Calculate Fast Fourier Transform (FFT) of the filtered signal.
Step 8. Compute each CSM at the locations of the space using (14).
Step 9. Assess the estimated performance.
End.

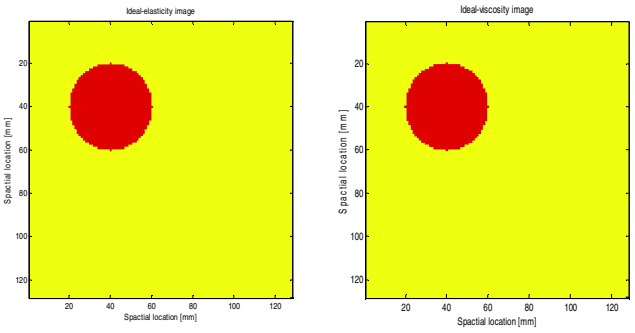
III. Numerical Simulation Results

Simulation parameters: The 2D medium has a size of 120×120 , which contains a tumor at the coordinate (40 mm, 40 mm), the radius of the tumor is 20 mm, the elasticity of the medium is μ_1 = 6000 Pa and the viscosity of the medium is η_1 = 1.2 Pa.s, the elasticity and viscosity of the tumor are μ_2 = 9000 Pa and η_2 = 1.8 Pa.s, vibration frequency of needle f = 200 Hz, bulk density of medium ρ = 1000 kg/m^3, amplitude of needle vibration 5 mm. The ideal images are shown in Fig. 3.

Normalized error of estimating methods of elasticity and viscosity corresponding to time steps is presented in Table 1. Corresponding to time steps of 500, 750, 1000, 1250, reduced normalized errors of elasticity estimated by the proposed method are 13.24%, 39.04%, 16.90%, 19.65% respectively, compared to the traditional method. Similarly, corresponding to time steps 500, 750, 1000, 1250, reduced normalized errors of viscosity estimated by the proposed method are 13%, 9.02%, 8.15%, 8.41%, respectively in comparison with the traditional method. Thus, the image quality estimated by the proposed method (both elasticity and viscosity) when using the band-pass filter is significantly improved, compared to the solution using the low-pass filter.

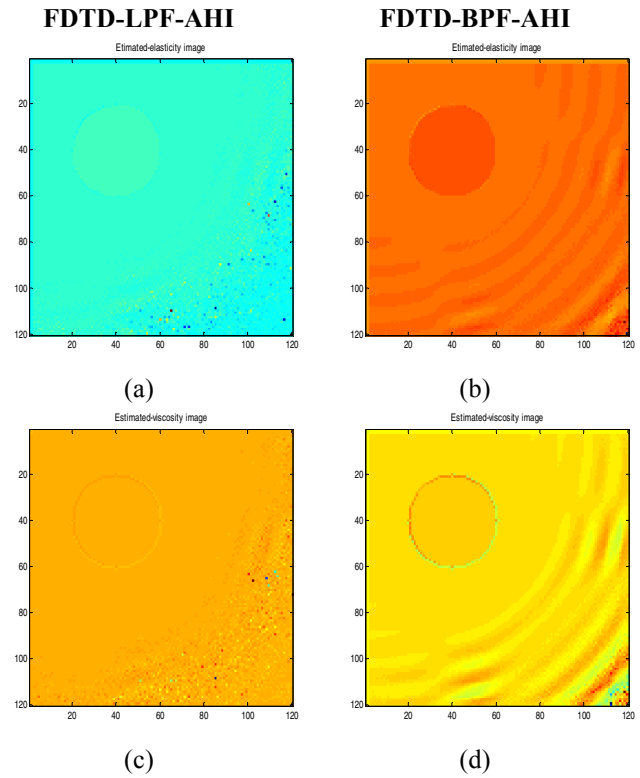
Table 1. Normalized error of estimated methods of elasticity and viscosity with different time steps

Methods	Time step	500	750	1000	1250
FDTD-BPF-AHI	Estimated elasticity	0.0710	0.0292	0.0284	0.0229
	Estimated viscosity	0.5100	0.2406	0.1889	0.1451
FDTD-LPF-AHI	Estimated elasticity	0.0616	0.0178	0.0236	0.0184
	Estimated viscosity	0.4437	0.2189	0.1735	0.1329



(a) (b)

Figure 3. Ideal initial image: (a) Elasticity; (b) Viscosity



(a) (b)

(c) (d)

Figure 4. The restored images by the traditional and proposed methods (in time step of 1500)

As shown in Figures 4 (a)-(d), the reconstructed quality is very good if the spatial points are near the vibration needle. The area of a quarter circle of radius of 120×120 mm centered around the needle position has good estimation. At spatial points outside of this area, the estimation is not good the particle velocity is decayed quickly. It can be seen that our approach offers better estimation than the conventional one in both these cases (inside and outside of the quarter circle of radius of 120×120 mm).

Our research showed that by filtering noise with BPF, the situation was nicely ameliorated and the estimation error was 1.85% for elasticity and 13.29% for viscosity. We may directly compare with an existing study [14] with the same simulation condition: the estimation error was 2.28% for elasticity and 14.64% for viscosity. In [14], the noise impact was reduced by applying a low-pass filter. It means that it only can remove the high frequency noise. In our approach, we only keep the frequency band that consist the exciting frequency.

Our approach also overcome the recent work reported in [15] in term of complexity. In [15], the authors applied least mean square (LMS) filter to reduce noise from the measured particle velocity. However, the LMS algorithms should be used at every spatial points that cause the high computation time. This limitation will prevent the method from real-time implementation.

IV. Conclusions

Elasticity and viscosity are two important parameters used to examine tissue structure, especially for tissue detection. In this paper, we have successfully applied a new solution which consists of FDTD model, band-pass filtering, and AHI. Band-pass filtering only keep the useful frequency bands in the measured particle velocity before AHI is used to estimate CSM directly. The quality of reconstructed images is significantly improved in the proposed solution.

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