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Synthesis of a nonlinear control law with efficiency energy for the self-balancing two wheeled vehicle

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Abstract. In the paper, we present a method of synthesizing the controller with optimum energy for two-wheeled self-balancing vehicles, based on the multi-purpose consumption function. The authors select the multi-purpose consumption function based on the quasi-optimality and minimum of energy. The synthesis of control laws based on the analytical design of aggregated controllers (ADAR) with manifold of quasi-optimality. The results of the proposed method are simulated and compared to other methods.

1. Introduction

A two-wheeled self-balancing vehicle has two wheels with two the same axles, built into a commercial product derived from the original idea of the Segway company. The principle of controlling a two-wheeled self-balancing vehicle is to combine the position control of the person on the vehicle and the automatic balance controller for the vehicle. When you want the car to move straight, the driver just needs to stand straight and lean forward slightly. If you want to increase the speed of the vehicle's movement, the person facing forward more. To stop the driver, just stand upright so that it is perpendicular to the platform, this time the car will automatically stop and still be balanced on the vehicle (fig. 1) [1].

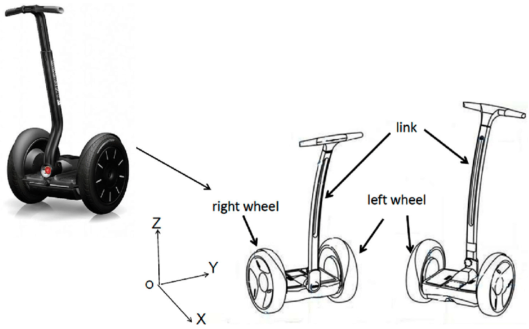

Fig. 1. Two-wheeled self-balancing vehicle

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This is a means of transportation that is used more and more widely with many advantages: compact, light, takes up little area when moving, does not cause environmental pollution, is easy to control, flexible movement in many places, easy to transport, easy to disassemble.

Two-wheeled self-balancing vehicle works based on the inverted pendulum. This is the standard model for developing control algorithms. There has been a lot of control algorithms for this object. In [2, 3] using traditional PID controllers are presented with various methods of determining controller parameters. The studies [4, 5] built the controller according to LQR method but did not consider the energy consumption through control signals. The study [6] proposed a method to use Type-2 fuzzy controller to simplify in building control laws. The studies [7, 8] proposed a control law with quasi-optimality, ensuring system quality, but is not energy efficient. A critical issue with vehicles is energy efficiency when operating. In this paper, the authors propose a method to synthesize the controller with energy efficiency base a synergistic approach by ADAR method to the target function and manifold with quasi-optimality.

2 The mathematical model of the self-balancing two wheeled vehicle

We consider the well known the self-balancing two wheeled vehicle [1,] which is schematically depicted in Fig. 2.

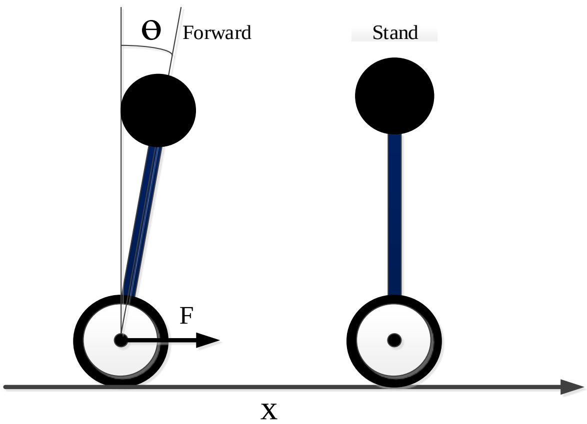

Fig. 2. The self-balancing two wheeled vehicle

The linear mathematical model of two-wheeled self-balancing vehicle at the equilibrium point is described in equations (1) which was presented in [7]:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \dfrac{k_e k_e(M_p lr-I_p-M_p l^2)}{Rr^2\alpha} & \dfrac{M_p l^2}{\alpha} & 0\\ 0 & 0 & 0 & 1\\ 0 & \dfrac{2k_e k_e(r\beta-M_p l)}{Rr^2\alpha} & \dfrac{M_p^2 g l^2\beta}{\alpha} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ \dfrac{2k_m(I_p+M_p l^2-M_p lr)}{Rr\alpha}\\ 0\\ \dfrac{2k_m(M_p l-r\beta)}{Rr\alpha}\end{bmatrix}V_a \quad (1)$$

where $\beta=(2M_\omega+\frac{2I_\omega}{r^2}+M_p)$; $\alpha=(I_p\beta+2M_p l^2(M_\omega+\frac{I_\omega}{r^2}))$; x - displacement (m); ẋ - velocity of vehicle (m/s); ẍ - acceleration of vehicle (m/s2); θ - angular displacement (rad); $\dot{\theta}$- angular velocity (rad/s);$\ddot{\theta}$- angular acceleration (rad/s2);Va- armature voltage (V); km - motor's torque constant (Nm/A); ke - back EMF constant (Vs/rad.); R - motor resistance (**Ω**); l - distance from wheel axle to center of chassis (m); g- gravitational constant (m/s2); Mp- mass of the chassis (kg); r- radius of wheels (m); Ip- moment of inertia of the chassis (Nm); Iω - moment of inertia of the wheels (Nm); Mω - mass of the wheels (kg).

In this article, we need to design a angular controller for your vehicle. The control system block diagram for two-wheeled self-balancing vehicle is as shown in Figure 3.

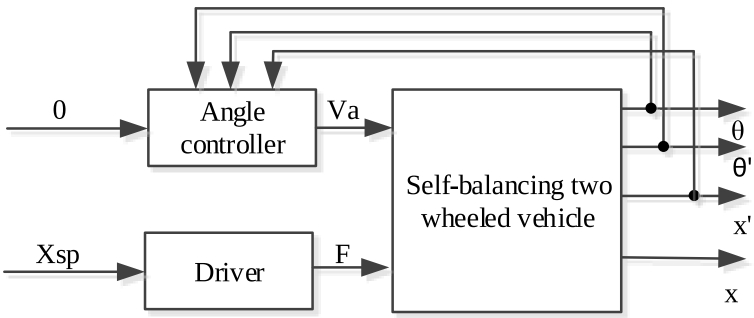

Fig. 3. The control system block diagram for two-wheeled self-balancing vehicle

From the control system block diagram, the driver will apply the force to the car so that the angle deviates from the zero balance point forward. Therefore, in this paper when designing the angle controller, from the system of equation (1), we get the state equation according to the angle ($x_1 = \theta, x_2 = \dot{\theta}$) of two-wheeled self-balancing vehicles as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{M_p{}^2 g l^2 \beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{2k_m(M_p l - r\beta)}{Rr\alpha} \end{bmatrix} V_a + \begin{bmatrix} 0 \\ \dfrac{2k_m k_e(r\beta - M_p l)}{Rr^2\alpha} \end{bmatrix} \dot{x} \tag{2}$$

3 Synthesis of energy efficient angular controller based on synergistic method ADAR

a. Synergistic approach to the analytical design of aggregated regulators.

The analytical design of aggregated controllers (ADAR) proposed in [9, 11] in the sense of ensuring asymptotic stability and given dynamic properties of transients is based on the introduction of functionals of the form:

$$J = \int_0^\infty F(\psi, \dot{\psi}) dt \tag{3}$$

where $F(\psi, \dot{\psi})$ – continuously differentiable function with all arguments; $\psi(x_1, \ldots x_n)$ – aggregated macro-variable, representing some arbitrary or piecewise of differentiable continuous function of phase coordinates $(x_1, \ldots, x_n)$, moreover $\psi(0, \ldots, 0) = 0$.

The essence of the synergetic approach is to construct an accompanying function in such a way as to determine in the phase space of the non-linear object under consideration a certain set of points to which all close motion paths are attracted. Such a set of points is usually called an attractor [10]. The above actually corresponds to the basic principle of synergetics - the principle of submission, on which the theory of self-organization of nonlinear dynamic systems is built [9].

In particular, for a non-linear SISO object, with scalar control, it is advisable to choose the following quadratic form as an integrand in (4):

$$F(\psi, \dot{\psi}) = m^2 \varphi^2(\psi) + c^2 \dot{\psi}^2(t) \tag{4}$$

with the function $\varphi(\psi)$ must satisfy the following conditions:

1. Single-valuedness, continuity and differentiability for all values;
2. $\varphi(0) = 0$;
3. $\varphi(\psi)\psi > 0$ at any $\varphi \neq 0$.

b. Synthesis of angle stability control laws for two-wheeled self-balancing vehicle that are energy efficient

The energy-saving for autonomous vehicles is current issue but there has not been much research. In order to synthesize control laws with energy optimization, studies have shown the target function of energy optimality. The requirement of controller is energy efficient and the system quality is the same as the system with the quasi-time optimal control law. The authors propose a method of synthesizing control laws based on synergistic method ADAR.

Select macro-variables with quasi-optimality $\psi = x_2 + \nu \frac{x_1}{\sqrt{x_1^2 + \varepsilon_1^2}}$. Select the target function for the equations (2) of the form (5):

$$J = \int_0^\infty (\varphi^2(\psi) + T^2\dot{\psi}^2(t))dt \quad (5)$$

where

$$\varphi(\psi) = \frac{\psi}{\sqrt{\psi^2 + \varepsilon_2^2}},\ \dot{\psi} = \frac{2k_m(M_p l - r\beta)}{Rr\alpha}V_a + \frac{M_p{}^2 g l^2\beta}{\alpha}x_1 + \frac{2k_m k_e(r\beta - M_p l)}{Rr^2\alpha}y + \frac{\nu\varepsilon_1^2}{(x_1^2 + \varepsilon_1^2)^{1.5}};$$

From the target function as chosen when away from the equilibrium point $\varphi^2(\psi) \to 1$, the target function becomes $J = \int_0^\infty (1 + T^2\dot{\psi}^2(t))dt$. Clearly, the target function contains the optimal component in terms of energy and the quasi-time optimization.

The law of synthetic control must satisfy the functional equation (6) [9,11]:

$$\varphi(\psi) + T\dot{\psi}(t) = 0 \quad (6)$$

Solving equation (6) and receiving the control law in analytic form has the following form [7]:

$$V_a = -\frac{Rr\alpha}{2k_m(M_p l - r\beta)}\left(\frac{M_p{}^2 g l^2\beta}{\alpha}x_1 + \frac{2k_m k_e(r\beta - M_p l)}{Rr^2\alpha}\dot{x} + \frac{\nu\varepsilon^2}{(x_1{}^2 + \varepsilon^2)^{1.5}} + \frac{1}{T}\varphi(\psi)\right) \quad (7)$$

4 Simulating results of the angle stability control law of two-wheeled self-balancing vehicle

The parameters of mathematical model (1) is determined by real model as follows [1]: km = 0.022(Nm/A), ke = 0.4(Vs/rad), R= 1.7(Ω), l = 0.05 (m), g = 9.81(m/s^2), Mp=1 (kg), r= 0.0325(m), Ip=0.0012($kg.m^2$), Iω=0.000016($kg.m^2$), Mω=0.03(kg). The value of the controller parameters include v=1, ε1=ε2=1, T=0.0025. The simulation was performed on Simulink-Matlab software with the initial condition of the angle of vehicle x1=0.6 (rad), x2=0 (rad/s) with the purpose of bringing the vehicle to the balance position (x1=0, x2=0) with the quasi-optimality and energy-saving. The simulation results were performed with two controllers shown in fig. 4 and fig. 5: Quasi-time optimal controller (KOB) and proposed controller (SI-ADAR). In particular, fig. 4 (a) is the angle response of vehicle, fig. 4 (b) is the velocity response of vehicle, fig.5 (a) is the phase plane of the system. The results show that the system response to both controllers is almost identical, which means that the proposed controller retains the quasi-optimal characteristics of the KOB controller. In fig. 5 (b) is the control signal given by KOB controllers and the proposed method, the results show that the value of the proposed controller is more energy efficient.

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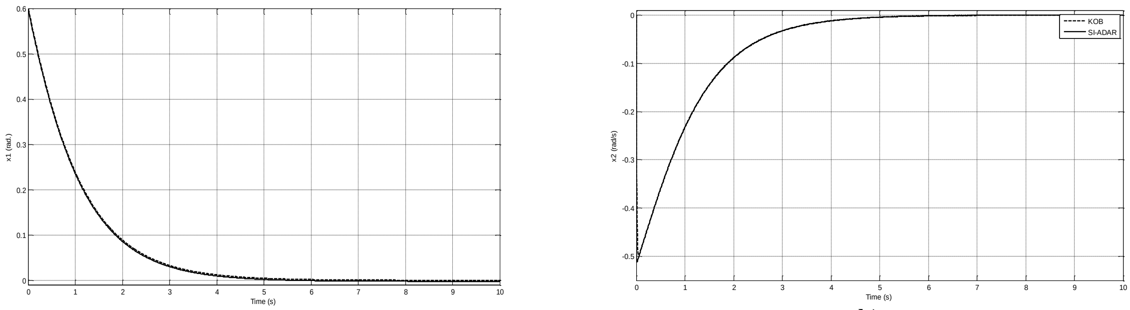
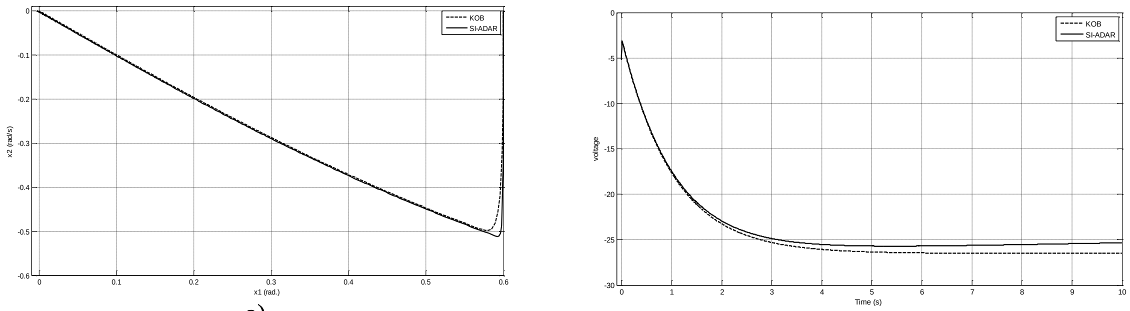


a) b)

Figure 4. a) The angle response of vehicle, b) The velocity response of vehicle



a) b)

Figure 5. a) The phase space of system, b) The control signal

5 Conclusion

Combining the ADAR method with the determined manifold method with quasi-optimality and the target function with energy efficient, the authors have built a control law that is both quasi-optimality and energy efficient. Comparisons with the quasi-time optimal controller show the effectiveness of the proposed method. The control signal power of the proposed method is smaller. Besides, we also see that with the choice of such a target function, when near the control signal equilibrium point there is no energy optimization. Therefore, in future studies, the authors will present methods to overcome these disadvantages.

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