

Distance-Based Mean Filter for Image Denoising

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ABSTRACT

In this paper, we propose distance-based mean filter (DBMF) to remove the salt and pepper noise. Although DBMF also uses the adaptive conditions like AMF, it uses distance-based mean instead of median. The distance-based mean focuses on similarity of pixels based on distance. It also skips noisy pixels from evaluating new gray value. Hence, DBMF works more effectively than AMF. In the experiments, we test on 20 images of the MATLAB library with various noise levels. We also compare denoising results of DBMF with other similar denoising methods based on the peak signal-to-noise ratio and the structure similarity metrics. The results showed that DBMF can effectively remove noise with various noise levels and outperforms other methods.

CCS CONCEPTS

• Computing methodologies → Image processing;

Keywords

Image restoration, Image denoising, Image Processing, Image Quality Assessment, Salt and pepper noise.

1. INTRODUCTION

The salt and pepper (SnP) noise [1, 2, 3, 4, 5] is a popular type of noise appearing in many electronics devices. SnP is caused by a sharp and sudden disturbance in the signal and it reduces image quality. This issue reduces performance for many automation and control systems [6, 7, 8, 9, 10]. Hence, noise removal is a very necessary task. Moreover, since noise reduces image quality and can cause many issues for pattern recognition, classification, and feature extraction, it is necessary to remove it. Therefore, the denoising problem has a wide range of applications. It is an important task of the preprocessing stage before further processing/analyzing.

Unlike other types of noise such as Gaussian noise [11, 12, 13] or Poisson noise [14], SnP noise has a simpler structure. It is a type of impulse noise. In a noisy image, noisy pixels are classified into two classes [15, 10]: salt pixels (white pixels) and pepper pixels (black pixels). The salt pixels have gray value equaling the maximum gray value of the image and the pepper pixels have gray value being the

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same as the minimum gray value of the image. For an 8-bits grayscale image, the maximum gray value is 255 and the minimum gray value is 0.

In this work, we proposed a distance-based mean filter to remove SnP noise. DBMF is developed based on Adaptive Median Filter (AMF). DBMF and AMF use similar adaptive conditions for processing high-density noise. However, AMF has two limitations: (1) for high-density noise, the median may reflect inaccurately the new gray value of the center pixel, and (2) the median does not take into account the similarity of pixels. Therefore, DBMF is proposed in this paper to improve the noise removal performance of AMF. Basically, DBMF does not use noisy pixels to evaluate a new gray value for the center pixel of an adaptive window. It mainly focuses on the similarity of pixels in the window by placing weights based on distance for pixels. Hence, evaluating a new gray value for the center pixel of the window is more accurately.

Our contributions focus on: (1) proposing a distance-based mean that takes into account weights based on distance of pixels, (2) improving Adaptive Median Filter by replacing the median by the distance-based mean in the algorithm implementation, and (3) implementing the proposed denoising algorithm, comparing denoising results with other similar methods and discussing on the obtained results.

In the experiments, we test the proposed denoising method on 20 images of the MATLAB library with various noise levels. We also compare denoising results with ones of other state-of-the-art denoising methods.

The paper is structured as follows. Section 2 is a literature review. Section 3 introduces the proposed denoising method. Section 4 presents the experiments and the comparison of denoising results with other similar methods. Finally, section 5 concludes.

2. LITERATURE REVIEW

To remove SnP noise, there are many approaches including filters (nonlinear, linear) [2, 16], regularization [10, 17, 15], wavelets: Empirical Bayes [18], False Discovery Rate [19] (FDR), Stein's Unbiased Risk Estimate [20] and machine learning [21]. In the paper, we mainly focus on filters.

There are two types of filters: linear filters and nonlinear filters. Linear filters can only remove low-density noise. They are not effective for medium-density and high-density noise. Inversely, nonlinear filters are more effective, especially for SnP noise. Among nonlinear filters, the Median Filter (MF) is a simple and effective filter for removing low-density noise (usually up to 20%). Adaptive Median Filter (AMF) [22] is an improved version of MF [23] that focuses on medium-density and high-density noise removal. AMF uses adaptive conditions based on the maximum gray value, the minimum gray value, the median value and gray

values of pixels in an adaptive window. If the conditions are satisfied, the median value of gray values of all pixels in the adaptive window will be assigned to the center pixel of the window. The size of an adaptive window also changes to remove noise more flexibly and effectively. In many works, the maximum size of the window is fixed by 9. The higher values of the size can slow down the denoising process.

Modified Decision Based Unsymmetric Trimmed Median Filter (MDBUTMF) [1] is another effective filter for SnP. MDBUTMF also uses a dynamic window to remove noise. Otherwise, it also excludes noisy pixels before evaluating the median value. Hence, it is an effective filter. When MDBUTMF process high-density noise, it can cause some defects. In recent years, another well-known filter is Based Pixel Density Filter (BPDF) [2]. BPDF works well on low-density and medium-density noise. For high-density noise, BPDF usually causes a raindrop effect. Adaptive Type-2 Fuzzy Filter (T2FF) [24] is another effective filter that is based on the theory of fuzzy decision. However, when processing high-density noise, T2FF usually does not preserve image structure well. Iterative Mean Filter [16] is another state-of-the-art filter. It can remove noise with various densities effectively. However, unlike other nonlinear filters, IMF is an iterative manner. Therefore, it requires tolerance. This matter can affect the speed of evaluation.

3. IMAGE DENOISING METHOD

3.1 Definitions and Noise Model

Let $[u_{ij}]_{m \times n}$, $[v_{ij}]_{m \times n}$, $[w_{ij}]_{m \times n}$ be a noise-free image, a corrupted image by SnP noise and a restored image, respectively, where m, n – numbers of pixels by the image width (horizon) and height (vertical), respectively.

Definition 1. Suppose that $[u_{min}, u_{max}]$ is a range of grey values of an image. SnP noise can be modeled as follows:

$$v_{ij} = \begin{cases} u_{min}, & \text{with probability } p \\ u_{max}, & \text{with probability } q \\ u_{ij}, & \text{with probability } 1 - (p + q) \end{cases},$$

where $p, q, p + q \in [0, 1]$.

Definition 2. Let $d \geq 1$ be a natural number. An adaptive window centered at a location (i, j) with size $2d + 1$ is defined as follows:

$$\mathcal{W}_{ij}(d) = \{(i', j') : |i - i'| \leq d, |j - j'| \leq d\}.$$

Definition 3. We denote the maximum value, the minimum value and the median value of an adaptive window $\mathcal{W}_{ij}(d)$ by $\mathcal{W}_{ij}^{max}(d)$, $\mathcal{W}_{ij}^{min}(d)$, $\mathcal{W}_{ij}^{med}(d)$, respectively.

Definition 4. Let (i_1, j_1) and (i_2, j_2) be locations of two pixels in an adaptive window $\mathcal{W}_{ij}(d)$. Euclidean distance between two pixels is defined as follows:

$$\mathcal{D}_{(i_1, j_1)}^{(i_2, j_2)} = \sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}.$$

Definition 5. Distance-based mean of an adaptive window $\mathcal{W}_{ij}(d)$ of an image $[w_{ij}]_{m \times n}$ is defined as follows:

$$\begin{aligned} & \mathcal{W}_{ij}^{dmean}(d) \\ &= \sum_{(i', j') \in \mathcal{W}_{ij}(d)} \frac{\delta_{i' j'} w_{i' j'}}{(2 + \mathcal{D}_{(i, j)}^{(i', j')})^2} / \sum_{(i', j') \in \mathcal{W}_{ij}(d)} \frac{\delta_{i' j'}}{(2 + \mathcal{D}_{(i, j)}^{(i', j')})^2}, \end{aligned}$$

where

$$\delta_{i' j'} = \begin{cases} 1, & \text{if } \mathcal{W}_{ij}^{min}(d) < v_{i' j'} < \mathcal{W}_{ij}^{max}(d) \\ 0, & \text{otherwise} \end{cases}.$$

3.2 Motivation of the proposed method

The goal of the proposed method based on the algorithm of the adaptive median filter. The AMF algorithm is presented in Algorithm 1. Note that, the notion $\&$ is for AND operation and the notion $\|$ is for OR operation. As can be seen, AMF uses adaptive conditions to detect the noisy pixels and if the conditions are satisfied, AMF uses median to evaluate the new gray value of the center pixel of the window. By this way, there are two drawbacks: the median value of AMF may contain gray value of noisy pixels and AMF skips similarity of gray value of neighbor pixels (i.e. adjacent pixels have closer gray value than nonadjacent pixels).

To remove the drawbacks, we propose a distance-based mean (see Definition 5). The distance-based mean will exclude noisy pixels from evaluating the new gray value for the center pixel of the window. Otherwise, it places weights based Euclidean distance between pixels in the adaptive window and the center pixel before evaluating the mean.

3.3 Salt and Pepper Denoising Method

Algorithm of DBMF is similar to one of AMF. However, we do not use the median, but we use the distance-based mean. Algorithm of DBMF is presented in Algorithm 2.

Algorithm 1. Adaptive Median Filter (AMF).

Input: The corrupted image v .

Output: The restored image w .

Initialize $w := v, d_{max}$

For each pixel (i, j) of the image w

For d from 1 to d_{max}

 Evaluate $\mathcal{W}_{ij}^{max}(d), \mathcal{W}_{ij}^{min}(d), \mathcal{W}_{ij}^{med}(d)$

If $\mathcal{W}_{ij}^{min}(d) < \mathcal{W}_{ij}^{med}(d) \& \mathcal{W}_{ij}^{med}(d) < \mathcal{W}_{ij}^{max}(d)$

If $w_{ij} = \mathcal{W}_{ij}^{min}(d) \| w_{ij} = \mathcal{W}_{ij}^{max}(d)$

 Set $w_{ij} := \mathcal{W}_{ij}^{med}(d)$

Break

End

End

End

Algorithm 2. Distance-Based Mean Filter (DBMF).

Input: The corrupted image v .

Output: The restored image w .

Initialize $w := v, d_{max}$

For each pixel (i, j) of the image w

For d from 1 to d_{max}

 Evaluate $\mathcal{W}_{ij}^{max}(d), \mathcal{W}_{ij}^{min}(d), \mathcal{W}_{ij}^{dmean}(d)$

If $\mathcal{W}_{ij}^{min}(d) < \mathcal{W}_{ij}^{dmean}(d) \& \mathcal{W}_{ij}^{dmean}(d) < \mathcal{W}_{ij}^{max}(d)$

If $w_{ij} = \mathcal{W}_{ij}^{min}(d) \| w_{ij} = \mathcal{W}_{ij}^{max}(d)$

 Set $w_{ij} := \mathcal{W}_{ij}^{dmean}(d)$

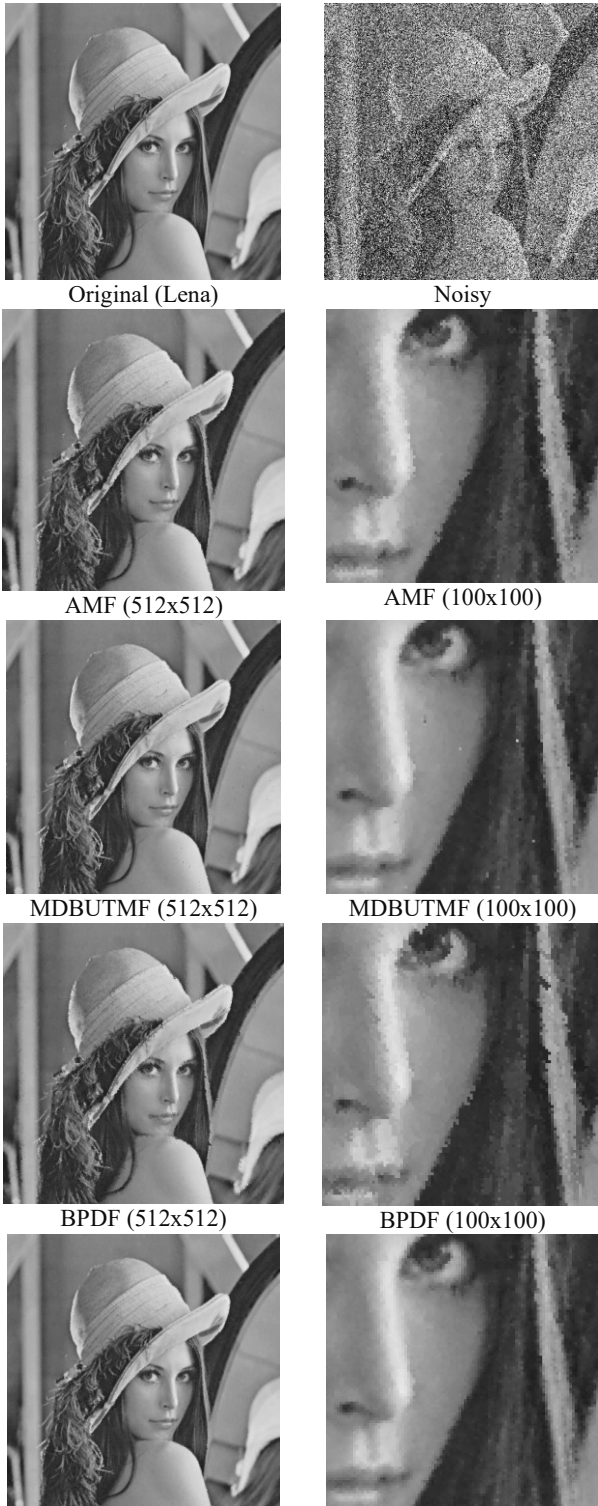
Break

End

End

End

End



By proposed method (512x512) By proposed method (100x100)
Figure 1. Denoising results on the Lena image with 50% of noise. PSNR/SSIM of the AMF, MDBUTMF, BPDF and DBMF methods are 30.2610/0.8905, 32.2601/0.9129, 28.4392/0.8693 and 33.4460/0.9216, respectively.

Let us explain the algorithm of DBMF works. DBMF will consider all pixels of the image. For each pixel (i, j) , it considers adaptive windows centered at a pixel location (i, j) with sizes varying from 1 to d_{max} . For the low-density noise, we can choose d_{max} to be

small, but for the high-density noise, we must choose d_{max} to be large enough. If value of d_{max} is too large, evaluation time is too long. In practice, we choose $d_{max} = 9$, i.e. sizes of adaptive windows are 3x3, 5x5 to 19x19. For each adaptive window, we evaluate the maximum value, the minimum value and the distance-based mean value. If these values satisfied the adaptive conditions, the value of the distance-based mean of the adaptive window will be assigned to gray value of the center pixel of the window. This process continues until all pixels of the image are considered. We must notice that, by using the distance-based mean, we can exclude the noisy pixels from evaluating new gray value by the help of weights $\delta_{i',j'}$. Otherwise, similarity of pixels is also preferred for computing new gray value of the center pixel by using weights $\mathcal{D}_{(i,j)}^{(i',j')}$ based the distance of two pixels (i, j) and (i', j') .

It is similar to the algorithm of AMF, complexity of DBMF is $\mathcal{O}(m \times n \times d_{max})$. If $m, n \gg d_{max}$, complexity of DBMF is $\mathcal{O}(m \times n)$, where $m \times n$ are a size by pixel of an image.

4. EXPERIMENTS

4.1 Full Reference Image Quality Assessment

Peak signal-to-noise ratio (PSNR) [25, 26, 27, 28] error metric is a quantitative metric and defined as follows:

$$PSNR = 10 \log_{10} \left(\frac{u_{max}^2}{MSE} \right), \quad MSE = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (w_{ij} - u_{ij})^2$$

is mean squared error, u is a noise-free image, u_{max} denotes the maximum gray value, e.g., for an 8-bit image $u_{max} = 255$. The higher PSNR value, the better image quality.

Structural similarity (SSIM) [25, 26, 27] is a qualitative metric and is proven to be a better error metric and its value is in $[0, 1]$. The higher SSIM value, the better image quality. This metric based on the characteristic of the human vision. The SSIM is computed between two images ω_1 and ω_2 ,

$$SSIM = \frac{(2\mu_{\omega_1}\mu_{\omega_2} + c_1)(2\sigma_{\omega_1\omega_2} + c_2)}{(\mu_{\omega_1}^2 + \mu_{\omega_2}^2 + c_1)(\sigma_{\omega_1}^2 + \sigma_{\omega_2}^2 + c_2)}$$

where μ_{ω_i} – the average of ω_i , $\sigma_{\omega_i}^2$ – the variance of ω_i , $\sigma_{\omega_1\omega_2}$ – the covariance, and c_1, c_2 numerical stabilizing parameters.

4.2 Dataset, Test Cases and Discussion

We test the proposed denoising method on 20 images of the MATLAB library: Lena, Cameraman, Barbara, Hill, Pirate, Boat, House, Baboon, Peppers, Flower, Parrot, Living Room, Lake, Plane, Bridge, Elaine, Flintstones, Dark-Haired Woman, Blonde Woman, and Einstein. All images are stored in TIF format, grayscale and with the size of 512x512 pixels.

For the first case, we implement denoising methods such as AMF, MDBUTMF, BPDF and the proposed method to remove noise of 50% on the Lena image. Denoising results are presented in Figure 1. We can see that noise damaged the Lena image and it is very hard to see all image contents. AMF and BPDF cannot preserve edges well. Otherwise, they also created artifacts. This can be seen clearly on the nose, the eyes and the hairs of Lena. MDBUTMF and the proposed method removed noise excellently. However, for MDBUTMF, there is a little noise on the result. DBMF worked perfectly. All noise was removed, edges were preserved well. All details were smoothed naturally. The PSNR/SSIM values of AMF, MDBUTMF, BPDF and DBMF are 30.2610/0.8905, 32.2601/0.9129, 28.4392/0.8693 and **33.4460/0.9216** (the highest), respectively. By both PSNR and SSIM, we can confirm that the denoising result of DBMF is the best.

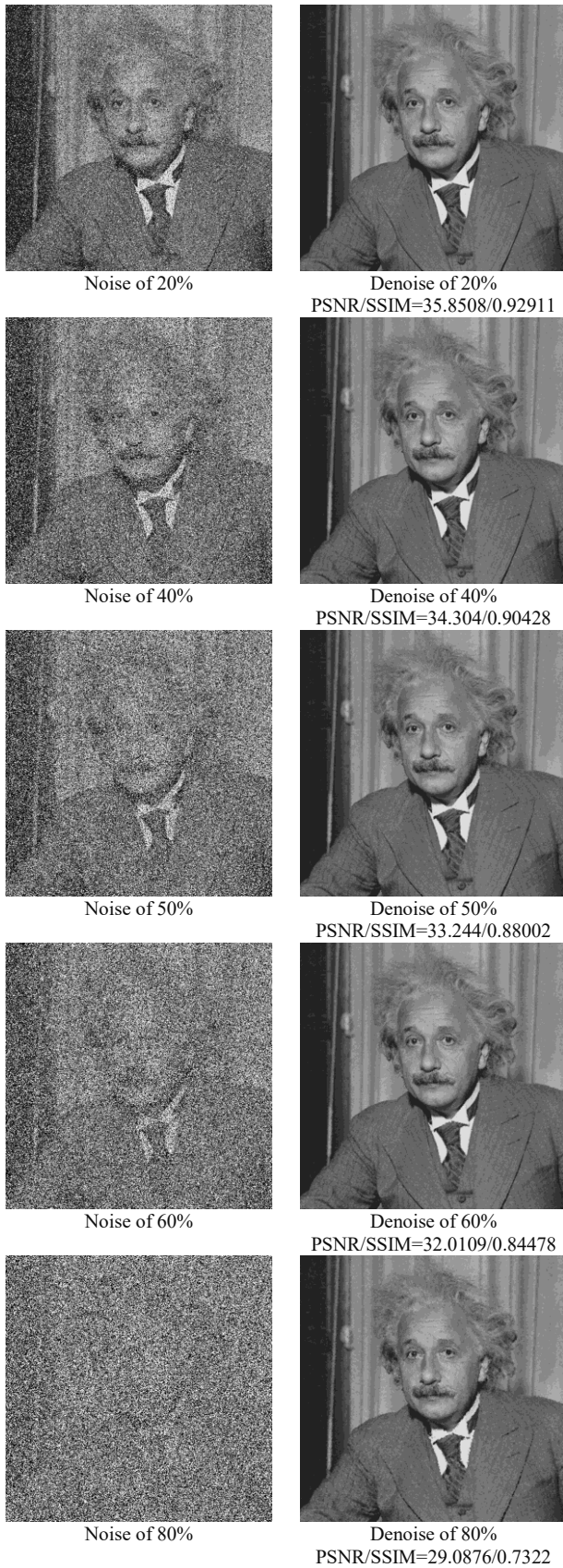


Figure 2. Denoising results of DBMF for the Einstein image with various noise levels (20%, 40%, 50%, 60%, 80%).

Table 1. PSNR values of denoising results of the methods for 20 images of the MATLAB library with various noise levels.

| Noise Level | AMF | MDBUTMF | BPDF | DBMF |
|-------------|---------|----------------|---------|----------------|
| 20% | 33.3153 | 36.3153 | 33.5589 | 33.4341 |
| 40% | 29.9258 | 32.1386 | 28.8353 | 32.3975 |
| 50% | 28.4183 | 29.9792 | 26.8029 | 31.3102 |
| 60% | 26.9479 | 27.0985 | 24.6268 | 29.9589 |
| 80% | 23.5413 | 19.3551 | 17.7594 | 26.704 |
| Mean | 28.4297 | 28.9773 | 26.3167 | 30.7609 |

Table 2. SSIM values of denoising results of the methods for 20 images of the MATLAB library with various noise levels.

| Noise Level | AMF | MDBUTMF | BPDF | DBMF |
|-------------|--------|---------------|--------|---------------|
| 20% | 0.9237 | 0.9706 | 0.9549 | 0.9276 |
| 40% | 0.8933 | 0.9183 | 0.886 | 0.9188 |
| 50% | 0.86 | 0.8891 | 0.836 | 0.9003 |
| 60% | 0.817 | 0.8037 | 0.7666 | 0.8711 |
| 80% | 0.6811 | 0.3883 | 0.5078 | 0.7655 |
| Mean | 0.835 | 0.794 | 0.7903 | 0.8767 |

The second test case is for denoising the Einstein image with various noise levels. We added noise with 20%, 40%, 50%, 60% and 80% over the image. After that, we denoised by DBMF. Denoising results are shown in Figure 2. DBMF removed noise efficiently for all noise levels. Moreover, DBMF can preserve textures on the blazer of Einstein very well even in the case of the high noise level of 80%.

The third test case is to implement the denoising methods on 20 images of the MATLAB library. We add noise of 20%, 40%, 50%, 60% and 80% over the images and denoise them by the methods. The average PSNR values for each noise level are presented in Table 1. The average SSIM values for each noise level are presented in Table 2. For the noise level of 20%, the average PSNR and SSIM values of DBMF are lower than ones of MDBUTMF and BPDF, but still higher than ones of AMF. For other noise levels (40%, 50%, 60%, 80%), DBMF gave the best denoising results (the average PSNR/SSIM values of DBMF is the highest). The average PSNR value and the average SSIM value for all noise levels of DBMF are also the highest. Hence, we can confirm that DBMF outperforms other compared denoising methods.

For execution time, all methods work very fast. They only take up to 3 seconds to process an 512x512 pixels.

5. CONCLUSION

In this work, we have proposed distance-based mean filter (DBMF) to remove the salt and pepper noise. DBMF is developed based on AMF: use adaptive conditions to detect noise, but DBMF uses distance-based mean instead of median. The distance-based mean can exclude gray values of noisy pixels. Otherwise, it also focuses on similarity of pixels based on distance. Hence, DBMF outperform AMF. From a variety of test cases, DBMF can compete with other state-of-the-art denoising methods.

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