An Investigation on Amplitude Distribution for Controlling Side-lobe Level of Sparse Cylindrical Sonar Arrays

Nguyen Dinh Tinh Faculty of Radio-Electronic Engineering Le Quy Don Technical University Hanoi, Vietnam tinhnd_k31@lqdtu.edu.vn

Abstract— This paper proposes a new solution calculating amplitude distribution for mitigating side-lobe level (SLL) of sparse cylindrical sonar array (SCSA) based on the explicit expression of the beam pattern, the separation of amplitude distribution into the row and the column, and simulation tools. With the proposed solution, the amplitude distribution is determined in order to satisfy the particular requirements.

Keywords—Phased array antenna, cylindrical array, sparse array, sonar application, amplitude distribution

I. INTRODUCTION

Cylindrical arrays have been used in various applications such as navigation, sonar, and mobile communication systems due to the potential of 360° coverage with an omnidirectional beam or multiple beams, or a narrow beam steering over 360° [1]. The sparse cylindrical sonar array (SCSA) [2] (also named the cylindrical arrays with triangular grid [3]) generated by removing elements from the fully cylindrical array has the advantage of reducing the number of elements while still nearly maintaining performance. Therefore, the SCSA has been utilized in sonar applications for mitigating complexity of hardware and operation [2].

Due to the geometry characteristic of the SCSA, it is challenging to reduce the side-lobe level (SLL) and the half power beamwidth (HPBW) when steering the beam to any desired direction. Although the amplitude distributions play a vital role, there is still a small amount of released research paying attention to them in detail. In [2, 4], simulated annealing method based on adding and removing deactivate elements of the array is used to determine the beam pattern. The method increases computation complexity and challenge of determining the explicit beam pattern. The authors in [5] used two subsequent circles for synthetizing the beam pattern in azimuth plane. The SLL in [2, 4, 5] is higher than -16 dB and difficult to control.

In this study, we propose a determination solution of amplitude distribution generating the low SLL by exploiting mathematical expression of the beam pattern and analyzing simulation results. With the solution, the optimal amplitude distribution is explicitly determined in order to control the SLL and the HPBW for the SCSA.

II. GEOMETRY MODEL OF SCSA

The full SCSA are shown as Fig. 1(a), which includes *P* circles with radius *R* and *N* elements in a circle. $\Delta \theta = 2\pi/N$ and *h* are the angle between two adjacent elements and the distance between two adjacent circles, respectively.

Trinh Dang Khanh Faculty of Radio-Electronic Engineering Le Quy Don Technical University Hanoi, Vietnam khanhtd_k31@lqdtu.edu.vn



Fig. 1. Illustration of SCSA

When steering the beam, the angle of active sector in each circle might be often chosen as 60°, 90° or 120° [1], and the number of active elements in each circle is Q ($Q \le N$) (Fig. 1b). The element at the point A₁ in Fig.1 (b) is chosen as the 1st element of the array. Therefore, the *n*th element in the *p*th ($1 \le p \le P$) circle and the *q*th ($1 \le q \le Q$) column will satisfy the condition of n = Q(p-1)+q, whose coordinates are determined as in [3].

$$x_n = R\cos\left(\left(q - 1 + \frac{1}{2}\left\lceil \frac{p}{2} - \left\lfloor \frac{p}{2} \right\rfloor\right]\right) \Delta\theta\right)$$
(1)

$$y_n = R\sin\left(\left(q - 1 + \frac{1}{2}\left\lceil \frac{p}{2} - \left\lfloor \frac{p}{2} \right\rfloor\right]\right) \Delta\theta\right)$$
(2)

$$z_r = (p-1)h \tag{3}$$

where $\lceil t \rceil$ and $\lfloor t \rfloor$ denote round functions toward integers of arbitrary real number t: $\lceil t \rceil = \min\{n \in \mathbb{Z}, n \ge t\}$ and $\lfloor t \rfloor = max\{n \in \mathbb{Z}, n \le t\}$.

III. DETERMINATION OF THE AMPLIDUDE DISTRIBUTION CONTROLLING THE SLL AND THE HPBW

With general coordinate formula of the SCSA, the beam pattern of the SCSA (array factor) can be mathematically analyzed when steering to a direction represented by the directive unit vector $\vec{u_0} = (\cos\theta_0 \cos\varphi_0, \sin\theta_0 \cos\varphi_0, \sin\varphi_0)$, which is given by [3].

$$AF(\theta,\varphi) = \sum_{n=1}^{p \circ Q} a_n \left(\exp\left(jkR \left(\cos\left(\left(q - 1 + \frac{1}{2} \left\lceil \frac{p}{2} - \left\lfloor \frac{p}{2} \right\rfloor \right] \right) \Delta \theta \right) - 1 \right) \left(\cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \right) \right) \\ \times \exp\left(jkR \sin\left(\left(q - 1 + \frac{1}{2} \left\lceil \frac{p}{2} - \left\lfloor \frac{p}{2} \right\rfloor \right] \right) \Delta \theta \right) \left(\sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0 \right) \right) \\ \times \exp\left(jk(p-1)h(\sin \varphi - \sin \varphi_0) \right)$$
(4)

Based on the explicit expression (4), the beam pattern of SCSA can be mathematically analyzed in order to choose the amplitude distribution satisfying particular requirements for the beam pattern. Thanks to the expression (4), the amplitude distribution a_n into the product of distributions on the row and the column as below.

$$a_n = a_p \times a_q \tag{5}$$

where a_p and a_q are the amplitude distributions on the column and the row, respectively.

In vertical direction, the SCSA can be considered as a uniform linear array antenna (ULAA) in which the Dolph-Chebyshev weights are chosen for controlling the SLL [1, 6]. Therefore, in order to control the SLL in the SCSA, the amplitude distribution on the column a_p is fixed by Dolph-Chebyshev distribution, and the amplitude distributions on the row a_q are investigated by the simulation tools for determining the optimal amplitude distribution satisfying particular requirements of a lower SLL than a requirement value and the narrowest HPBW according to the SLL.

IV. SIMULATION RESULTS

We consider an example of SCSA with 48 elements (N = 48) on a circle and 16 circles (P = 16). When the active sector is chosen as 120°, the number of active elements on a circle is 17 (Q = 17), and the total number of active elements in the array is $16 \times 17 = 272$ elements.

Assuming that the sound speed in sea water is 1500 m/s, carrier frequency is 30 kHz ($\lambda = 5$ cm). The distances between two adjacent circles and between two adjacent elements on a circle is chosen as distances with $h = 0.5 \times \lambda = 2.5$ (cm) and $d = 0.8 \times \lambda = 4$ (cm), respectively. Therefore, the radius of a circle in the SCSA is R = 30.6 (cm). Considering the desired steering angles in azimuth and elevation planes are $\theta_0 = 60^\circ$ and $\phi_0 = 0^\circ$ respectively.

The amplitude distribution a_p is chosen as Dolph-Chebyshev weights with the side-lobe attenuation (SLA) -25 dB. In order to reduce the SLL in the azimuth plane to less than -22 dB, which is lower than the SLL released in [2, 4, 5], the amplitude distributions a_q chosen include Dolph-Chebyshev window (SLA = -45 dB), Gaussian window with the standard deviation σ =2.1, Hanning window, and Kaiser

comparison of the parameters.

The distributions can be generated by simulation tools as the MATLAB software. The simulation results of the beam pattern of the SCSA are shown in Fig.2 and Table I, which are analyzed, compared to determine the optimal distribution according to the SLL and the HPBW.

window with β =4.6. A uniform window is also chosen for



Fig. 2. The beam patterns of SCSA with amplitude distributions

TABLE I. THE PARAMETERS OF THE SCSA BEAM PATTERNS WITH AMPLITUDE DISTRIBUTIONS

| Distribution for a_q | SLL (dB) | HPBW (°) |
|--|----------|----------|
| Uniform | -10.14 | 4.20 |
| Dolph-Chebyshev ($SLA = -45 \text{ dB}$) | -22.98 | 5.94 |
| Gaussian ($\sigma = 2.1$) | -23.29 | 5.74 |
| Hanning | -22.73 | 6.76 |
| Kaiser ($\beta = 4.6$) | -22.01 | 5.98 |

It can be seen that in these cases, the uniform window can give the narrowest HPBW but generate an uncontrollably high SLL (-10.14 dB). With the amplitude distributions satisfying the SLL less than -22 dB, Hanning window provides the SLL -22.73 dB but generates the widest HPBW (6.76°) and the uncontrollable SLL. Gaussian window with σ =2.1 provides both the lowest SLL (-23.29 dB) and the narrowest HPBW (5.74°). The others generate both a higher SLL and a wider HPBW than Gaussian window. Therefore, the Gaussian window is the optimal amplitude distribution providing the lowest SLL and the narrowest HPBW.

V. CONCLUSION

The paper has investigated amplitude distribution for the SCSA and determined the amplitude distribution providing the lower SLL than -22 dB with the narrowest HPBW. With the proposed solution, the SLL and the HPBW of the SCSA have been controlled to satisfy particular requirements for both the SLL and the narrowest HPBW.

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