Evolutionary Intelligence
https://doi.org/10.1007/s12065-019-00297-4

RESEARCH PAPER

Improving many objective optimisation algorithms using objective dimensionality reduction

Xuan Hung Nguyen[1] · Lam Thu Bui[1] · Cao Truong Tran[1]

Received: 25 November 2018 / Revised: 12 August 2019 / Accepted: 31 August 2019

Abstract
Many-objective optimisation problems (MaOPs) have recently received a considerable attention from researchers. Due to the large number of objectives, MaOPs bring serious difficulties to existing multi-objective evolutionary algorithms (MOEAs). The major difficulties includes the poor scalability, the high computational cost and the difficulty in visualisation. A number of many-objective evolutionary algorithms (MaOEAs) has been proposed to tackle MaOPs, but existing MaOEAs have still faced with the difficulties when the number of objectives increases. Real-world MaOPs often have redundant objectives that are not only inessential to describe the Pareto-optimal front, but also deteriorate MaOEAs. A common approach to the problem is to use objective dimensionality reduction algorithms to eliminate redundant objectives. By removing redundant objectives, objective reduction algorithms can improve the search efficiency, reduce computational cost, and support for decision making. The performance of an objective dimensionality reduction strongly depends on nondominated solutions generated by MOEAs/MaOEAs. The impact of objective reduction algorithms on MOEAs and vice versa have been widely investigated. However, the impact of objective reduction algorithms on MaOEAs and vice versa have been rarely investigated. This paper studies the interdependence of objective reduction algorithms on MaOEAs. Experimental results show that combining an objective reduction algorithm with an MOEA can only successfully remove redundant objectives when the total number of objectives is small. In contrast, combining the objective reduction algorithm with an MaOEA can successfully remove redundant objectives even when the total number of objectives is large. Experimental results also show that objective reduction algorithms can significantly improve the performance of MaOEAs.

Keywords Evolutionary multi-objective optimisation · Many-objective optimisation · Objective dimensionality reduction · Principal component analysis

1 Introduction

In the real world, there often exist problems with more than one objective, which are referred as multi-objective problems (MOPs) [1, 31]. In a MOP, different solutions are likely to have an advantage over other objectives, so the Pareto dominance concept is commonly-used to compare different solutions. Multi-objective optimisation is the process to approximate the objective space Pareto Front so that no further enhancement on any objective is able to achieve without spoiling the rest of objectives [31, 34].

MOPs with four or more objectives are informally regarded as many-objective problems (MaOPs) [19]. MaOPs appear in many real-world applications, such as automotive engine calibration [20], dynamic airspace sectorisation [40], water distribution system design [21], sensornet protocol optimisation [41], and sequence alignment of proteins [17]. MaOPs are increasingly recognised as a key research topic in the multi-objective optimisation community [31, 50].

Evolutionary algorithms (EAs) are computer applications which mimic biological processes in order to solve complex problems. EAs are population-based, black-box optimisation methods and do not require particular assumptions such as continuity or differentiability [9, 49]. Therefore, EAs are

✉ Cao Truong Tran
truongct@lqdtu.edu.vn

Xuan Hung Nguyen
nguyenxuanhung@outlook.com

Lam Thu Bui
lambt@lqdtu.edu.vn

[1] Research Group of Computational Intelligence, Le Quy Don Technical University, 236 Hoang Quoc Viet St, Hanoi, Vietnam

very appropriate for addressing MOPs, and plenty of multi-objective evolutionary algorithms (MOEAs) have been proposed to deal with MOPs [50]. However, because of the large number of objectives, when tackling with MaOPs, conventional MOEAs encounter serious difficulties [31, 38].

There are three main difficulties associated with MaOPs. The first difficulty is the poor scalability of most Pareto dominance based MOEAs such as NSGA-II [15] and SPEA-II [51]. The underlying reason is that when there are a large number of objectives, almost the entire population become non-dominated. This phenomenon makes the Pareto dominance-based selection ineffective, so the convergence of MOEAs is seriously degraded [38]. The second difficulty is the high computation cost because the size of population needed to approximate Pareto-optimal front (POF) increase exponentially to the number of objectives [38]. The third difficulty is the obstacle of the visualisation of solutions owning to the increase in the number of objectives. As a result, this makes very difficult for users to choice a final solution in MaOPs [38].

The approaches to tackling with MaOPs can be categorised into preference-ordering approaches and objective reduction approaches [31]. The preference-ordering approaches suppose that there is no redundant objectives in the given problem, and aim to induce a preference ordering over the nondominated solutions to reduce selection pressure for convergence. Many-objective evolutionary algorithms (MaOEAs) such as NSGA-III [14] belong to these approaches. In contrast, the objective reduction approaches support that there exist redundant objectives in the given problem, and aim to identify a smallest subset of conflicting objectives which generates the same POF as the original problem [35, 38].

By removing redundant objectives and keeping only essential objectives, objective reduction approaches bring potential benefits. If the number of essential objectives could be reduced to less than four, objective reduction could make an unsolvable problem (many-objective) solvable by using any of the existing MOEAs. Even if the number of essential objectives are four or more, objective reduction still could improve search efficiency, lower computational cost and make visualisation and decision-making easier [38, 39].

An objective reduction method operates on the objective vectors of the nondominated solutions which are obtained by an MOEA or an MaOEA. Consequently, the effectiveness of the objective reduction method strongly depends on the ability of the an MOEA or an MaOEA to search for temporary nondominated solutions. The effectiveness of existing objective reduction approaches have been widely evaluated on MOEAs [33, 38], but have not been evaluated on MaOEAs. Therefore, how MaOEAs influence and benefit from objective reduction approaches should be investigated.

The overall goal of this paper is to investigate how MaOEAs can benefit from objective reduction algorithms when dealing with MaOPs. In order to achieve this goal, firstly, the paper examines the influence of temporary nondominated solutions obtained by MaOEAs on objective reduction algorithms. After that, the paper evaluates MaOEAs on the given problem with selected objectives obtained by reduction algorithms to figure out the advantages which MaOEAs can receive when integrating with these objective reduction algorithms.

The rest of this paper is organised as follows. Section 2 shows an overview of related work. Section 3 describes the method and experiment design. Section 4 presents results and discussions. Finally, Sect. 5 makes conclusions and states future work.

2 Related work

This section presented related work including multi-objective optimisation, quality measurements of multi-objective optimisation algorithms, many-objective optimisation and objective dimensionality reduction approaches.

2.1 Multi-objective optimisation

Multi-objective optimisation problem as defined as follows [34]:

$$\begin{aligned} &\text{minimize } \mathbf{f} = \left\{ f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x}) \right\} \\ &\text{subject to } \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where there are k (≥ 2) objective function $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$. The decision vectors $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ belongs to (nonempty) feasible region Ω, which is a subset of decision variable space $\mathbb{R}^n$. All of k objective functions need to be minimised simultaneously. It is assumed that there does not exist a single solution that is optimal with respect to every objective function. The image of region by $\mathbb{Z} = f(\Omega)$, which is a subset of the objective space $\mathbb{R}^k$ is called the feasible objective region. The elements of $\mathbb{Z}$ are called objective (function) vectors and denoted by $\mathbf{f}(x)$ or $\mathbf{z} = (z_l, z_2, \dots, z_k)^T$, where $z_i = f_i(\mathbf{x})$ for all $i = 1, \dots, k$ are objective (function) values.

Techniques for solving MOPs can be divided into weighted sum techniques and evolutionary computation-based techniques. Weighted sum techniques solve a MOP by converting the problem into a single objective optimisation problem. After converting, the new problem has a single objective function, then it can be solved by using developed theory and methods for single objective optimisation. The advantages of weighted sum techniques are easy to understand and implement. The fitness combination technique is also computationally efficient. The main disadvantage of weighted sum techniques is that the results depends on determination of weighting coefficients which is not easy to determine in advanced [9].

Evolutionary computation-based techniques solves a MOP by using evolutionary algorithms to approximate optimal solutions for the problem. By evolving a population of solutions, multi-objective evolutionary algorithms (MOEAs) are able to approximate a set of optimal solutions in a single run. In the past few decades, researchers have proposed plenty of MOEAs. Some well-known MOEAs are nondominated sorting genetic algorithm II (NSGA-II) [15], strength pareto evolutionary algorithm (SPEA) [52], pareto archived evolution strategy (PAES) [30].

The Pareto dominance relation is widely used to compare solutions in MOEAs. Based on the Pareto dominance relation, Pareto Optimal Solution, Pareto Set (PS), and Pareto Front (PF) are further defined. These terms are defined as follows:

Definition 1 A vector $\mathbf{u} = (u_1, \dots, u_m)^T$ is said to dominate another vector $\mathbf{v} = (v_1, \dots, v_m)^T$, denoted as $\mathbf{u} \prec \mathbf{v}$, if $\forall i \in \{1, \dots, m\}, u_i \leq v_i$ and $\exists j \in \{1, \dots, m\}\, u_j < v_j$.

Definition 2 A feasible solution $\mathbf{x}^* \in \Omega$ of problem (1) is called a Pareto optimal solution, if $\nexists\, \mathbf{y} \in \Omega$ such that $\mathbf{f}(\mathbf{y}) \prec \mathbf{f}(\mathbf{x}^*)$.

Definition 3 The set of all the Pareto optimal solutions is called the Pareto set (PS), denoted as $PS = \left\{\mathbf{x} \in \Omega \mid \nexists \mathbf{y} \in \Omega, \mathbf{f}(\mathbf{y}) \prec \mathbf{f}(\mathbf{x})\right\}$

Definition 4 The image of the Pareto set in the objective space is called the Pareto front (PF), denoted as PF $=\{\mathbf{f}(\mathbf{x}) \mid \mathbf{x} \in PS\}$.

2.2 Quality measurements of multi-objective optimisation algorithms

There exist over 50 metrics to compare the performance of different MOEAs [36]. Generational distance (GD) and inverted generational distance (IGD) are the most commonly-used metrics because they still work well when the number of objectives increases. GD is a value representing how 'far' PF_{know}[1] is from PF_{true}[2] and is defined as in Eq. (2):

$$GD = \frac{\left(\sum_{i=1}^{m} d_i^p\right)^{1/p}}{m} \tag{2}$$

where m is the number of vectors in $PF_{know}, p = 2$, and d_i is the Euclidean distance (in objective space) between each vector and the nearest member of PF_{true}. This measurement reflects the convergence aspect of MOEAs. The smaller GD value is, the better algorithm is [36].

GD often does not work well when an MOEA generates very few nondominated solutions. IGD is proposed to alleviate the issue, and is defined as in Eq. (3):

$$IGD = \frac{\left(\sum_{j=1}^{n} d_j^p\right)^{1/p}}{n} \tag{3}$$

where n is the number of vectors in PF_{true}, p is often set to 2, and d_j is the Euclidean distance between each vector in PF_{true} and its nearest vector in PF_{know}. IGD can reflect both the convergence and diversity of an MOEA [36].

2.3 Many-objective optimisation

Multi-objective optimisation problems, which have more than three objectives, are considered as many-objective optimisation problems (MaOPs). There exist a number of many-objective optimisation evolutionary algorithms (MaOEAs) which are proposed to solve MaOPs. Some well-known MaOEAs are reference-point based non-dominated sorting (NSGA-III) [14], grid-based evolutionary algorithm (GrEA) [45], knee point driven evolutionary algorithm (KnEA) [48].

When dealing with these MaOPs, MOEAs encounter serious difficulties. One difficulty is that when applying a well-known and frequently-used Pareto-dominance-based MOEAs such as NSGAII [15] and SPEA2 [51] to MaOPs, a large portion of population becomes non-dominated. This means these solutions cannot be compared to select for next generation (most of them are "good"). When using none-Pareto-based MOEAs such as aggregation-based and indicator-based approaches, they still have to search simultaneously in an exponentially increasing number of directions [27, 31]. The second difficulty is that the size of population has to increase exponentially to describe the front result [27]. Moreover, when deal with MaOPs, we encounter difficulties to visualise the solution set in order to help decision maker to choose the final solution [27].

Approaches to MaOPs can be categorised into preference-ordering approaches and objective reduction approaches. The preference-ordering approaches modify the classical MOEAs for MaOPs and can be further divided into four groups:

– Modification (also called relaxed or improvement) of Pareto dominance approach aims at enlarging the dominating area of non-dominated solutions so that some of them are more likely to be dominated by others. As

[1] The final set of solutions returned by MOEA at termination.

[2] Is implicitly defined by the functions composing an MOP.

a result, this reduces portion of non-dominated solution set. An example of this approach is Grid-based Evolutionary Algorithm (GrEA) [45]. Instead of using Pareto-dominance, GrEA introduces grid-dominance and grid-difference in order to exploits the potential of the grid-based approach to strengthen the selection pressure towards the optimal direction while maintaining an extensive and uniform distribution among solutions.
- Aggregation-based approach decomposes a MaOP into many single-objective sub-problems, so these MOEAs do not rely on the Pareto dominance when conducting the selection. In this approach, some algorithms are proposed such as weighted sum (MOEA/D) [47], weighted min-max and vector angle distance scaling (MSOPS) [25]). MOEA/D decomposes a multiobjective optimisation problem into a number of scalar optimisation subproblems and optimizes them simultaneously, each subproblem is optimized by only using information from its several neighboring subproblems. According to both the vector angle distance scaling and weighted Tchebycheff methods, MSOPS ranks individuals in population and enables users to analyse a MaOP at hand, especially in terms of bounds and discontinuities of the Pareto Front.
- Reference set based approach uses a set of reference solutions to measure the quality of solutions. NSGA-III [14] is an example. Beside selecting individuals/points in good layers, it prioritises individuals/points near to reference lines which contructed by ideal point and referent points evenly distribued in hyperplane. Another example is TwoArch2 algorithm [44]. The algorithm selects solutions from historical or current populations to construct the reference set (called convergence set in this algorithm). Thus, the search process is guided by the solutions in the reference solution set.
- Indicator-based approach which uses indicator values such as hypervolume indicator in HypE [2] to guide the search process for solving MaOPs. The authors in HypE based on that hypervolume indicator is the only single set quality measure that is known to be strictly monotonic with regard to Pareto dominance, so they proposed a fast search algorithm that can do many-objective problems become feasible.

Along with preference-ordering approaches, objective reduction approaches for solving MaOPs is presented in Sect. 2.4.

2.4 Objective dimensionality reduction

Dimensionality reduction is a machine learning technique which removes redundant features from original feature set. Dimensionality reduction are widely used in classification, regression and clustering. Dimensionality reduction has remarkable benefits such as improving the model performance saving the storage space required and fastening the time required for computation [13, 43].

In evolutionary many-objective optimisation, dimensionality reduction is usually called objective dimensionality reduction. In real applications, there exist problems having many objectives,[3] in which some of objectives conflict (or conflict partially) each others (real conflict), but some do not conflict each other, even correlated. If the absence of an objective does not affect the Pareto front, then that objective is considered redundant. Different from approaches mentioned in Sect. 2.3, objective dimensionality reduction aims at removing redundant objectives. Instead of directly solving a MaOP having redundant objectives, objective dimensionality reduction algorithms are proposed to solve these problems by eliminating redundant objectives [4, 39].

Objective dimensionality reduction approaches can be categorised into three groups: dominance structure based approach, correlation based approach, and feature-based one [3, 39]. The first group tries to retain the dominance relations as much as possible when removing objectives in the given nondominated solutions. Brockhoff and Zitzler [6] introduced the problem of computing a minimum subset of objectives without lossing information (MOSS). They also introduced a general notion of conflicts between objective sets, and proposed an exact algorithm and a greedy heuristic for the NP-hard MOSS problem. In [4, 5], two problems $\delta - MOSS$[4] and $k - EMOSS$[5] were introduced. In these studies, Brockhoff and Zitzler proposed a greedy algorithms to solve these two problems. Singh et al. [39] proposed the PCSEA algorithm. Instead of using non-dominated sorting and crowding distance for finding non-dominated solution in whole space like NSGA-II [15], PCSEA uses a corner-sort ranking for finding the corner solutions. After that, a heuristic technique is performed to determine the critical objectives and eliminate redundant ones. Gu et al. [22] presented a novel measure for measuring the capacity of preserving the dominance structure of an objective set, then they proposed a fast algorithm to find a minimum set of objectives preserving the dominance structure as much as possible.

The second group aims to keep the most conflict objectives and remove the objectives that are low conflict, or non-conflict each other. Deb and Saxena [16] proposed a principal component analysis (PCA) based evolutionary multi-objective optimization procedure, for dimensionality reduction. The main assumption is that if two objectives are

[3] Objectives in evolutionary many-objective optimisation are considered features in dimensionality reduction.

[4] Computation of of an objective subset of minimum size, yielding a (change) dominance structure with given error.

[5] Computation of an objective subset of given size with the minimum error.

negatively correlated (taking the generated Pareto front as the data set), then these objectives are in conflict with each other. However, when the data points live on a non-linear manifold, PCA is often ineffective in revealing the underlying dimensionality. To overcome the issue, Saxena et al. proposed two new non-linear dimensionality reduction algorithms for evolutionary multi-objective optimization, namely C-PCA-NSGAII and MVU-PCA-NSGA-II in [37], L-PCA and NL-MVU-PCA in [38].

The third group based on unsupervised feature selection techniques. Jaimes et al. [33] proposed dimensionality reduction schemes for solving $\delta - MOSS$ and $k - EMOSS$ problems. The main idea of these algorithms is to divide the objective set into homogeneous neighbourhoods around each objective, then retain the center of the most compact one and discard its neighbour. This dimensionality reduction scheme was later integrated into a MOEA by Jaimes et al. [28] to form reduction genetic algorithm.

Another way to categorise objective dimensionality reduction is online approaches and offline approaches. Since objective dimensionality reductions often integrate with MaOEAs, this classification bases on the timing of integrating. For offline methods, objective dimensionality reduction is carried out after obtaining a set of Pareto optimal solutions [8, 33, 38, 39]. With online ones, by iteratively obtaining solution sets and invoking the objective dimensionality reduction, the number of objectives can be reduced gradually during the search process [7, 23, 24, 28].

Along with feature selection, feature extraction techniques are also used to perform dimensionality reduction. The main purpose of feature selection is to find a small subset of the given features in order to represent the given data best, is discussed above. Feature extraction (or feature construction) aims at creating novel features from the original features to explain data. In Cheung and Gu [11], formulated the essential objective as a linear combination of the original objectives with the combination weights determined based on the correlations of each pair of the essential objectives. In Cheung et al. [12], proposed an objective extraction which formulates the reduced objective as a linear combination of the original objectives to maximize the conflict between the reduced objectives, and minimize the correlation between each pair of reduced objectives.

Algorithm 1: Framework for linear objective reduction algorithm

Input: $t = 0$ and $F_t = \{f_1, f_2, ..., f_M\}$

1 **begin**

2 Run an MOEA/MaOEA, obtain a set of non-dominated solutions corresponding to F_t.

3 Compute a positive semi-definite matrix R(M×M)

$$R = \frac{1}{M} X X^T$$

4 From matrix R, compute the eigenvalues $\lambda_1, \lambda_2 ..., \lambda_M$; eigenvectors: $V_1, V_2, ...V_M$; normalise eigenvalues, and sort them descending together with eigenvector

5 Perform the Eigenvalue Analysis to identify the set of important objectives $F_e \subseteq F_t$

6 Perform the Reduced Correlation Matrix Analysis to identify the identically correlated subset (S) in F_e. If there no such subset, $F_s = F_e$

7 Apply the selection scheme to identify the most significant objective in each S, to arrive at F_s, such that $F_s \subseteq F_e \subseteq F_t$

8 Computation of error

9 **if** $F_s = F_t$ **then**

10 Stop and declare F_t as the essential objective set;

11 Set T = t and compute the total error

12 **else**

13 set t=t+1, $F_t = F_s$, and go to Step 2

14 **end**

15 **end**

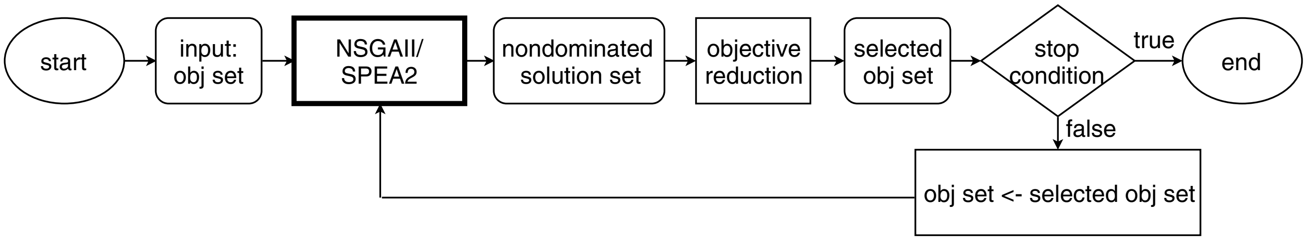
Algorithm 1 shows framework of Linear Principal Component Analysis Algorithm (L-PCA algorithm) proposed by Saxena et al. [38] for linear objective reduction algorithms. This framework will be utilised in the method in the next section.

3 Method and experiment design

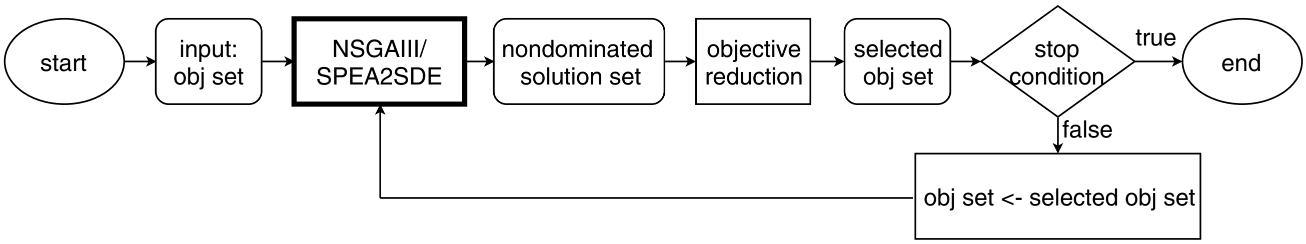
This section firstly describes how we investigate the impact of a many-objective evolutionary algorithm on an objective dimensionality reduction algorithm, and vice versa. Secondly, it presents the design of experiments including test problems and experimental settings.

3.1 The method

This study is designed to investigate the interaction between many-objective evolutionary algorithms and objective dimensionality reduction algorithms. Firstly, the study examines how a many-objective evolutionary affects the performance of an object dimensionality reduction algorithm. The study also evaluates what benefits a many-objective

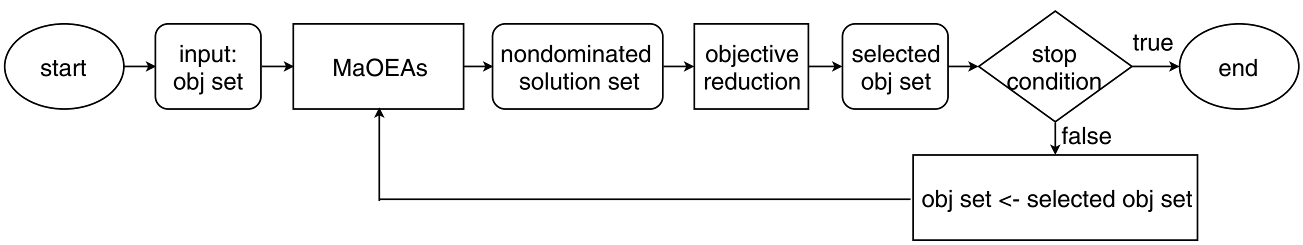


(a) The combination of a *multi-objective* evolutionary algorithm with an objective dimensionality algorithm.

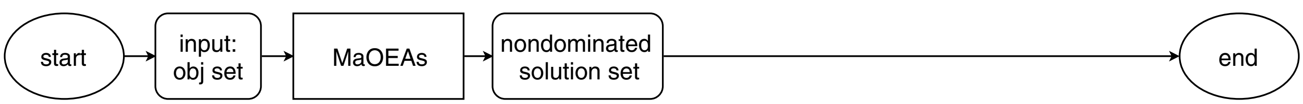


(b) The combination of a *many-objective* evolutionary algorithm with an objective dimensionality algorithm.

Fig. 1 The integration of MOEAs/MaOEAs on objective dimensionality reduction algorithms



(a) Integrating an objective dimensionality reduction into a many-objective evolutionary algorithm for dealing with many-objective problems.



(b) Using a many-objective evolutionary algorithm for dealing with many-objective problems.

Fig. 2 Two ways using many-objective evolutionary algorithms to deal with many-objective problems

evolutionary algorithm can obtain when it combines with an object reduction algorithm.

In order to demonstrate the impact of a many-objective evolutionary algorithm on an objective dimensionality reduction algorithm, we design experiments to compare the performance of an objective dimensionality reduction algorithm when it combines with MOEAs and combines with MaOEAs. Figure 1 shows the integration of MOEAs/MaOEAs on an objective dimensionality reduction algorithm. Figure 1a describes the combination of a *multi-objective* evolutionary algorithm with an objective dimensionality algorithm while Fig. 1b describes the combination of a *many-objective* evolutionary algorithm with an objective dimensionality algorithm. L-PCA algorithm as showed in Algorithm 1 is used for as an objective reduction algorithm. Two MOEAs—NSGAII [15] and SPEA2 [51]—are used in Fig. 1a while two MaOEAs—NSGAIII [14] and SPEA2SDE [32]—are used in Fig. 1b.

In order to examine whether an many-objective evolutionary algorithm can obtain advantages when it combines with an objective reduction algorithm, we design experiments to compare the performance of the integration of an many-objective evolutionary algorithm with an objective reduction algorithm against the performance of the many-objective evolutionary algorithm alone. Figure 2 shows two ways using many-objective evolutionary algorithms to deal with many-objective problems. Figure 2a shows the integration of an objective dimensionality reduction into a many-objective evolutionary algorithm while Fig. 2b shows a common way to use a many-objective evolutionary algorithm for dealing with a many-objective problem. We also use L-PCA as showed in Algorithm 1 for removing redundant objectives. Five well-known MaOEAs algorithms—GrEA [45], KnEA [48], NSGAIII [14], RVEA* [10], θ-DEA [46]—are used in Fig. 2a and b to search for nondominated solutions.

3.2 Test problems and experimental design

To study, we use DTLZ5(I,M) problem [26], it is defined as:

$$\begin{aligned}
&min\, f_1(\mathbf{x}) \quad = (1+100g(\mathbf{x}_M))cos(\theta_1)cos(\theta_2)\dots cos(\theta_{M-2})cos(\theta_{M-1})\\
&min\, f_2(\mathbf{x}) \quad = (1+100g(\mathbf{x}_M))cos(\theta_1)cos(\theta_2)\dots cos(\theta_{M-2})sin(\theta_{M-1})\\
&min\, f_3(\mathbf{x}) \quad = (1+100g(\mathbf{x}_M))cos(\theta_1)cos(\theta_2)\dots sin(\theta_{M-2})\\
&\dots\\
&min\, f_{M-1}(\mathbf{x}) = (1+100g(\mathbf{x}_M))cos(\theta_1)sin(\theta_2)\\
&min\, f_M(\mathbf{x}) \quad = (1+100g(\mathbf{x}_M))sin(\theta_1)\\
&\text{where } \theta_i = \frac{\pi}{2}x_i \text{ for } i = 1,2,\dots,(I-1)\\
&\qquad \theta_i = \frac{\pi}{4(1+g(\mathbf{x}_M))}(1+2g(\mathbf{x}_M)x_i) \text{ for } i = I,\dots,(M-1)\\
&\qquad g = \sum\nolimits_{x_i\in\mathbf{x}_M}(x_i-0.5)^2\\
&\qquad 0 \le x_i \le 1 \text{ for } i = 1,2,\dots,n
\end{aligned}$$

The first property of the problem is that the dimensionality (I) of the Pareto-optimal front can be changed by setting I to an integer between two and M. The second one is that Pareto-optimal front is non-convex and follows the relationship: $\sum_{i=1}^{M}(f_i^*) = 1$. The another property is that there are $M-I$ first objectives correlated, while the others and one of $M-I$ first objective are conflict each other. The experiments are performed on seven versions of DTLZ5(I,M) problem: DTLZ5(2,5), DTLZ5(3,5), DTLZ5(5,10), DTLZ5(7,10), DTLZ5(5,20) and DTLZ5(7,20).

The experiments use LPCA in [38] for doing objective dimensionality reduction. In LPCA, the threshold θ, which is used to decide which objectives should be included, is set to 0.997 as suggested in [38].

All of MOEAs and MaOEAs used the experiments are implemented by PlatEMO — an Evolutionary Multi-Objective Optimisation Platform [42]. The population size is set to 200, and the the number of generation is set to 2000. The probability of crossover and mutation is set to 0.9 and 0.1, respectively. The distribution index for crossover is set to 5, and the distribution index for mutation is set to 20 [38]. The quality of POF provided by the different algorithms is evaluated by using generational distance (GD) and inverted generational distance (IGD) [36].

4 Results

This section firstly presents the results and analysis to demonstrate the impact of multi-objective evolutionary algorithms and many-objective evolutionary algorithms on object dimensionality reduction algorithms. After that, it presents results and analysis to show the benefits which an many-objective evolutionary algorithm can achieve when combining with an objective reduction algorithm.

4.1 The impact of MOEAs and MaOEAs on objective dimentionality reduction

In order to show the impact of MOEAs and MaOEAs on objective dimensionality reduction, we examine the impact of two pairs of MOEA/MaOEA—NSGAII/NSGAIII and SPEA2/SPEA2SDE—on LPCA for objective reduction. Firstly, we show some case studies to illustrate how an MOEA/MaOEA affects to an objective reduction algorithm on a specific problem—DTLZ5(6,8). Subsequently, we show the impact of the pairs on different test problems.

4.1.1 Case study

This section illustrates step by step how LPCA combined with MOEA/MaOEA for reducing redundant objectives on DTLZ5IM(6,8).

Pair of SPEA2SDE and SPEA2: We show the performance of LPCA when combining with SPEA2SDE and SPEA2 on DTLZ5(6,8).

Table 1 shows the matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with SPEA2SDE on DTLZ5(6,8). The correlation matrix R of population results is depicted in Table 1a, and the corresponding eigenvalues and eigenvectors[6] are presented in Table 1b.

Next, the number of significant eigenvectors (V) as determined as the smallest number of element eigenvalues (N_v) such that $\sum_{j=1}^{N_v} e_j \ge \theta$. We use $\theta = 0.997$ as recommended in [38]. Firstly, an object f_j which has the highest contribution to V_j by magnitude is picked. If there exists at least one other objective having opposite-sign with the selected objective, then the objectives (with opposite-sign) are picked. If not (all objectives have the same sign), an objective with the second highest contribution by magnitude is selected. Following these steps, a set of objectives $F_e = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8\}$ is selected. Then, from F_e we identify the subsets of identically correlated objectives (RCM-Reduced Correlation Matrix [38]) with the same size

[6] Eigenvalues are normalised, eigenvalues and eigenvectors are sorted descending together based on eigenvalues.

Table 1 The matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with SPEA2SDE on DTLZ5 (6,8)

(a) The correlation matrix R

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_1	1.00E+00	1.00E+00	1.00E+00	− 1.50E−01	− 1.68E−01	− 1.73E−01	− 1.73E−01	− 1.82E−01
f_2	1.00E+00	1.00E+00	1.00E+00	− 1.50E−01	− 1.68E−01	− 1.73E−01	− 1.73E−01	− 1.82E−01
f_3	1.00E+00	1.00E+00	1.00E+00	− 1.50E−01	− 1.68E−01	− 1.73E−01	− 1.73E−01	− 1.82E−01
f_4	− 1.50E−01	− 1.50E−01	− 1.50E−01	1.00E+00	− 1.65E−01	− 2.08E−01	− 1.64E−01	− 1.35E−01
f_5	− 1.68E−01	− 1.68E−01	− 1.68E−01	− 1.65E−01	1.00E+00	− 1.52E−01	− 2.08E−01	− 1.37E−01
f_6	− 1.73E−01	− 1.73E−01	− 1.73E−01	− 2.08E−01	− 1.52E−01	1.00E+00	− 1.97E−01	− 9.28E−02
f_7	− 1.73E−01	− 1.73E−01	− 1.73E−01	− 1.64E−01	− 2.08E−01	− 1.97E−01	1.00E+00	− 2.20E−01
f_8	− 1.82E−01	− 1.82E−01	− 1.82E−01	− 1.35E−01	− 1.37E−01	− 9.28E−02	− 2.20E−01	1.00E+00

(b) The eigenvalues(e) and eigenvector (V) of the correlation matrix R

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
3.94E−01	1.58E−01	1.49E−01	1.44E−01	1.33E−01	2.15E−02	6.26E−12	2.74E−13
V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
−5.62E−01	1.51E−02	− 1.94E−03	− 3.37E−03	− 7.90E−03	1.30E−01	1.74E−01	− 7.98E−01
−5.62E−01	1.51E−02	− 1.94E−03	− 3.37E−03	− 7.90E−03	1.30E−01	− 7.78E−01	2.48E−01
−5.62E−01	1.51E−02	− 1.94E−03	− 3.37E−03	− 7.90E−03	1.30E−01	6.04E−01	5.49E−01
8.49E−02	− 2.93E−01	6.70E−01	− 3.34E−01	4.07E−01	4.27E−01	− 2.04E−08	2.17E−08
1.01E−01	2.33E−01	2.41E−01	8.31E−01	− 2.17E−02	4.31E−01	− 1.68E−07	2.44E−10
1.06E−01	4.36E−01	− 5.25E−01	− 1.91E−01	5.50E−01	4.29E−01	− 1.07E−07	1.29E−07
9.80E−02	− 7.00E−01	− 4.48E−01	5.66E−02	− 2.49E−01	4.84E−01	− 7.64E−08	− 4.32E−08
1.16E−01	4.23E−01	1.31E−01	− 3.98E−01	− 6.85E−01	4.04E−01	7.64E−07	3.19E−08

as R except columns not in F_e. We determine potential identically correlated subset $\hat{S_1} = \hat{S_2} = \hat{S_3} = \{f_1, f_2, f_3\}$. Threshold cut T_{cor} is calculated[7] equal to 0.8522. Correlation satisfy condition greater than or equals to T_{cor} then $S_1 = S_2 = S_3 = \{f_1, f_2, f_3\}$. In each subset S, we retain the objective with the highest selection score, and eliminating the others.[8] Therefore, we have $sc = \{sc_1, sc_2, sc_3\} = \{0.228611361, 0.228611498, 0.228611267\}$ then we select objective 2 and remove objectives 1 and 3. So we retain $F_s = \{f_2, f_4, f_5, f_6, f_7, f_8\}$

Table 2 shows The matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *SPEA2* on DTLZ5(6,8). The correlation matrix R is presented in Table 2a, and the eigenvalues and eigenvector of the correlation R are shown in Table 2b. Based on Table 2b, all of eight principal components have to be included to account for $\theta = 0.997$, leading to $F_e = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8\}$. When analyzing Reduced Correlation Matrix, we have three potential identically correlated subsets $\hat{S_1} = \hat{S_2} = \hat{S_3} = \{f_1, f_2, f_3\}$. Due to all values $R_{1,2} = 0.16826, R_{1,3} = 0.08172, R_{2,3} = 0.09392$ are less than $T_{cor} = 0.9751$, the subset of identically correlated objectives is empty. As a result, we cannot reduce any objective.

In short, the combination of LPCA with SPEA2 cannot reduce any redundant objective while the combination of LPCA with SPEA2SDE can correctly reduce redundant objectives in DTLZ5(6,8).

Pair of NSGAII and NSGA-III: We show the performance of LPCA when combining with NSGA-III and NSGAII on DTLZ5(6,8).

Table 3 shows the matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *NSGAII* on DTLZ5(6,8). According to data in Table 3, all principal components (eight components) need to accumulate to have total which is greater than or equal to $\theta = 0.997$. Based on eigenvalues and eigenvectors in Table 3b, set F_e (conflicting objectives) is determined as $\{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8\}$. Then we determine the subsets of identically correlated objectives $\hat{S}$ in F_e: $\hat{S_1} = \hat{S_2} = \{f_1, f_2\}$. Due to all values $R_{1,2} = 0.171481$ is less than $T_{cor} = 0.9751$, the subset of identically correlated objectives is empty. As a result, we cannot remove any objective.

Table 4 shows the matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *NSGAIII* on DTLZ5(6,8). The

[7] $T_{cor} = 1.0 - e_1(1.0 - M_{2\alpha}/M)$ in which $e_1 = 0.39416, M_{2\alpha} = 5, M = 8$.

[8] Selection score for each objective is calculated $sc_i = \sum_{j=1}^{N_v} e_j |f_{ij}|$.

Table 2 The matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *SPEA2* on DTLZ5(6,8)

(a) The correlation matrix R

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_1	1.00E+00	1.68E−01	8.17E−02	− 1.55E−01	− 1.66E−01	− 1.68E−01	− 1.48E−01	− 1.50E−01
f_2	1.68E−01	1.00E+00	9.39E−02	− 1.65E−01	− 1.47E−01	− 1.38E−01	− 1.99E−01	− 1.30E−01
f_3	8.17E−02	9.39E−02	1.00E+00	− 1.69E−01	− 2.30E−01	− 1.35E−01	− 1.29E−01	− 1.72E−01
f_4	− 1.55E−01	− 1.65E−01	− 1.69E−01	1.00E+00	− 1.40E−01	− 1.53E−01	− 9.95E−02	− 6.30E−02
f_5	− 1.66E−01	− 1.47E−01	− 2.30E−01	− 1.40E−01	1.00E+00	− 1.17E−01	− 1.21E−01	− 9.45E−02
f_6	− 1.68E−01	− 1.38E−01	− 1.35E−01	− 1.53E−01	− 1.17E−01	1.00E+00	− 1.18E−01	− 1.52E−01
f_7	− 1.48E−01	− 1.99E−01	− 1.29E−01	− 9.95E−02	− 1.21E−01	− 1.18E−01	1.00E+00	− 1.18E−01
f_8	− 1.50E−01	− 1.30E−01	− 1.72E−01	− 6.30E−02	− 9.45E−02	− 1.52E−01	− 1.18E−01	1.00E+00

(b) The eigenvalues(e) and eigenvector (V) of the correlation matrix R

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
2.00E−01	1.49E−01	1.45E−01	1.37E−01	1.31E−01	1.14E−01	1.03E−01	2.06E−02
V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
− 5.04E−01	1.19E−01	7.08E−02	1.19E−01	− 7.38E−02	4.82E−01	6.35E−01	2.78E−01
− 5.11E−01	− 3.04E−02	2.04E−01	− 3.04E−02	− 3.08E−02	2.58E−01	− 7.45E−01	2.66E−01
− 4.73E−01	1.87E−03	− 2.30E−01	1.87E−03	7.36E−02	− 7.72E−01	1.06E−01	3.29E−01
2.41E−01	− 3.37E−01	− 2.65E−01	− 3.37E−01	− 5.97E−01	7.71E−03	− 2.88E−02	3.84E−01
2.82E−01	3.98E−01	6.57E−01	3.98E−01	− 2.96E−01	− 2.07E−01	4.50E−02	4.08E−01
1.50E−01	− 5.56E−01	− 7.93E−02	− 5.56E−01	8.56E−02	1.50E−01	4.72E−02	3.99E−01
2.26E−01	6.09E−01	− 5.80E−01	6.09E−01	1.68E−01	1.97E−01	− 1.50E−01	3.76E−01
2.22E−01	− 1.82E−01	2.36E−01	− 1.82E−01	7.14E−01	− 2.91E−03	4.67E−02	3.60E−01

conflicting objectives along six significant principal components are determined as $F_e = \{f_1,f_2,f_3,f_4,f_5,f_6,f_7,f_8\}$. Correlation $\hat{S_1} = \hat{S_2} = \hat{S_3} = \{f_1,f_2,f_3\}$. $T_{cor} = 1.0 - 0.4083(1 - 5/8) = 0.8469$, and all values $R_{1,2} = R_{1,3} = R_{2,3} = 1$ are greater than T_{cor}, so there are three identically correlated set $S_1 = S_2 = S_3 = \{f_1,f_2,f_3\}$. We calculate $sc = \{0.2591124, 0.2591125,\ 0.2591124\}$ and retain the index of maximum value, so we retain f_2 and remove others. As a result, we retain $F_s = \{f_2, f_4, f_5, f_6, f_7, f_8\}$

In summary, the combination of LPCA with NSGAII cannot reduce any redundant objective while the combination of LPCA with NSGAIII can correctly reduce redundant objectives in DTLZ5(6,8).

4.1.2 The succeed of objective dimensionality reduction when combined with MOEAs/MaOEAs

Table 5 shows the mean and standard deviation of the number of retained objectives which is done by the combinations of LPCA and MOEAs/MaOEAs — NSGAII, SPEA2, NSGAIII and SPEA2SDE — for removing redundant objectives in 20 running times. It also shows the number of times which the algorithms correctly retain essential objectives.

It is clear from Table 5 that with nondominated solutions obtained from NSGAII and SPEA2, the objective dimensionality reduction algorithm only can successfully remove redundant objectives when the number of original objectives is small. For example, the combinations of LPCA and NSGAII/SPEA2 can exactly remove redundant objectives on DTLZ5(2,5) and DTLZ5(3,5). However, when the number of original objectives increases, the combination of LPCA with the MOEAs cannot successfully remove redundant objectives. For example, the combination of LPCA with NSGAII/SPEA2 is not successful any time in removing redundant features on DTLZ5(7,10), DTLZ5(5,20) and DTLZ5(7,20).

In contrast, with nondominated solutions obtained from NSGAIII and SPEA2SDE, the objective dimensionality reduction can successfully remove redundant objectives even when the number of original objectives increases. For example, the combination of LPCA with SPEA2SDE can successfully remove redundant objectives in all the six test problems while the combination of LPCA with SPEA2SDE only cannot successfully remove redundant objectives in two cases of DTLZ5(5,20).

In summary, the quality of MOEAs/MaOEAs plays important roles to the performance of an objective dimensionality reduction algorithm. The combination of an objective reduction algorithm with MaOEAs can successfully remove redundant objectives even if the number of original

Table 3 The matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *NSGAII* on DTLZ5(6,8)

(a) The correlation matrix R

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_1	1.00E+00	1.71E−01	5.69E−02	− 6.12E−02	− 6.58E−02	− 8.24E−02	− 1.08E−01	− 1.86E−01
f_2	1.71E−01	1.00E+00	8.45E−02	− 1.02E−01	− 2.00E−01	− 9.16E−02	− 7.13E−02	− 1.58E−01
f_3	5.69E−02	8.45E−02	1.00E+00	2.48E−03	− 1.28E−01	− 9.47E−02	− 1.99E−01	− 2.28E−01
f_4	− 6.12E−02	− 1.02E−01	2.48E−03	1.00E+00	− 5.87E−02	− 1.68E−01	− 1.44E−01	− 1.72E−01
f_5	− 6.58E−02	− 2.00E−01	− 1.28E−01	− 5.87E−02	1.00E+00	− 1.09E−02	− 1.85E−01	− 2.19E−01
f_6	− 8.24E−02	− 9.16E−02	− 9.47E−02	− 1.68E−01	− 1.09E−02	1.00E+00	− 1.99E−01	− 1.25E−01
f_7	− 1.08E−01	− 7.13E−02	− 1.99E−01	− 1.44E−01	− 1.85E−01	− 1.99E−01	1.00E+00	− 6.98E−02
f_8	− 1.86E−01	− 1.58E−01	− 2.28E−01	− 1.72E−01	− 2.19E−01	− 1.25E−01	− 6.98E−02	1.00E+00

(b) The eigenvalues(e) and eigenvector (V) of the correlation matrix R

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1.88E−01	1.68E−01	1.48E−01	1.36E−01	1.20E−01	1.12E−01	9.94E−02	2.84E−02
V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
−4.54E−01	1.40E−01	− 1.67E−01	− 2.04E−01	4.86E−01	2.67E−01	6.00E−01	1.93E−01
−4.41E−01	3.70E−01	− 2.48E−01	− 4.37E−02	7.42E−02	1.61E−01	− 7.16E−01	2.48E−01
−4.92E−01	1.36E−02	8.36E−02	3.08E−01	− 3.11E−01	− 6.50E−01	1.93E−01	3.15E−01
−1.36E−01	− 1.59E−01	7.10E−01	2.54E−01	− 9.99E−02	5.12E−01	− 3.74E−02	3.38E−01
5.12E−02	− 6.22E−01	7.25E−02	− 3.91E−01	3.88E−01	− 2.91E−01	− 2.41E−01	3.99E−01
5.42E−02	− 3.96E−01	− 5.91E−01	9.09E−02	− 4.72E−01	3.48E−01	1.13E−01	3.54E−01
2.85E−01	4.45E−01	1.66E−01	− 6.04E−01	− 3.58E−01	− 5.59E−02	1.31E−01	4.23E−01
5.02E−01	2.75E−01	− 1.33E−01	5.21E−01	3.87E−01	− 9.17E−02	3.08E−02	4.74E−01

objectives is large. However, the combination of an objective reduction algorithm with MOEAs often only can remove redundant objectives when the number of original objectives is small.

4.2 The impact of objective dimensionality reduction on many-objective evolutionary algorithms

In order to demonstrate the benefits of an objective dimensionality reduction algorithm with an MaOEA, we compare the performance of the MaOEA combining the objective reduction against the MaOEA alone. Generational distance (GD) and inverted generational distance (IGD) are used to examine the algorithms.

Table 6 shows the mean and standard deviation (in parentheses) of GD and IGD of five MaOEAs including GrEA, KnEA, NSGAIII, RVEA*, and θ-DEA. IGD_1 and GD_1 refer to IGD and GD of the MaOEAs without combining with any objective dimensionality reduction algorithm, respectively. IGD_2 and GD_2 refer to IGD and GD of the MaOEAs combining with LPCA for removing redundant objectives, respectively. The table also shows the mean and standard deviation of the number of objectives which are retained after carrying out objective reduction.

To investigate whether results of the MaOEAs using objective reduction are significant different to the MaOEAs in a statistical sense, Wilcoxon rank-sum test is performed [18]. The null hypothesis is that the performance of the two methods are similar with significant level at 0.05, and the alternative hypothesis is that the performance of the two methods is significant different. In Table 6. The results are given at the end of each cells with ↑, or ↓ symbols. The cells ending with ↑ or ↓ show that the null hypothesis is rejected. ↑ means that IGD_2 or GD_2 are significant better than IGD_1 or GD_1 while ↓ means that IGD_2 or GD_2 are significant worse than IGD_1 or GD_1.

As can be seen from Table 6 that the performance of the combination of MaOEAs and objective reduction is significantly better than MaOEAs alone in almost all cases. In detail, IGD_2 is significant better than IGD_1 on 22 of 30 cases, and GD_2 is also significant better than GD_1 in 22 of 30 cases. Moreover, IGD_2 is only significant worse than IGD_1 on 1 of 30 cases, and GD_2 is significant worse than GD_1 in 2 of 30 cases.

In conclusion, combining with an objective dimensionality reduction algorithm to remove redundant objectives, an many-objective evolutionary algorithm can achieve significant better performance than using the algorithm alone.

Table 4 The matrix R with its corresponding eigenvalues and eigenvectors of LPCA when combining with *NSGAIII* on DTLZ5(6,8)

(a) The correlation matrix R

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_1	1.00E+00	1.00E+00	1.00E+00	4.07E−01	−6.93E−02	−4.72E−02	−1.11E−01	−7.15E−02
f_2	1.00E+00	1.00E+00	1.00E+00	4.07E−01	−6.93E−02	−4.72E−02	−1.11E−01	−7.15E−02
f_3	1.00E+00	1.00E+00	1.00E+00	4.07E−01	−6.93E−02	−4.72E−02	−1.11E−01	−7.15E−02
f_4	4.07E−01	4.07E−01	4.07E−01	1.00E+00	−2.11E−01	3.59E−02	−2.47E−01	−3.90E−02
f_5	−6.93E−02	−6.93E−02	−6.93E−02	−2.11E−01	1.00E+00	−2.38E−01	−2.05E−01	−2.90E−01
f_6	−4.72E−02	−4.72E−02	−4.72E−02	3.59E−02	−2.38E−01	1.00E+00	−3.19E−01	−2.95E−01
f_7	−1.11E−01	−1.11E−01	−1.11E−01	−2.47E−01	−2.05E−01	−3.19E−01	1.00E+00	−3.05E−01
f_8	−7.15E−02	−7.15E−02	−7.15E−02	−3.90E−02	−2.90E−01	−2.95E−01	−3.05E−01	1.00E+00

(b) The eigenvalues(e) and eigenvector (V) of the correlation matrix R

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
4.08E−01	1.74E−01	1.63E−01	1.49E−01	9.17E−02	1.36E−02	1.44E−12	−1.23E−16
V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
−5.44E−01	9.63E−02	−1.92E−02	−1.15E−02	1.65E−01	−1.83E−02	4.28E−01	−6.95E−01
−5.44E−01	9.63E−02	−1.92E−02	−1.15E−02	1.65E−01	−1.83E−02	−8.16E−01	−2.36E−02
−5.44E−01	9.63E−02	−1.92E−02	−1.15E−02	1.65E−01	−1.83E−02	3.88E−01	7.19E−01
−3.11E−01	−2.81E−01	3.41E−02	8.09E−02	−8.74E−01	−2.31E−01	−1.50E−06	−3.42E−10
6.48E−02	3.79E−01	2.52E−01	−7.53E−01	−8.17E−02	−4.64E−01	−7.37E−07	−6.43E−10
3.31E−03	−4.55E−01	6.46E−01	2.42E−01	3.16E−01	−4.67E−01	−6.83E−07	−5.03E−10
1.03E−01	6.00E−01	−1.48E−01	5.84E−01	−4.46E−02	−5.15E−01	−1.51E−06	−7.77E−10
3.42E−02	−4.29E−01	−7.03E−01	−1.66E−01	2.14E−01	−4.97E−01	−1.17E−06	−6.91E−10

5 Conclusion and future work

This paper investigated the interaction between objective dimensionality reduction algorithms and many-objective evolutionary algorithms. The experiments were designed to evaluate the impact of many-objective evolutionary algorithms on the performance of objective dimensionality reduction algorithms. The paper then examined the benefits which many-objective evolutionary algorithms can achieve when combining with objective reduction algorithms to removing redundant objectives. The results showed that the performance of an objective dimensionality reduction algorithm strongly depends on algorithms which generate non-dominated solutions. By combining with an many-objective evolutionary algorithm, an objective dimensionality reduction can successfully remove redundant objective even when the number of original objectives is large. The results also demonstrated that combining with an objective reduction algorithm to remove redundant objectives can significantly improve the performance of many-objective evolutionary algorithms.

This paper focused on LPCA for performing objective dimensionality reduction. There exist other objective reduction methods, but the performance of these methods has not been examined when combining with many-objective evolutionary computation algorithms. Therefore, the future work could investigate how these objective reduction methods can improve the performance of many-objective evolutionary computation algorithms.

Acknowledgements This research is funded by Ministry of Science and Technology under Bilateral and Multilateral Research Programs (the grant for Face Recognition).

Appendix

This section further investigates the proposed methods when using a clustering method for objective dimensionality reduction.

Integrating clustering objective dimensionality reduction algorithm into MaOEAs

Based on correlation coefficient (where $\rho(x, y)$ is the correlation coefficient between random variables x and y, the range of ρ is from -1 to 1), Jaimes et al. [33] used $(1-\rho) \in [0, 2]$ to measure the degree of correlation between two objectives

Table 5 The number of objectives retained and number of success when integrate objective dimensionality reduction (LPCA) into MOEAs/MaOEAs

	DTLZ5IM(2,5)		DTLZ5IM(3,5)		DTLZ5IM(5,10)		DTLZ5IM(7,10)		DTLZ5IM(5,20)		DTLZ5IM(7,20)	
	Retain	# Success	Retain	# Success	Retain	# Success	Retain	# Success	Retain	# Success	Retain	# Success
NSGAII	2.00 ± 0.00	20	3.00 ± 0.00	20	9.25 ± 1.83	3	10.0 ± 0.00	0	20.00 ± 0.00	0	20.00 ± 0.00	0
SPEA2	2.00 ± 0.00	20	3.00 ± 0.00	20	9.25 ± 1.83	3	10.0 ± 0.00	0	20.00 ± 0.00	0	20.00 ± 0.00	0
NSGAIII	2.00 ± 0.00	20	3.00 ± 0.00	20	5.00 ± 0.00	20	7.00 ± 0.00	20	4.90 ± 0.31	18	7.00 ± 0.00	20
SPEA2SDE	2.00 ± 0.00	20	3.00 ± 0.00	20	5.00 ± 0.00	20	7.00 ± 0.00	20	5.00 ± 0.00	20	7.00 ± 0.00	20

in approximation set of Pareto Front in MOPs. In which, zero value indicates that objectives x and y are completely positively correlated and a value of 2 indicates that x and y are completely negatively correlated. A negative correlation between two objectives means that one objective increases while the other decreases and vice versa. On the other hand, if the correlation is positive, then both objectives increase or decrease at the same time. Following this way, the more negative correlation between two objectives leads to the more conflict between the objectives. In [33], based on a correlation matrix of a non-dominated set obtained using an evolutionary algorithm, the objective set is first divided into homogeneous neighborhoods. The distance between the objectives is considered as the conflict between the objectives. Thereafter, the most compact neighborhood is chosen, and all the objectives in it except the center one are removed, as they are the least conflicting.

MICA-NORMOEA and OC-ORA algorithms are developed in [23, 24], respectively. In these algorithms, interdependence coefficient matrix is calculated, then PAM clustering algorithm [29] and NSGA-II [15] are invoked iteratively to reduce the redundant objectives until criterion is satisfied. The main different between these methods with LPCA is the relationship between pair of objectives. While LPCA use linear relationship, the method represents nonlinear one.

The framework of these algorithms (MICA-NORMOEA and OC-ORA) is shown

Step 1. Set an iteration counter $t = 0$; original objective set is $F_t = f_1, f_2, \ldots, f_M$, and the number of predefined clusters is k.

Step 2. Initialize a random population P_t run NSGA-II corresponding to F_t and obtain a non-dominated set A_t

Step 3. Calculate the interdependence coefficient matrix based on the non-dominated set A_t and use the PAM clustering algorithm to divide the objective set F_t into k clusters.

Step 4. According to the clusters of objective set F_t obtained in Step 3, remove one of the redundant or the most interdependent objective from F_t according to the above objective reduction rules, and the remaining objective set is denoted as F_{t+1}

Step 5. If $F_t = F_{t+1}$ then stop; else $t := t + 1$; $F_t := F_{t+1}$; return to Step 2.

The results

These are done same as in Sect. 3. When calculating interdependence, the number of subintervals is set as 20, and the threshold θ is set as 0.9.

Table 7 shows the mean and standard deviation (in parentheses) of GD and IGD of five MaOEAs including GrEA, KnEA, NSGAIII, RVEA*, and θ-DEA. IGD_1 and GD_1 refer

Table 6 The values of IGD, GD of true Pareto (IGD_1, GD_1); number of objective retain (R) and IGD, GD (IGD_2, GD_2) after carrying out objective dimensionality reduction (LPCA)

Problem	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM
I	2	3	5	7	5	7
M	5	5	10	10	20	20
GrEA						
Retain	2.000E+00±0.000E−15	3.000E+00±0.000E−15	4.950E+00±2.179E−01	7.000E+00±0.000E−15	1.865E+01±3.623E+00	1.805E+01±4.642E+00
IGD_1	2.680E−03 ± 0.000E−15	5.946E−03 ± 2.236E−04	2.033E−02 ± 2.500E−03	2.387E−02 ± 2.133E−03	1.476E+00 ± 8.208E−01	1.212E+00 ± 7.224E−01
IGD_2	**2.522E−03 ± 0.000E−15↑**	**5.334E−03 ± 0.000E−15↑**	**1.462E−02 ± 1.497E−02↑**	**1.887E−02 ± 5.477E−04↑**	1.500E+00 ± 7.864E−01	1.211E+00 ± 7.230E−01
GD_1	1.322E−04 ± 0.000E−15	2.144E−03 ± 0.000E−15	1.047E−02 ± 8.944E−04	1.809E−02 ± 3.873E−04	1.032E+01 ± 2.840E+00	9.390E+00 ± 4.004E+00
GD_2	**1.247E−04 ± 0.000E−15↑**	*2.232E−03 ± 0.000E−15↓*	9.877E−03 ± 2.366E−03	**1.641E−02 ± 5.000E−04↑**	1.040E+01 ± 2.816E+00	9.390E+00 ± 4.004E+00
KnEA						
Retain	2.000E+00 ± 0.000E−15	3.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15
IGD_1	2.111E−04 ± 0.000E−15	2.809E−03 ± 0.000E−15	1.287E−02 ± 1.000E−03	2.271E−02 ± 1.803E−03	1.172E−02 ± 1.162E−03	2.380E−02 ± 2.958E−03
IGD_2	*3.220E−04 ± 0.000E−15↓*	2.851E−03 ± 0.000E−15	1.320E−02 ± 2.335E−03	**2.218E−02 ± 2.510E−03↑**	2.029E−02 ± 2.213E−02	**2.164E−02 ± 1.360E−03↑**
GD_1	1.345E−04 ± 0.000E−15	2.217E−03 ± 0.000E−15	1.279E−02 ± 2.291E−03	5.611E−02 ± 4.067E−02	1.178E−02 ± 1.500E−03	4.902E−02 ± 2.560E−02
GD_2	1.308E−04 ± 0.000E−15	**1.997E−03 ± 0.000E−15↑**	**1.281E−02 ± 5.950E−03↑**	**2.951E−02 ± 1.047E−02↑**	**9.049E−01 ± 2.131E+00↑**	**2.595E−02 ± 1.073E−02↑**
NSGAIII						
Retain	2.000E+00 ± 0.000E−15	3.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	4.900E+00 ± 3.000E−01	6.950E+00 ± 2.179E−01
IGD_1	4.261E−03 ± 1.183E−03	7.297E−03 ± 6.325E−04	2.165E−02 ± 3.066E−03	2.789E−02 ± 5.822E−03	3.447E−02 ± 6.771E−03	3.928E−02 ± 2.837E−03
IGD_2	**1.113E−03 ± 0.000E−15↑**	**5.786E−03 ± 0.000E−15↑**	2.063E−02 ± 8.367E−04	**2.501E−02 ± 9.220E−04↑**	3.290E−02 ± 2.828E−03	**3.821E−02 ± 7.717E−03↑**
GD_1	1.133E−04 ± 0.000E−15	2.446E−03 ± 0.000E−15	1.468E−02 ± 1.658E−03	2.297E−02 ± 2.012E−03	9.310E−01 ± 2.145E+00	3.197E−02 ± 2.729E−02
GD_2	*1.338E−04 ± 0.000E−15↓*	**2.407E−03 ± 3.873E−04↑**	1.532E−02 ± 1.265E−03	**2.041E−02 ± 1.323E−03↑**	**4.355E−01 ± 1.831E+00↑**	2.096E−02 ± 5.263E−03
*RVEA**						
Retain	2.000E+00 ± 0.000E−15	3.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	4.850E+00 ± 6.538E−01	7.000E+00 ± 0.000E−15
IGD_1	1.296E−03 ± 4.472E−04	3.917E−03 ± 0.000E−15	1.873E−02 ± 1.396E−03	2.204E−02 ± 8.367E−04	2.306E−02 ± 5.196E−03	2.892E−02 ± 1.936E−03
IGD_2	**1.062E−04 ± 0.000E−15↑**	**1.773E−03 ± 0.000E−15↑**	**9.925E−03 ± 3.162E−04↑**	**1.391E−02 ± 2.236E−04↑**	**1.483E−02 ± 7.849E−03↑**	**2.413E−02 ± 1.107E−02↑**
GD_1	3.151E−04 ± 0.000E−15	3.246E−03 ± 0.000E−15	1.874E−02 ± 1.500E−03	2.294E−02 ± 7.416E−04	2.730E−02 ± 5.710E−03	3.598E−02 ± 1.746E−03
GD_2	**1.054E−04 ± 0.000E−15↑**	**1.809E−03 ± 0.000E−15↑**	**1.131E−02 ± 5.916E−04↑**	**1.404E−02 ± 2.236E−04↑**	**9.721E−03 ± 2.898E−03↑**	**1.073E−02 ± 3.755E−03↑**
θ-DEA						
Retain	2.000E+00 ± 0.000E−15	3.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	4.950E+00 ± 2.179E−01	7.000E+00 ± 0.000E−15
IGD_1	8.212E−03 ± 1.072E−03	1.057E−02 ± 1.140E−03	2.617E−02 ± 2.739E−03	2.803E−02 ± 1.643E−03	3.696E−02 ± 3.302E−03	4.131E−02 ± 2.748E−03
IGD_2	**2.600E−06 ± 0.000E−15↑**	**1.155E−05 ± 0.000E−15↑**	**9.050E−06 ± 0.000E−15↑**	**1.180E−04 ± 0.000E−15↑**	**5.956E−04 ± 2.490E−03↑**	**1.400E−03 ± 5.784E−03↑**
GD_1	1.193E−04 ± 0.000E−15	1.852E−03 ± 0.000E−15	1.584E−02 ± 1.673E−03	2.356E−02 ± 2.500E−03	1.646E−02 ± 3.399E−03	2.361E−02 ± 4.533E−03
GD_2	**1.950E−06 ± 0.000E−15↑**	**1.155E−05 ± 0.000E−15↑**	**9.050E−06 ± 0.000E−15↑**	**1.180E−04 ± 0.000E−15↑**	**4.959E−04 ± 2.074E−03↑**	**2.605E−03 ± 1.100E−02↑**

Bold indicates that the proposed method is better than the benchmark methods

Table 7 The values of IGD, GD of true Pareto (IGD_1, GD_1); the number of objective retain (R) and IGD, GD (IGD_2, GD_2) after carrying out clustering objective dimensionality reduction

Problem	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM
I	2	3	5	7	5	7
M	5	5	10	10	20	20
GrEA						
Retain	2.250E+00 ± 4.330E−01	3.000E+00 ± 0.000E−15	5.000E+00 ± 0.000E−15	7.050E+00 ± 2.179E−01	5.300E+00 ± 1.418E+00	8.550E+00 ± 2.673E+00
IGD_1	2.591E−03 ± 0.000E−15	5.812E−03 ± 0.000E−15	1.877E−02 ± 1.432E−03	2.279E−02 ± 1.304E−03	2.938E−01 ± 4.426E−01	6.942E−01 ± 6.047E−01
IGD_2	**2.344E−03 ± 0.000E−15↑**	**5.001E−03 ± 0.000E−15↑**	**1.010E−02 ± 3.162E−04↑**	**1.863E−02 ± 8.367E−04↑**	**4.494E−02 ± 4.519E−02↑**	**2.056E−01 ± 4.208E−01↑**
GD_1	1.260E−04 ± 0.000E−15	2.080E−03 ± 0.000E−15	9.498E−03 ± 2.236E−04	1.792E−02 ± 4.472E−04	4.240E+00 ± 1.917E+00	7.165E+00 ± 3.227E+00
GD_2	**1.215E−04 ± 0.000E−15↑**	2.134E−03 ± 0.000E−15↓	9.770E−03 ± 0.000E−15↓	**1.608E−02 ± 5.477E−04↑**	**1.125E+00 ± 1.413E+00↑**	**1.707E+00 ± 3.027E+00↑**
KnEA						
Retain	2.250E+00 ± 4.330E−01	2.950E+00 ± 2.179E−01	5.050E+00 ± 2.179E−01	6.900E+00 ± 4.359E−01	5.050E+00 ± 2.179E−01	6.850E+00 ± 3.571E−01
IGD_1	1.980E−04 ± 0.000E−15	2.706E−03 ± 2.236E−04	1.206E−02 ± 1.183E−03	2.217E−02 ± 2.820E−03	1.162E−02 ± 1.000E−03	2.241E−02 ± 3.017E−03
IGD_2	3.026E−04 ± 0.000E−15↓	3.663E−03 ± 3.975E−03	1.175E−02 ± 9.220E−04	2.128E−02 ± 1.342E−03	**1.138E−02 ± 1.549E−03↑**	2.312E−02 ± 6.708E−03
GD_1	1.280E−04 ± 0.000E−15	2.139E−03 ± 0.000E−15	1.213E−02 ± 2.191E−03	5.609E−02 ± 4.084E−02	1.164E−02 ± 1.072E−03	4.740E−02 ± 2.459E−02
GD_2	**1.230E−04 ± 0.000E−15↑**	**1.904E−03 ± 2.236E−04↑**	**1.050E−02 ± 1.162E−03↑**	**2.536E−02 ± 6.979E−03↑**	**2.079E−02 ± 4.735E−02↑**	**2.626E−02 ± 9.290E−03↑**
NSGAIII						
Retain	2.000E+00 ± 0.000E−15	3.150E+00 ± 3.571E−01	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	5.150E+00 ± 1.276E+00	6.850E+00 ± 1.314E+00
IGD_1	4.118E−03 ± 1.183E−03	7.339E−03 ± 7.746E−04	2.061E−02 ± 2.530E−03	2.728E−02 ± 5.895E−03	1.764E−02 ± 7.420E−03	2.892E−02 ± 9.327E−03
IGD_2	**3.300E−06 ± 0.000E−15↑**	**2.491E−03 ± 5.745E−03↑**	**2.595E−04 ± 3.162E−04↑**	**2.317E−03 ± 0.000E−15↑**	4.798E−02 ± 7.772E−03↓	2.425E−02 ± 1.893E−02
GD_1	9.980E−05 ± 0.000E−15	2.463E−03 ± 0.000E−15	1.358E−02 ± 1.072E−03	2.247E−02 ± 1.483E−03	1.486E+00 ± 4.120E+00	2.830E−02 ± 2.145E−02
GD_2	**3.300E−06 ± 0.000E−15↑**	**4.385E−04 ± 7.746E−04↑**	**2.718E−04 ± 3.873E−04↑**	**2.354E−03 ± 3.162E−04↑**	**8.419E−02 ± 2.939E−01↑**	**4.547E−03 ± 6.477E−03↑**
*RVEA**						
Retain	2.250E+00 ± 4.330E−01	3.050E+00 ± 2.179E−01	5.000E+00 ± 0.000E−15	7.000E+00 ± 0.000E−15	3.600E+00 ± 9.165E−01	6.000E+00 ± 1.449E+00
IGD_1	1.283E−03 ± 3.162E−04	3.768E−03 ± 0.000E−15	1.756E−02 ± 5.916E−04	2.123E−02 ± 0.000E−15	1.364E−02 ± 7.981E−03	2.390E−02 ± 6.719E−03
IGD_2	**1.394E−04 ± 0.000E−15↑**	**1.780E−03 ± 3.873E−04↑**	**9.755E−03 ± 5.916E−04↑**	**1.409E−02 ± 3.873E−04↑**	5.424E−02 ± 1.822E−02↓	4.328E−02 ± 1.584E−02↓
GD_1	3.098E−04 ± 0.000E−15	3.121E−03 ± 0.000E−15	1.741E−02 ± 3.162E−04	2.216E−02 ± 2.236E−04	1.981E−02 ± 7.416E−03	3.160E−02 ± 6.535E−03
GD_2	**1.223E−04 ± 0.000E−15↑**	**1.839E−03 ± 3.162E−04↑**	**1.135E−02 ± 3.162E−04↑**	**1.450E−02 ± 3.162E−04↑**	**1.195E−03 ± 3.122E−03↑**	**4.194E−03 ± 5.362E−03↑**
θ-DEA						
Retain	2.000E+00 ± 0.000E−15	3.150E+00 ± 3.571E−01	5.050E+00 ± 2.179E−01	7.000E+00 ± 0.000E−15	4.500E+00 ± 8.062E−01	6.850E+00 ± 3.571E−01
IGD_1	8.000E−03 ± 1.095E−03	1.050E−02 ± 1.025E−03	2.472E−02 ± 1.830E−03	2.674E−02 ± 6.325E−04	2.942E−02 ± 9.937E−03	3.900E−02 ± 3.033E−03
IGD_2	**3.300E−06 ± 0.000E−15↑**	**9.810E−04 ± 2.168E−03↑**	**1.173E−03 ± 4.324E−03↑**	**1.667E−03 ± 0.000E−15↑**	**1.472E−02 ± 1.990E−02↑**	**7.132E−03 ± 1.588E−02↑**

Table 7 (continued)

Problem	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM	DTLZ5IM
I	2	3	5	7	5	7
M	5	5	10	10	20	20
GD_1	1.091E−04 ± 0.000E−15	1.841E−03 ± 0.000E−15	1.410E−02 ± 1.449E−03	2.264E−02 ± 2.655E−03	1.317E−02 ± 4.685E−03	2.117E−02 ± 4.237E−03
GD_2	**3.300E−06 ± 0.000E−15↑**	**4.296E−04 ± 7.416E−04↑**	**9.326E−04 ± 3.332E−03↑**	**1.667E−03 ± 0.000E−15↑**	**2.368E−03 ± 6.281E−03↑**	**2.664E−03 ± 3.240E−03↑**

Bold indicates that the proposed method is better than the benchmark methods

to IGD and GD of the MaOEAs without combining with objective dimensionality reduction algorithm, respectively. IGD_2 and GD_2 refer to IGD and GD of the MaOEAs combining with clustering objective dimensionality reduction (OCA-ORA) for removing redundant objectives, respectively. The table also shows the mean and standard deviation of the number of objectives which are retained after carrying out objective reduction. The table indicates the performance of the combination of MaOEAs and clustering objective reduction is significantly better than MaOEAs alone in almost all cases. In detail, IGD_2 is significant better than IGD_1 on 21 of 30 cases, and GD_2 is also significant better than GD_1 in 28 of 30 cases. Moreover, IGD_2 is only significant worse than IGD_1 on 4 of 30 cases, and GD_2 is significant worse than GD_1 in 2 of 30 cases.

In summary, the proposed methods can be combined with different objective dimensionality reduction methods to improve evolutionary computation many objective optimisation algorithms.

References

1. Antonelli M, Ducange P, Lazzerini B, Marcelloni F (2009) Multi-objective evolutionary learning of granularity, membership function parameters and rules of mamdani fuzzy systems. Evol Intel 2(1–2):21
2. Bader J, Zitzler E (2011) Hype: an algorithm for fast hypervolume-based many-objective optimization. Evol Comput 19(1):45–76
3. Bechikh S, Elarbi M, Said LB (2017) Many-objective optimization using evolutionary algorithms: a survey. In: Recent advances in evolutionary multi-objective optimization, Springer, Berlin, pp 105–137
4. Brockhoff D, Zitzler E (2006) Are all objectives necessary? on dimensionality reduction in evolutionary multiobjective optimization. In: Parallel Problem Solving from Nature-PPSN IX, pages 533–542. Springer
5. Brockhoff D, Zitzler E (2006) Dimensionality reduction in multiobjective optimization with (partial) dominance structure preservation: generalized minimum objective subset problems. TIK Report, 247
6. Brockhoff D, Zitzler E (2006) On objective conflicts and objective reduction in multiple criteria optimization. TIK Report, 243
7. Brockhoff D, Zitzler E (2007) Offline and online objective reduction in evolutionary multiobjective optimization based on objective conflicts. TIK Report, 269
8. Brockhoff D, Zitzler E (2009) Objective reduction in evolutionary multiobjective optimization: theory and applications. Evol Comput 17(2):135–166
9. Coello CAC, Lamont GB, Van Veldhuizen DA (2002) Evolutionary algorithms for solving multi-objective problems. Springer, Berlin, p 800
10. Cheng R, Jin Y, Olhofer M, Sendhoff B (2016) A reference vector guided evolutionary algorithm for many-objective optimization. IEEE Trans Evolut Comput 20(5):773–791
11. Cheung YM, Gu F (2014) Online objective reduction for many-objective optimization problems. In: 2014 IEEE congress on evolutionary computation (CEC), pp 1165–1171. IEEE
12. Cheung Y-M, Gu F, Liu H-L (2016) Objective extraction for many-objective optimization problems: algorithm and test problems. IEEE Trans Evol Comput 20(5):755–772
13. Cunningham JP, Ghahramani Z (2015) Linear dimensionality reduction: survey, insights, and generalizations. J Mach Learn Res 16:2859–2900
14. Deb K, Jain H (2014) An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: solving problems with box constraints. IEEE Trans Evolut Comput 18(4):577–601
15. Deb K, Pratap A, Agarwal S, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: Nsga-ii. IEEE Trans Evol Comput 6(2):182–197
16. Deb K, Saxena D (2006) Searching for pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. In: Proceedings of the world congress on computational intelligence (WCCI-2006), pp 3352–3360
17. DeRonne KW, Karypis G (2013) Pareto optimal pairwise sequence alignment. IEEE/ACM Trans Comput Biol Bioinform (TCBB) 10:481–493
18. Erceg-Hurn DM, Mirosevich VM (2008) Modern robust statistical methods: an easy way to maximize the accuracy and power of your research. Am Psychol 63:591
19. Farina M, Amato P (2002) On the optimal solution definition for many-criteria optimization problems. In: Proceedings of the NAFIPS-FLINT international conference, pp 233–238
20. Fleming PJ, Purshouse RC, Lygoe RJ (2005) Many-objective optimization: an engineering design perspective. In: International conference on evolutionary multi-criterion optimization, pp 14–32
21. Fu G, Kapelan Z, Kasprzyk JR, Reed P (2012) Optimal design of water distribution systems using many-objective visual analytics. J Water Resour Plan Manag 139:624–633
22. Gu F, Liu H-L, Cheung Y-m (2017) A fast objective reduction algorithm based on dominance structure for many objective

optimization. In: Asia-Pacific conference on simulated evolution and learning, Springer, pp 260–271
23. Guo X, Wang X, Wang M, Wang Y (2012) A new objective reduction algorithm for many-objective problems: employing mutual information and clustering algorithm. In: 2012 eighth international conference on computational intelligence and security (CIS), IEEE, pp 11–16
24. Guo X, Wang Y, Wang X (2013) Using objective clustering for solving many-objective optimization problems. Mathematical Problems in Engineering 2013
25. Hughes EJ (2003) Multiple single objective pareto sampling. Congr Evolut Comput 2003:2678–2684
26. Ishibuchi H, Masuda H, Nojima Y (2016) Pareto fronts of many-objective degenerate test problems. IEEE Trans Evolut Comput 20:807–813
27. Ishibuchi H, Tsukamoto N, Nojima Y (2008) Evolutionary many-objective optimization. In: 3rd international workshop on genetic and evolving systems, 2008. GEFS 2008, IEEE, pp 47–52
28. Jaimes AL, Coello CAC, Barrientos JEU (2009) Online objective reduction to deal with many-objective problems. In: International conference on evolutionary multi-criterion optimization, Springer, Berlin, pp 423–437
29. Kaufman L, Rousseeuw P (1987) Clustering by means of medoids. North-Holland, Amsterdam
30. Knowles JD, Corne DW (2000) Approximating the nondominated front using the pareto archived evolution strategy. Evol Comput 8(2):149–172
31. Li B, Li J, Tang K, Yao X (2015) Many-objective evolutionary algorithms: a survey. ACM Comput Surv (CSUR) 48(1):13
32. Li M, Yang S, Liu X (2014) Shift-based density estimation for pareto-based algorithms in many-objective optimization. IEEE Trans Evol Comput 18(3):348–365
33. López Jaimes A, Coello CCA, Chakraborty D (2008) Objective reduction using a feature selection technique. In: Proceedings of the 10th annual conference on genetic and evolutionary computation, ACM, New York, pp 673–680
34. Miettinen K (1999) Nonlinear multiobjective optimization, volume 12 of international series in operations research and management science
35. Pei Y, Takagi H (2013) Accelerating iec and ec searches with elite obtained by dimensionality reduction in regression spaces. Evol Intel 6(1):27–40
36. Riquelme N, Von Lücken C, Baran B (2015) Performance metrics in multi-objective optimization. In: Computing conference (CLEI), 2015 Latin American, IEEE, pp 1–11
37. Saxena DK, Deb K (2007) Non-linear dimensionality reduction procedures for certain large-dimensional multi-objective optimization problems: employing correntropy and a novel maximum variance unfolding. In: International conference on evolutionary multi-criterion optimization, Springer, Berlin, pp 772–787
38. Saxena DK, Duro JA, Tiwari A, Deb K, Zhang Q (2013) Objective reduction in many-objective optimization: linear and nonlinear algorithms. IEEE Trans Evol Comput 17(1):77–99
39. Singh HK, Isaacs A, Ray T (2011) A pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems. IEEE Trans Evol Comput 15(4):539–556
40. Tang J, Alam S, Lokan C, Abbass HA (2012) A multi-objective evolutionary method for dynamic airspace re-sectorization using sectors clipping and similarities. In: 2012 IEEE congress on evolutionary computation (CEC), pp 1–8
41. Tate J, Woolford-Lim B, Bate I, Yao X (2012) Evolutionary and principled search strategies for sensornet protocol optimization. IEEE Trans Syst Man Cybernet Part B (Cybernet) 42:163–180
42. Tian Y, Cheng R, Zhang X, Jin Y (2017) Platemo: a matlab platform for evolutionary multi-objective optimization [educational forum]. IEEE Comput Intell Mag 12(4):73–87
43. Tran CT, Zhang M, Andreae P, Xue B (2016) Improving performance for classification with incomplete data using wrapper-based feature selection. Evol Intel 9(3):81–94
44. Wang H, Jiao L, Yao X (2015) Two_arch2: an improved two-archive algorithm for many-objective optimization. IEEE Trans Evol Comput 19(4):524–541
45. Yang S, Li M, Liu X, Zheng J (2013) A grid-based evolutionary algorithm for many-objective optimization. IEEE Trans Evol Comput 17(5):721–736
46. Yuan Y, Xu H, Wang B, Yao X (2016) A new dominance relation-based evolutionary algorithm for many-objective optimization. IEEE Trans Evol Comput 20(1):16–37
47. Zhang Q, Li H (2007) Moea/d: a multiobjective evolutionary algorithm based on decomposition. IEEE Trans Evol Comput 11(6):712–731
48. Zhang X, Tian Y, Jin Y (2015) A knee point-driven evolutionary algorithm for many-objective optimization. IEEE Trans Evol Comput 19(6):761–776
49. Zhang Z (2011) Artificial immune optimization system solving constrained omni-optimization. Evol Intel 4(4):203–218
50. Zhou A, Qu B-Y, Li H, Zhao S-Z, Suganthan PN, Zhang Q (2011) Multiobjective evolutionary algorithms: a survey of the state of the art. Swarm Evolut Comput 1(1):32–49
51. Zitzler E, Laumanns M, Thiele L (2001) Spea2: improving the strength pareto evolutionary algorithm. TIK-report, 103
52. Zitzler E, Thiele L (1999) Multiobjective evolutionary algorithms: a comparative case study and the strength pareto approach. IEEE Trans Evol Comput 3(4):257–271