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Research Article

On Performance of Full-Duplex Decode-and-Forward Relay Systems with an Optimal Power Setting under the Impact of Hardware Impairments

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In this paper, we analyze the performance of in-band full-duplex (IBFD) relay systems that use the decode-and-forward (DF) protocol at the relay under the impact of imperfect self-interference cancellation and hardware impairments. Three practical relay scenarios are considered in our analysis: (i) there is no direct link from the source to the destination; (ii) there is a direct link, but the signal from the source is considered interference; and (iii) there is a direct link, and the signal from the source is cooperatively combined with that from the relaying path. Specifically, we derive exact and asymptotic expressions for the outage probability (OP) of the IBFD system. Based on the OP, the exact expression for symbol error probability (SEP) is also derived. Moreover, in order to cope with the effect of imperfect self-interference cancellation (SIC) due to the full-duplex mode, we propose optimal and suboptimal power calculation methods for the relay to minimize the OP and SEP. A performance evaluation shows that the IBFD relay system is significantly affected by both imperfect SIC and hardware impairments. However, the optimal power values can help to improve the system performance, significantly.

1. Introduction

The last decade has witnessed significant development of wireless technologies. In particular, the evolution of the fifth generation (5G) of mobile communications allows deployment of massive Internet of Things (IoT) devices to provide data access for various applications such as precision agriculture, disaster warning, smart retail, health care, smart home, and industrial innovation. It is expected that there will be billions of IoT devices connected to the cellular networks in the coming years. The main features of the massive IoT devices include low cost, extreme coverage, narrow band, and small volume of data. In order to meet these requirements, multiple antennas are not used in the devices but wireless relaying is desired to extend the coverage. It also requires that spectrum is efficiently used to support massive connections within the limited radio frequency band.

In the literature, there were a large number of works studying the relay communications for the wireless networks.

With the help of a relay node, it is possible to extend the network range as well as increase the channel capacity. Relay communication has now found its application in various types of wireless networks such as the long-term evolution advanced (LTE-A) mobile communication, ad hoc network, and vehicle-to-vehicle (V2V) networks. Meanwhile, in-band full-duplex (IBFD) communication, which allows for simultaneous transmission and reception over the same frequency, has attracted increasing attention due to its capability to double the channel capacity and increase the spectrum efficiency. Benefits of using both IBFD and relay communications have been studied widely [1–11]. Recently, the combination of IBFD and relay cooperation has also been considered for the 5th generation (5G) heterogeneous networks (HetNets) [12].

Most studies on IBFD relay networks tackle the problem of residual self-interference (RSI) due to imperfect self-interference cancellation (SIC) (note that this abbreviation differs from that used for successive interference cancellation

in CDMA, NOMA, and MIMO systems) at the full-duplex relay node [2, 4, 13–15]. A paper [4] showed that the outage performance of the IBFD relay system is significantly affected by RSI. In this work, the authors have also successfully derived the closed form of the outage probability (OP) of the IBFD relay network. In [6], the authors analyzed the spectrum efficiency of the IBFD relay systems by using the sum rate as a function of the distance between users and the relay. Performance of the IBFD systems was shown to depend on the interference suppression capability and location of the relay. The work in [16] considered an IBFD multiuser decode-and-forward (DF) relay system with self-interference management. In order to maximize the signal-to-noise ratio (SNR) at the receiver, the authors in [17] investigated the IBFD amplify-and-forward (AF) system with the direct link using the minimum mean squared error decision feedback equalizer (MMSE-DFE). Their results showed that the outage performance of the IBFD system could be significantly improved if the direct link is appropriately combined with the relaying link at the destination.

In [18], relay selection was proposed for a full-duplex decode-and-forward (FD-DF) relay network with Naka-gami-m fading channels. The exact OP expressions were investigated, and their results were shown that the achievable diversity depends on the shape factors of the fading channels and the power scaling scheme of the relay. In [19], the system quality of the DF opportunistic relay network was analyzed in the presence of cochannel interference in the case of cooperative communications. Their results showed that the opportunistic relay selection based on the signal-to-interference-plus-noise ratio (SINR) provides more benefits with an increasing number of relays. In [5], the outage performance of the FD cooperative system quality was analyzed under asymmetric generalized fading conditions. The authors demonstrated that the system performance is much affected by the fading conditions of both the direct and relay links.

Although the outage performance of IBFD relay networks was extensively studied in previous works, the common assumption was that the transceivers were ideal. In practice, most hardware equipment is not ideal and is often influenced by various factors such as manufacturing errors, phase oscillator noises, imbalance in the modulator, nonlinear distortion of the high-power amplifier (HPA) at the transmitter, and the low-noise amplifier (LNA) at the receiver. These hardware impairments degrade the performance and thus should not be neglected in the system design and performance analysis. In fact, the impact of hardware impairments was thoroughly analyzed for the half-duplex relay system in the literature. In [20], the authors analyzed the impact of hardware impairments at both the transmitter and receiver in a dual-hop relay network using AF and DF protocols. In [8], the authors examined the effect of hardware impairments on the system performance of a relay network using switch-and-examine combining with a postselection scheduling scheme in the Rician fading and shadowing channel. Hardware impairments were shown to cause an irreducible outage floor at the high SINR region. The work in [21]

investigated and compared the outage performance of the one-way and two-way relay networks under the impact of both hardware impairments and imperfect channel estimation. It was shown in [21] that the one-way relay network has a lower outage floor than the counterpart network. Recently, HI was also considered in the IBFD relay systems [22–27]. In particular, the impact of HI on the spectral efficiency of a multipair massive MIMO two-way IBFD relay system was analyzed in [22]. The spectral efficiency of a similar system using zero forcing at the relay was evaluated in [23]. A gradient projection-based algorithm to obtain the optimal relay function together with nonalternating and alternating suboptimal solutions was proposed for IBFD AF relay systems with multiple antennas in [24]. The alternating optimization was then extended to design the bidirectional IBFD MIMO OFDM systems with linear precoding and decoding [26]. The performance of the IBFD relay system under the impact of hardware impairments, imperfect channel state information, and imperfect self-interference cancellation was analyzed in [25]. Antenna selection was proposed as a solution to compensate for the performance saturation of the MIMO spatial modulation IBFD relay system [27]. Although the impact of hardware impairments on the performance of the IBFD relay systems was widely analyzed, the case with direct link between the source and the destination nodes was still neglected. Moreover, the exact solution for relay power allocation has not been reported.

Motivated by the previous results, in this paper, we analyze the OP and symbol error probability (SEP) of the IBFD decode-and-forward (DF) relay networks under the effect of the aggregate transceiver impairments and different cooperation scenarios. The contributions of this paper are summarized as follows:

  (i) Inspired by the 5G HetNet structure [12], we develop a new system model for a more realistic IBFD relay network with hardware impairments and RSI. Based on this model, performance of the IBFD relay network is analyzed for realistic cooperation situations

 (ii) Exact expressions of the signal-to-interference-plus-noise-and-distortion ratio (SINDR) and outage performance are derived for three cases, i.e., (i) there is no direct link from the source to the destination; (ii) there is a direct link, but the signal from the source is considered interference; and (iii) there is a direct link, and the signal from the source is cooperatively combined with that from the relaying path

(iii) In order to cope with the effect of imperfect self-interference cancellation (SIC) due to the full-duplex mode, we propose optimal and suboptimal power calculation methods for the relay to minimize the OP. Unlike previous works about a power allocation scheme for the IBFD transmission mode, we obtain the exact optimal transmission power value for the IBFD relay that minimizes the OP

and SEP of the IBFD relay network. Our results can significantly reduce the computational complexity in the deployment of the IBFD relay system

The remainder of this paper is organized as follows. Section 2 presents the system model of the IBFD relay network. Performance analysis is given in Section 3, followed by the proposed optimal/suboptimal power calculation methods in Section 4. The corresponding numerical results are presented in Section 0. Finally, conclusions are drawn in Section 0. For reading convenience, a summary list of the frequently used mathematical notations is given in Table 1.

2. System Model

Let us consider the case of an IBFD DF relay system as shown in Figure 1. In this system, the two terminal nodes S_1 and S_2 transmit to the relay node R one at a time while R can forward the received signal from one terminal node to the other at the same time using the same carrier frequency. Due to the symmetrical communications, we will limit our presentation for the link from S_1 to S_2 via R. The source node S_1 and the destination node S_2 are equipped with a single antenna while the relay R has two antennas, one for reception and one for transmission. Unlike in the ideal hardware system, the transmitted signal from a relay system with hardware impairment is distorted due to the impact of the transceiver errors such as amplification gain instability, phase noise (PN), high-power amplifier (HPA) nonlinearity, or in-phase/quadrature-phase (I/Q) imbalance. In order to tackle the problem of hardware impairments, various compensation solutions using both analog and digital signal processing have been reported in the literature [21, 28–30]. However, the measurement results demonstrated that the residual impairments after compensation still affect the system as the additive distortion noises which can be modeled by a complex Gaussian variable with zero mean and finite variance [21, 28–30]. As can be seen from Figure 1, there are two distortion sources in the system, which are denoted by $\eta_i(t)$, $i = \{1, R\}$, where t denotes the time slot. In this paper, we follow the distortion model in [30] which combines the distortion source at the transmitter with that at the receiver. Thus, $\eta_1(t)$ denotes the combined distortions at both the transmitter of S_1 and the receiver of R and $\eta_R(t)$ for those at both the transmitter of R and receiver of S_2. These combined distortion sources have the following distributions $\eta_1 \sim \mathcal{CN}(0, k_{S_1R}^2 P_1)$ and $\eta_R \sim \mathcal{CN}(0, k_{RS_2}^2 P_R)$; P_1 and P_R are the average signal power from the source and the relay node, respectively; $k_{S_1R}^2 = (k_{S_1}^t)^2 + (k_R^r)^2$; $k_{RS_2}^2 = (k_R^t)^2 + (k_{S_2}^r)^2$, where $k_{S_1}^t$ and k_R^t and k_R^r and $k_{S_2}^r$ are, respectively, the level of the impairments at the transmitters of S_1 and R and the receivers of R and S_2. Note that when $k_{S_1R} = k_{RS_2} = 0$, we have an ideal hardware system. The transmit signal from each transmitter can be expressed as $s_i(t) + \eta_i(t)$. We assume that the processing delay at the relay R equals one symbol period. Therefore, a symbol

TABLE 1: List of the frequently used mathematical notations.

Notation	Description
P_1	The average transmit power at the source node S_1
P_R	The average transmit power at the relay node R
Prob$\{\mathscr{A}\}$	The probability of an event $\mathscr{A}$
OP	Outage probability
$\text{OP}^{(\ell)}{}_{ex}$	Exact OP of the case ℓ with hardware impairments
$\text{OP}^{(\ell)}{}_{\_ex\_id}$	Exact OP of the case ℓ with ideal hardware
$\text{OP}^{(\ell)}{}_{\_ap}$	Approximate OP of the case ℓ with hardware impairments
$F_\rho(.)$	Cumulative distribution function (CDF)
$f_\rho(.)$	Probability density function (PDF)
$\mathscr{CN}(\mu, \sigma^2)$	Complex Gaussian distribution with mean μ and variance of σ^2
$\mathbb{E}\{\cdot\}$	The expectation operator
$\rho = |h|^2$	The instantaneous channel gain
$\Omega = \mathbb{E}\{\rho\}$	The average power of the channel gain
SEP	Symbol error probability

which was received by R at time slot $t-1$ will be transmitted at time slot t. It means that $s_R(t) = s_1(t-1)$ if R decodes successfully the received signal. The received signal at the relay and the destination is then given, respectively, by

$$y_R(t) = h_{1R}\tilde{s}_1(t) + \tilde{h}_{RR}\tilde{s}_R(t) + z_R(t), \quad (1)$$

$$y_2(t) = h_{R2}\tilde{s}_R(t) + h_{12}\tilde{s}_1(t) + z_2(t), \quad (2)$$

where $\tilde{s}_1(t)$ and $\tilde{s}_R(t)$, respectively, denote the transmitted signals from the source and the relay with $\tilde{s}_1(t) \triangleq s_1(t) + \eta_1(t)$ and $\tilde{s}_R(t) \triangleq s_R(t) + \eta_R(t)$; $\eta_1(t)$ and $\eta_R(t)$ denote the effect of hardware impairments at the transceivers with $\eta_1 \sim \mathcal{CN}(0, k_{S_1R}^2 P_1)$ and $\eta_R \sim \mathcal{CN}(0, k_{RS_2}^2 P_R)$; h_{1R}, h_{R2}, and h_{12} are the fading coefficients of the links from S_1 to R, from R to S_2, and from S_1 to S_2, respectively; $\tilde{h}_{RR}$ is the RSI channel from the transmitting antenna to the receiving antenna at the FD relay R after self-interference cancellation; and $z_R(t)$ and $z_2(t)$ denote the AWGN with zero mean and variance σ_R^2 and σ_2^2, respectively, $z_R \sim \mathcal{CN}(0, \sigma_R^2)$ and $z_2 \sim \mathcal{CN}(0, \sigma_2^2)$.

Note from (1) that the term $\tilde{h}_{RR}\tilde{s}_R(t)$ represents the SI at R and its power is significantly larger than that of the intended signal due to small distance between transmitter and receiver antennas of R. In order for the relay to decode the intended signal successfully, three popular SIC techniques can be applied to make the SI to the noise level, including those in the propagation, analog and digital domains [4, 31]. The RSI after these SIC techniques can be modeled by a Gaussian random variable, denoted
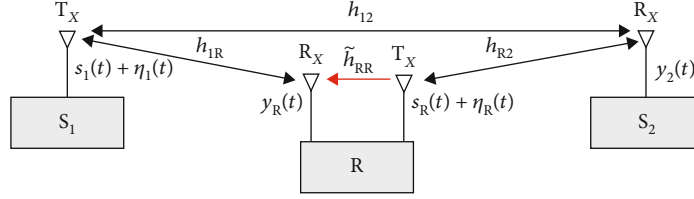
FIGURE 1: Block diagram of the IBFD relay system with transceiver impairments.

by I_R with zero mean and variance of σ_{RSI}^2, i.e., $I_R \sim C\mathcal{N}(0, \sigma_{RSI}^2)$ [4, 31]. The received signal at R after SIC can be now expressed as

$$y_R(t) = h_{1R}\tilde{s}_1(t) + I_R(t) + z_R(t). \tag{3}$$

In practice, due to the distance variation and shadowing, the transmitted signal from the source can reach the destination in some cases. Therefore, the system quality in realistic conditions varies according to the following three cases: (1) there is no direct link from S_1 to S_2; (2) there is a direct link from S_1 to S_2, but its signal is considered interference at S_2; and (3) there is a direct link from S_1 to S_2, and its signal is combined with that of the relaying link using cooperative communication.

Note that when the DF relaying protocol is used, the end-to-end SINDR of the system is the minimum of the SINDRs of the links from the source to the relay and from the relay to the destination. Thus, the end-to-end SINDR of the DF relay system is given by

$$\gamma = \min \{\gamma_R, \gamma_D\}, \tag{4}$$

where γ_R and γ_D are the SINDRs at the relay and the destination, respectively.

2.1. Case 1: Without the Direct Link. In this case, the transmitted signal from the source S_1 does not reach the destination S_2 due to long distance. The destination receives only the relayed signal from R. Using (3) and (2), we can obtain the SINDRs at the relay R and the destination S_2 for Case 1, respectively, as

$$\gamma_R^{(1)} = \frac{|h_{1R}|^2 P_1}{|h_{1R}|^2 k_{S_1R}^2 P_1 + \tilde{\Omega}_R P_R + \sigma_R^2} = \frac{\rho_1 P_1}{\rho_1 k_{S_1R}^2 P_1 + \sigma_{RSI}^2 + \sigma_R^2}, \tag{5}$$

$$\gamma_D^{(1)} = \frac{|h_{R2}|^2 P_R}{|h_{R2}|^2 k_{RS_2}^2 P_R + \sigma_2^2} = \frac{\rho_2 P_R}{\rho_2 k_{RS_2}^2 P_R + \sigma_2^2}, \tag{6}$$

where $\rho_1 = |h_{1R}|^2$ and $\rho_2 = |h_{R2}|^2$ are the channel gains of the links from S_1 to R and from R to S_2, respectively; σ_{RSI}^2 denotes the RSI at the relay after SIC. It is noted that,

after SIC, the RSI can be modeled by a complex Gaussian distributed random variable [2–6, 9, 10, 32, 33] with zero mean and variance $\sigma_{RSI}^2 = \tilde{\Omega}_R P_R$, where $\tilde{\Omega}_R$ is defined as the SIC capability of the relay node.

2.2. Case 2: With the Direct Link but No Cooperative Communication. In this case, the destination receives the transmitted signal from the source via both the relaying and direct links. However, due to time misalignment, the signal via the direct link is considered interference. Therefore, the SINDRs at the relay and the destination are, respectively, given by

$$\gamma_R^{(2)} = \gamma_R^{(1)} = \frac{\rho_1 P_1}{\rho_1 k_{S_1R}^2 P_1 + \sigma_{RSI}^2 + \sigma_R^2}, \tag{7}$$

$$\begin{aligned} \gamma_D^{(2)} &= \frac{|h_{R2}|^2 P_R}{|h_{R2}|^2 k_{RS_2}^2 P_R + |h_{12}|^2 P_1\left(1 + k_{S_1R}^2\right) + \sigma_2^2} \\ &= \frac{\rho_2 P_R}{\rho_2 k_{RS_2}^2 P_R + \rho_3 P_1\left(1 + k_{S_1R}^2\right) + \sigma_2^2}, \end{aligned} \tag{8}$$

where $\rho_3 = |h_{12}|^2$ is the channel gain of the link from S_1 to S_2.

2.3. Case 3: With the Direct Link and Cooperative Communication. In this case, the signal from the source via the relaying link and that via the direct link are aligned (the system is thus delay limited) and combined in order to maximize the SINDR at the destination and remove the intersymbol interference (ISI) due to the processing delay at the relay node. For such purpose, we adopt the MMSE-DFE proposed in [17] at the destination. In order to estimate the transmitted symbol, the equalizer needs to be trained and operates in an adaptive fashion. This results in training delay and requires that the channel be slowly varying. Moreover, since the transmitted signals from the source and relay nodes also contain HI distortions $\eta_1(t)$ and $\eta_R(t)$, these components are considered additive noises by the MMSE-DFE. The SINDRs at the relay and the destination are then given, respectively, by

$$\gamma_R^{(3)} = \gamma_R^{(1)} = \frac{\rho_1 P_1}{\rho_1 k_{S_1R}^2 P_1 + \sigma_{RSI}^2 + \sigma_R^2}, \tag{9}$$

$$\gamma_D^{(3)} = \frac{|h_{R2}|^2 P_R + |h_{12}|^2 P_1}{|h_{R2}|^2 k_{S_2}^2 P_R + |h_{12}|^2 k_{S_1R}^2 P_1 + \sigma_2^2}$$
$$= \frac{\rho_2 P_R + \rho_3 P_1}{\rho_2 k_{RS_2}^2 P_R + \rho_3 k_{S_1R}^2 P_1 + \sigma_2^2}. \tag{10}$$

3. System Performance

3.1. Outage Probability. The outage probability is the probability of a failure in any communication link between S_1, R, and S_2. An outage event occurs when the mutual information is lower than the minimum required data rate $\mathcal{R}_j$, $(j = 1, 2)$, i.e.,

$$\log_2\left(1 + \gamma_j\right) < \mathcal{R}_j, \tag{11}$$

where γ_j and $\mathcal{R}_j$ are the SINDR and the minimum required data rate of link j, respectively. Let $\mathcal{R}_1$ be the minimum required data rate from S_1 to R and $\mathcal{R}_2$ be that from R to S_2. Also, for simplicity, let $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}$. As a result, an outage event occurs when

$$\log_2(1 + \gamma_R) < \mathcal{R}$$
$$\text{or } \log_2(1 + \gamma_D) < \mathcal{R}, \tag{12}$$

which is equivalent to

$$\gamma_R < 2^{\mathcal{R}} - 1$$
$$\text{or } \gamma_D < 2^{\mathcal{R}} - 1. \tag{13}$$

The outage probability is then defined as $\text{OP} = \text{Prob}(\gamma < 2^{\mathcal{R}} - 1)$. Let $x = 2^{\mathcal{R}} - 1$, and using the rule of addition $\text{Prob}\{\mathcal{A} \cup \mathcal{B}\} = \text{Prob}\{\mathcal{A}\} + \text{Prob}\{\mathcal{B}\} - \text{Prob}\{\mathcal{A}\}\text{Prob}\{\mathcal{B}\}$ for independent events $\mathcal{A}$ and $\mathcal{B}$, we can obtain the OP as follows:

$$\text{OP} = \text{Prob}\{(\gamma_R < x) \cup (\gamma_D < x)\} = \text{Prob}\{\min[\gamma_R, \gamma_D] < x\}$$
$$= \text{OP}_R + \text{OP}_D - \text{OP}_R\text{OP}_D, \tag{14}$$

where $\text{OP}_R = \text{Prob}\{\gamma_R < x\}$ and $\text{OP}_D = \text{Prob}\{\gamma_D < x\}$.

Under the Rayleigh fading channel, the cumulative distribution function (CDF) $F_\rho(.)$ and probability density function (PDF) $f_\rho(.)$ of the channel gains $(\rho_l = |h_l|^2, l = 1, 2, 3)$ are given by

$$F_{\rho_l}(x) = 1 - e^{-(x/\Omega_l)}, \quad x \geqslant 0,$$
$$f_{\rho_l}(x) = \frac{1}{\Omega_l} e^{-(x/\Omega_l)}, \quad x \geqslant 0, \tag{15}$$

where $\Omega_l = \mathbb{E}\{|h_l|^2\}$.

Under the effect of both Rayleigh fading and hardware impairments, using the above CDF and PDF of the channel gains, we can get the OPs of the system for each cooperation situation in the following parts.

3.1.1. Exact OP

(1) Case 1: Without the Direct Link. Denoting $\text{OP}_R^{(1)\text{-ex}} = \text{Prob}\{\gamma_R^{(1)} < x\}$ and $\text{OP}_D^{(1)\text{-ex}} = \text{Prob}\{\gamma_D^{(1)} < x\}$, we have

$$\text{OP}_R^{(1)\text{-ex}} = \text{Prob}\left\{\gamma_R^{(1)} < x\right\}$$
$$= \begin{cases} F_{\rho_1}\left(\frac{(\sigma_R^2 + \sigma_{RSI}^2)x}{P_1\left(1 - k_{S_1R}^2 x\right)}\right), & x < \frac{1}{k_{S_1R}^2}, \\ 1, & x \geqslant \frac{1}{k_{S_1R}^2}, \end{cases} \tag{16}$$
$$= \begin{cases} 1 - e^{-\alpha_1(x)}, & x < \frac{1}{k_{S_1R}^2} \\ 1, & x \geqslant \frac{1}{k_{S_1R}^2}. \end{cases}$$

Similarly,

$$\text{OP}_D^{(1)\text{-ex}} = \text{Prob}\left\{\gamma_D^{(1)} < x\right\} = \begin{cases} 1 - e^{-\alpha_2(x)}, & x < \frac{1}{k_{RS_2}^2}, \\ 1, & x \geqslant \frac{1}{k_{RS_2}^2}. \end{cases} \tag{17}$$

Using (14), the OP of the system becomes

$$\text{OP}^{(1)\text{-ex}} = \begin{cases} 1 - e^{-\alpha_1(x) - \alpha_2(x)}, & x < d, \\ 1, & x \geqslant d, \end{cases} \tag{18}$$

where $\alpha_1(x) = (\sigma_R^2 + \sigma_{RSI}^2)x/\Omega_1 P_1(1 - k_{S_1R}^2 x)$, $\alpha_2(x) = \sigma_2^2 x/\Omega_2 P_R(1 - k_{RS_2}^2 x)$, and $d = \min(1/k_{S_1R}^2, 1/k_{RS_2}^2)$.

Note that for the case of ideal hardware, the OP is given by

$$\text{OP}^{(1)\_\text{ex}\_\text{id}} = 1 - e^{-\alpha_1^{\text{id}}(x) - \alpha_2^{\text{id}}(x)}, \tag{19}$$

where $\alpha_1^{\text{id}}(x) = (\sigma_R^2 + \sigma_{\text{RSI}}^2)x/\Omega_1 P_1$ and $\alpha_2^{\text{id}}(x) = \sigma_2^2 x/\Omega_2 P_R$.

(2) Case 2: With the Direct Link but No Cooperative Communication. Since $\gamma_R^{(1)} = \gamma_R^{(2)} = \gamma_R^{(3)}$, we have

$$\text{OP}_R^{(1)\_\text{ex}} = \text{OP}_R^{(2)\_\text{ex}} = \text{OP}_R^{(3)\_\text{ex}}. \tag{20}$$

In order to calculate $\text{OP}_D^{(2)\_\text{ex}}$ using (8), we have

$$F_{\gamma_D^{(2)}}(x) = \text{Prob}\left\{\gamma_D^{(2)} < x\right\} = \text{Prob}\left\{\rho_2 P_R\left(1 - k_{\text{RS}_2}^2 x\right) \\ < \left[\rho_3 P_1\left(1 + k_{\text{S}_1 R}^2\right) + \sigma_2^2\right]x\right\}. \tag{21}$$

This is the joint CDF of two independent random variables ρ_2 and ρ_3. First, consider the case $1 - k_{\text{RS}_2}^2 x \leq 0$, which means $x \geq 1/k_{\text{RS}_2}^2$. In this case, the inequality $\rho_2 P_R(1 - k_{\text{RS}_2}^2 x) < [\rho_3 P_1(1 + k_{\text{S}_1 R}^2) + \sigma_2^2]x$ is always true due to the fact

that $\rho_2 P_R(1 - k_{\text{RS}_2}^2 x) \leq 0$ and $(\rho_3 P_1(1 + k_{\text{S}_1 R}^2) + \sigma_2^2)x > 0$. Therefore, $F_{\gamma_D^{(2)}}(x) = 1$. Next, we consider the case $1 - k_{\text{RS}_2}^2 x > 0$, which means $\rho_2 P_R(1 - k_{\text{RS}_2}^2 x) > 0$. Since ρ_2 and ρ_3 are the instantaneous channel gains, we have $\rho_2, \rho_3 \in (0, \infty)$. To derive the CDF in this case, we change the above probability to a new equivalent one which has only one random variable. Using the property of a function of two random variables in Section 5.8 of [34], we can convert the joint CDF of these two independent variables into a function with double integral in which one variable is considered a constant while calculating the other. Then, by applying the result in Section 5.7 of [34], we can obtain

$$F_{\gamma_D^{(2)}}(x) = \int_0^\infty \text{Prob}\left\{\rho_2 < \frac{\left[\rho_3 P_1\left(1 + k_{\text{S}_1 R}^2\right) + \sigma_2^2\right]x}{P_R\left(1 - k_{\text{RS}_2}^2 x\right)} \mid \rho_3\right\} f_{\rho_3}(\rho_3) d\rho_3 \\ = \int_0^\infty F_{\rho_2}\left\{\frac{\left[\rho_3 P_1\left(1 + k_{\text{S}_1 R}^2\right) + \sigma_2^2\right]x}{P_R\left(1 - k_{\text{RS}_2}^2 x\right)}\right\} f_{\rho_3}(\rho_3) d\rho_3. \tag{22}$$

In summary, we have $\text{OP}_D^{(2)\_\text{ex}}$ given by

$$\text{OP}_D^{(2)\_\text{ex}} = \begin{cases} \int_0^\infty F_{\rho_2}\left(\frac{\left[\sigma_2^2 + \rho_3 P_1\left(1 + k_{\text{S}_1 R}^2\right)\right]x}{P_R\left(1 - k_{\text{RS}_2}^2 x\right)}\right) f_{\rho_3}(\rho_3) d\rho_3, & x < \frac{1}{k_{\text{RS}_2}^2} \\ 1, & x \geq \frac{1}{k_{\text{RS}_2}^2} \end{cases} = \begin{cases} 1 - A_2(x)e^{-\alpha_2(x)}, & x < \frac{1}{k_{\text{RS}_2}^2}, \\ 1, & x \geq \frac{1}{k_{\text{RS}_2}^2}, \end{cases} \tag{23}$$

where $A_2(x) = \Omega_2 P_R(1 - k_{\text{RS}_2}^2 x)/(\Omega_2 P_R(1 - k_{\text{RS}_2}^2 x) + \Omega_3 P_1(1 + k_{\text{S}_1 R}^2)x)$.

Thus, OP of the system for this case is then given by

$$\text{OP}^{(2)\_\text{ex}} = \begin{cases} 1 - A_2(x)e^{-\alpha_1(x) - \alpha_2(x)}, & x < d, \\ 1, & x \geq d. \end{cases} \tag{24}$$

In the case of ideal hardware, the system OP is given by

$$\text{OP}^{(2)\_\text{ex}\_\text{id}}(x) = 1 - A_2^{\text{id}}(x)e^{-\alpha_1^{\text{id}}(x) - \alpha_2^{\text{id}}(x)}, \tag{25}$$

where $A_2^{\text{id}}(x) = \Omega_2 P_R/(\Omega_2 P_R + \Omega_3 P_1 x)$.

(3) Case 3: With the Direct Link and Cooperative Communication. In this case, using the same derivation for $\text{OP}_D^{(2)\_\text{ex}}$, we can have $\text{OP}_D^{(3)\_\text{ex}}$ as follows:

$$\text{OP}_D^{(3)\_\text{ex}} = \text{Prob}\left\{\gamma_D^{(3)} < x\right\} \\ = \text{Prob}\left\{\rho_2 P_R\left(1 - k_{\text{RS}_2}^2 x\right) + \rho_3 P_1\left(1 - k_{\text{S}_1 R}^2 x\right) < \sigma_2^2 x\right\} \\ = \begin{cases} 1 - A_3(x)e^{-\alpha_2(x)} - B_3(x)e^{-\alpha_3(x)}, & x < d, G(x) \neq 0, \\ 1 - (1 + \alpha_2(x))e^{-\alpha_2(x)}, & x < d, G(x) = 0, \\ 1 - A_3(x)e^{-\alpha_2(x)}, & \frac{1}{k_{\text{S}_1 R}^2} \leq x < \frac{1}{k_{\text{RS}_2}^2}, \\ 1 - B_3(x)e^{-\alpha_3(x)}, & \frac{1}{k_{\text{RS}_2}^2} \leq x < \frac{1}{k_{\text{S}_1 R}^2}, \\ 1, & x \geq d', \end{cases} \tag{26}$$

where $A_3(x) = \Omega_2 P_R(1 - k_{\text{RS}_2}^2 x)/(\Omega_2 P_R(1 - k_{\text{RS}_2}^2 x) - \Omega_3 P_1(1 - k_{\text{S}_1 R}^2 x))$, $B_3(x) = \Omega_3 P_1(1 - k_{\text{S}_1 R}^2 x)/(\Omega_3 P_1(1 - k_{\text{S}_1 R}^2 x) - \Omega_2 P_R(1 - k_{\text{RS}_2}^2 x))$, $d' = \max(1/k_{\text{S}_1 R}^2, 1/k_{\text{RS}_2}^2)$, $\alpha_3(x) = \sigma_2^2 x/\Omega_3 P_1(1 - k_{\text{S}_1 R}^2 x)$, and $G(x) = \Omega_3 P_1(1 - k_{\text{S}_1 R}^2 x) - \Omega_2 P_R(1 - k_{\text{RS}_2}^2 x)$.

Combining $\mathrm{OP}_D^{(3)\_ex}$ in (26) with $\mathrm{OP}_R^{(3)\_ex}$ in (20), we have the OP for the case of cooperative communication $\mathrm{OP}^{(3)\_ex}$ as follows:

$$\mathrm{OP}^{(3)\_ex} = \begin{cases} 1 - A_3(x)e^{-\alpha_1(x)-\alpha_2(x)} - B_3(x)e^{-\alpha_1(x)-\alpha_3(x)}, & x < d, G(x) \neq 0, \\ 1 - (1 + \alpha_2(x))e^{-\alpha_1(x)-\alpha_2(x)}, & x < d, G(x) = 0, \\ 1 - B_3(x)e^{-\alpha_1(x)-\alpha_3(x)}, & \dfrac{1}{k_{RS_2}^2} \leqslant x < \dfrac{1}{k_{S_1R}^2}, \\ 1, & x \geqslant \dfrac{1}{k_{S_1R}^2}. \end{cases} \tag{27}$$

In the case of ideal hardware, we have

$$\mathrm{OP}^{(3)\_ex\_id} = \begin{cases} 1 - A_3^{id}e^{-\alpha_1^{id}(x)-\alpha_2^{id}(x)} - B_3^{id}e^{-\alpha_1^{id}(x)-\alpha_3^{id}(x)}, & G \neq 0, \\ 1 - \left(1 + \alpha_2^{id}(x)\right)e^{-\alpha_1^{id}(x)-\alpha_2^{id}(x)}, & G = 0, \end{cases} \tag{28}$$

where $A_3^{id} = \Omega_2 P_R/(\Omega_2 P_R - \Omega_3 P_1)$, $B_3^{id} = \Omega_3 P_1/(\Omega_3 P_1 - \Omega_2 P_R)$, $\alpha_3^{id}(x) = \sigma_2^2 x/\Omega_3 P_1$, and $G = \Omega_3 P_1 - \Omega_2 P_R$.

3.1.2. Asymptotic OP. In order to gain a better insight into the effect of hardware impairments on the system performance of the IBFD relay network, we derive the asymptotic OP using the assumption that the transmit power is extremely large. Using the Taylor series expansion, $e^x \approx 1 + x$ when $x \longrightarrow 0$ and let the transmit power increase to infinity, i.e., $P_j \longrightarrow \infty, j = 1, R$, and we assume that $P_1 = P_R$. Since $\lim_{P_j \to \infty} \alpha_i(x) = 0, i = 1, 2, 3$, we have

$$\lim_{P_j \to \infty} \mathrm{OP}^{(1)\_ex} = \begin{cases} 0, & x < d, \\ 1, & x \geqslant d, \end{cases} \tag{29}$$

$$\lim_{P_j \to \infty} \mathrm{OP}^{(2)\_ex} = \begin{cases} 1 - \dfrac{\Omega_2\left(1 - k_{RS_2}^2 x\right)}{\Omega_2\left(1 - k_{RS_2}^2 x\right) + \Omega_3\left(1 + k_{S_1R}^2\right)x}, & x < d, \\ 1, & x \geqslant d, \end{cases} \tag{30}$$

$$\lim_{P_j \to \infty} \mathrm{OP}^{(3)\_ex} = \begin{cases} 1 - A_3^\infty(x) - B_3^\infty(x), & x < d, G(x) \neq 0, \\ 0, & x < d, G(x) = 0, \\ 1 - B_3^\infty(x), & \dfrac{1}{k_{RS_2}^2} \leqslant x < \dfrac{1}{k_{S_1R}^2}, \\ 1, & x \geqslant \dfrac{1}{k_{S_1R}^2}, \end{cases} \tag{31}$$

where $A_3^\infty(x) = \Omega_2(1 - k_R^2 x)/(\Omega_2(1 - k_{RS_2}^2 x) - \Omega_3(1 - k_{S_1R}^2 x))$ and $B_3^\infty(x) = \Omega_3(1 - k_{S_1R}^2 x)/\Omega_3(1 - k_{S_1R}^2 x) - \Omega_2(1 - k_{RS_2}^2 x)$.

On the other hand, we have

$$\mathrm{OP}^{(1)\_ap} \approx \begin{cases} \alpha_1(x) + \alpha_2(x), & x < d, \\ 1, & x \geqslant d, \end{cases} \tag{32}$$

$$\mathrm{OP}^{(2)\_ap} \approx \begin{cases} 1 - A_2(x)[1 - \alpha_1(x) - \alpha_2(x)], & x < d, \\ 1, & x \geqslant d, \end{cases} \tag{33}$$

$$\mathrm{OP}^{(3)\_ap} \approx \begin{cases} \alpha_1(x) + A_3(x)\alpha_2(x) + B_3(x)\alpha_3(x), & x < d, G(x) \neq 0, \\ 1 - (1 + \alpha_2(x))[1 - \alpha_1(x) - \alpha_2(x)], & x < d, G(x) = 0, \\ 1 - B_3(x)[1 - \alpha_1(x) - \alpha_3(x)], & \dfrac{1}{k_{RS_2}^2} \leqslant x < \dfrac{1}{k_{S_1R}^2}, \\ 1, & x \geqslant \dfrac{1}{k_{S_1R}^2}. \end{cases} \tag{34}$$

From (29), (30), and (31), we can see clearly that hardware impairments cause the OP to have an irreducible error floor. Thus, careful design must be taken during the process of manufacturing transceivers in order to avoid this outage floor.

3.2. System Throughput. The throughput of the IBFD DF relay system can be determined from the OP as follows:

$$\mathscr{T}(x) = \mathscr{R}(1 - \mathrm{OP}), \tag{35}$$

where $\mathscr{R}$ is the given transmission rate (bit/s/Hz) and OP is the outage probability of the system, which is given in (18), (24), and (27) for the three cases of cooperation.

3.3. Symbol Error Probability. For most common modulation schemes, the SEP of the system is given by [35]

$$\begin{aligned} \mathrm{SEP}_\ell &= \alpha\mathbb{E}\left\{Q\left(\sqrt{\beta\gamma_\ell}\right)\right\} \\ &= \dfrac{\alpha}{\sqrt{2\pi}}\int_0^\infty F_\ell\left(\dfrac{t^2}{\beta}\right)e^{-(t^2/2)}dt, \quad \ell = 1, 2, 3, \end{aligned} \tag{36}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$ is the Gaussian function, γ_ℓ is the received SINDR obtained from (5), (6), (7), (8), (9), and (10), and α and β are decided by the employed modulation schemes, e.g., $\alpha = \beta = 1$ for the orthogonal binary frequency-shift keying (BFSK) modulation and $\alpha = 1, \beta = 2$ for the binary phase-shift keying (BPSK) modulation [35]. $F_\ell(x)$ are the cumulative distribution functions (CDFs) of SINDR. Replace $x = t^2/\beta$, and the SEP_ℓ for each cooperation situation is calculated as

$$\mathrm{SEP}_\ell = \dfrac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}}\int_0^\infty \dfrac{e^{-\beta x/2}}{\sqrt{x}}F_\ell(x)dx. \tag{37}$$

Using the definition of outage probability, we can replace CDFs $F_\ell(x)$ by OPs given by (18), (24), and (27). Since O $P^{(\ell)\text{–ex}}$ is complicated, it is not possible to obtain the closed form or approximation for (37) even using Mathematica software. However, it is worth noting that since SEP_ℓ does not contain any random variables, it can certainly be calculated numerically using software packages such as MATLAB.

4. Optimal Power Calculation Method for the FD Mode

Since the system performance depends much on the RSI, finding the optimal power for the transmit power at the FD relay node is important. In addition to the reduction of RSI, the optimal power also helps to save power for the relay node. Therefore, in this section, we propose an optimal power calculation method for the FD mode to select the transmit power P_R according to the SIC capability ($\tilde{\Omega}_R$), the transmit power of the source node (P_1), the hardware impairments (k_{S_1R} and k_{RS_2}), the distance between the nodes ($\Omega_1, \Omega_2, \Omega_3$), the threshold ($x$), and the AWGN at the destination node (σ_2^2). The optimal transmit power that is optimized with respect to minimum OP of the system is derived in the following sections.

4.1. Case 1: Without Direct Link. The optimization problem for the relay transmit power of the FD mode can be formulated as follows:

$$\begin{aligned} &\min \ \text{OP}^{(1)\text{–ex}} \\ &\text{s.t.} P_R > 0 \text{ and } x < d. \end{aligned} \tag{38}$$

In order to solve this problem, we can use a linear programming method to get the closed-form expression of the relay optimal power, denoted by P_R^*, as follows:

$$P_R^* = \arg \ \min_{P_R} \text{OP}^{(1)\text{–ex}}, \tag{39}$$

where $\text{OP}^{(1)\text{–ex}}$ is determined using (18). The power calculation method for Case 1 is summarized in Algorithm 1.

Note that when the system has no optimal power, i.e., $P_R^* = \varnothing$, choose P_R based on the resulting expression of $\partial \text{O} P^{(1)\text{–ex}}/\partial P_R$; herein, $\varnothing$ denotes the empty set.

Taking the derivative of $\text{OP}^{(1)\text{–ex}}$ with respect to P_R gives us

$$\begin{aligned} \frac{\partial \text{OP}^{(1)\text{–ex}}}{\partial P_R} = &- \exp\left(-\frac{\left(\sigma_R^2 + \tilde{\Omega}_R P_R\right)x}{\Omega_1 P_1\left(1 - k_{S_1R}^2 x\right)} - \frac{\sigma_2^2 x}{\Omega_2 P_R\left(1 - k_{RS_2}^2 x\right)}\right) \\ &\times \left[-\frac{\tilde{\Omega}_R x}{\Omega_1 P_1\left(1 - k_{S_1R}^2 x\right)} + \frac{\sigma_2^2 x}{\Omega_2 P_R^2\left(1 - k_{RS_2}^2 x\right)}\right]. \end{aligned} \tag{40}$$

Optimal power calculation algorithm
1: Solve $\partial \text{OP}^{(1)\text{–ex}}/\partial P_R = 0$ for $P_R = P_0$;
2: if $\begin{cases} P_0 > 0 \\ \partial \text{OP}^{(1)\text{–ex}}/\partial P_R < 0 \text{ for } P_R < P_0 \\ \partial \text{OP}^{(1)\text{–ex}}/\partial P_R > 0 \text{ for } P_R > P_0 \end{cases}$
3: then
4: Output optimal power $P_R^* = P_0$;
5: else
6: Output optimal power $P_R^* = \varnothing$;
7: end

Algorithm 1:

From (40), it can be easily shown that $\partial \text{OP}^{(1)\text{–ex}}/\partial P_R = 0$ when $P_R = P_0$, $\partial \text{OP}^{(1)\text{–ex}}/\partial P_R < 0$ when $P_R < P_0$, and $\partial \text{O} P^{(1)\text{–ex}}/\partial P_R > 0$ when $P_R > P_0$. Therefore, P_R^* is the optimal transmit power of the relay node, which is given by

$$P_R^* = P_0 = \sqrt{\frac{\Omega_1 P_1 \sigma_2^2\left(1 - k_{S_1R}^2 x\right)}{\Omega_2 \tilde{\Omega}_R\left(1 - k_{RS_2}^2 x\right)}}. \tag{41}$$

4.2. Case 2: With the Direct Link but No Cooperative Communication. Similar to Case 1, we can create a similar algorithm as Algorithm 1 by replacing $\text{OP}^{(1)\text{–ex}}$ by $\text{OP}^{(2)\text{–ex}}$. The calculation method for the optimal power for Case 2 can be summarized as follows. Solving $\partial \text{OP}^{(2)\text{–ex}}/\partial P_R = 0$ (for the case $x < d$) results in the following equation:

$$A P_R^3 + B P_R^2 + C P_R + D = 0, \tag{42}$$

and the optimal power P_R^* is given by

$$P_R^* = \frac{2\sqrt{\Delta}\cos\left(\arccos\left(K\right)/3\right) - B}{3A}, \tag{43}$$

where $A = \Omega_2^2(1 - k_{RS_2}^2 x)^2 \tilde{\Omega}_R x$, $B = \Omega_2(1 - k_{RS_2}^2 x)\Omega_3 P_1(1 + k_{S_1R}^2)\tilde{\Omega}_R x^2$, $C = -\Omega_1 P_1(1 - k_{S_1R}^2 x)\Omega_2(1 - k_{RS_2}^2 x)[\Omega_3 P_1(1 + k_{S_1R}^2)x + \sigma_2^2 x]$, $D = -\Omega_1 P_1(1 - k_{S_1R}^2 x)\Omega_3 P_1(1 + k_{S_1R}^2)\sigma_2^2 x^2$, $\Delta = B^2 - 3AC$, and

$$K = \frac{9ABC - 2B^3 - 27A^2 D}{2\sqrt{\Delta^3}}. \tag{44}$$

4.3. Case 3: With the Direct Link and Cooperative Communication. Similar to the above method, in this case, for the first case ($x < d$, $G(x) \neq 0$) of equation (27), due to the
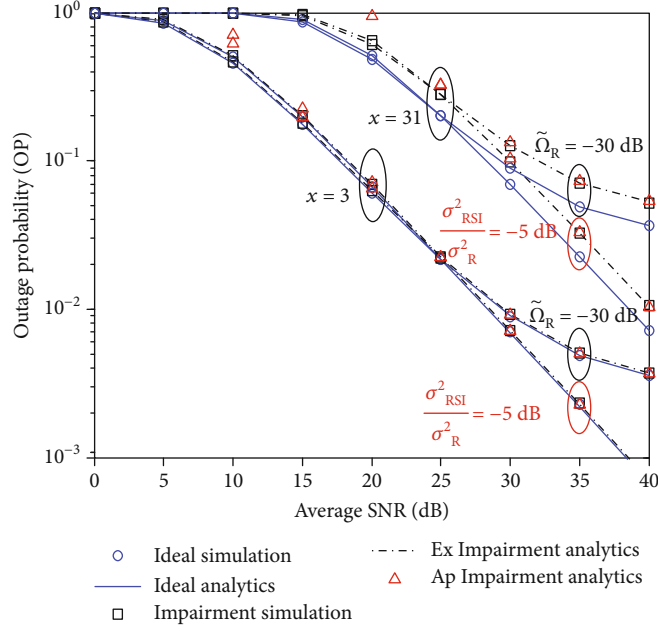
FIGURE 2: OP of the system for the case without the direct link. $\Omega_1 = \Omega_2 = 1$, $\sigma_R^2 = \sigma_2^2 = 1$, and $k_{S_1R} = k_{RS_2} = 0.1$. Fixed and varied RSI as $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB and $\tilde{\Omega}_R = -30$ dB, respectively. $\mathcal{R} = 2$ and 5 bits/s/Hz.
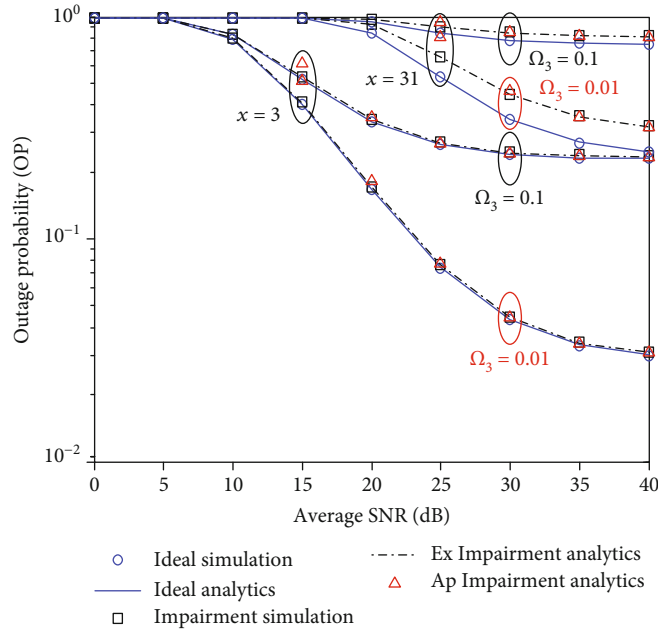

FIGURE 3: OP of the system for the case with the direct link but without cooperation. $\Omega_1 = \Omega_2 = 1$ and $\sigma_R^2 = \sigma_2^2 = 1$. $k_{S_1R} = k_{RS_2} = 0.1$ and $\sigma_{RSI}^2/\sigma_R^2 = 5$ dB. Two power levels of interference: $\Omega_3 = 0.01$ and $\Omega_3 = 0.1$. $\mathcal{R} = 2$ and 5 bits/s/Hz.

complexity of the OP, it is not possible to find the critical points. However, we can find the suboptimal points using some mathematical manipulations. In this case, the OP can be rewritten as follows:

$$\text{OP}^{(3)\_\text{ex}} = 1 - \frac{e^{-\alpha_1(x)-\alpha_2(x)}}{a-b}\left(a - be^{\alpha_2(x)-\alpha_3(x)}\right), \quad (45)$$

where $a = \Omega_2 P_R(1 - k_{RS_2}^2 x)$ and $b = \Omega_3 P_1(1 - k_{S_1R}^2 x)$. For $a > b$, we have $\alpha_2(x) < \alpha_3(x)$; thus, $e^{\alpha_2(x)-\alpha_3(x)} < 1$. Therefore, $a - be^{\alpha_2(x)-\alpha_3(x)} > 0$. So, the suboptimal point is the one that maximizes $e^{-\alpha_1(x)-\alpha_2(x)}$. From here, we get the suboptimal power P_R^* as given by (41). For $a < b$, using the similar method, we can also get the suboptimal power P_R^* as in (41). As a result, for the first case of (27), P_R^* is
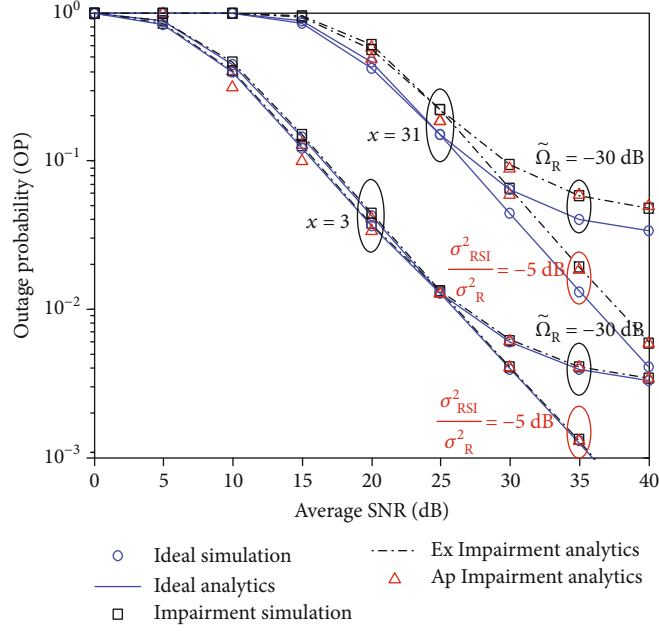
FIGURE 4: OP for the case with the direct link and using cooperation. $\Omega_1 = \Omega_2 = 1$ and $\sigma_R^2 = \sigma_2^2 = 1$. $k_{S_1R} = k_{RS_2} = 0.1$ and $\Omega_3 = 0.1$. Fixed and varied RSI as $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB and $\tilde{\Omega}_R = -30$ dB, respectively. $\mathcal{R} = 2$ and 5 bits/s/Hz.

determined by (41). For the second case $(x < d, G(x) = 0)$, there is only one P_R that makes $G(x) = 0$. For the third case $(1/k_{RS_2}^2 \leqslant x < 1/k_{S_1R}^2)$, the OP of the system is minimized when P_R approaches zero.

5. Performance Evaluation and Discussions

In this section, we present performance evaluation of the system using numerical calculation. Monte Carlo simulations using generated random channels are also used to verify our theoretical analysis. In our simulations, we set $\Omega_1 = \Omega_2 = 1$ and $\sigma_R^2 = \sigma_2^2 = \sigma^2 = 1$. The average SNR is defined as $\text{SNR} = P_1/\sigma^2$. In the case of no optimization, the transmit power at the relay is set as $P_R = P_1$. In the case of applying optimal transmit power, $P_R = P_1$ is computed as shown in Section 4. In order to obtain the OP, two thresholds are used for the OP, which are $x = 2^2 - 1 = 3$ and $x = 2^5 - 1 = 31$ with the minimum required data rates of 2 bits/s/Hz and 5 bits/s/Hz, respectively. To verify our theoretical analysis, simulated results are also plotted together with numerical ones for comparison.

Figure 2 shows the OP for the case without the direct link from the source to the destination, i.e., Case 1. The plotted curves were obtained by numerical calculations using equations (18), (19), and (32), while the marker symbols by computer simulations with $k_{S_1R} = k_{RS_2} = 0.1$, $\Omega_1 = \Omega_2 = 1$, and $\sigma_R^2 = \sigma_2^2 = 1$. The RSI was considered for two cases: (i) fixed at $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB and (ii) varying with transmit power of the relay node ($\sigma_{RSI}^2 = \tilde{\Omega}_R P_R$ with $\tilde{\Omega}_R = -30$ dB). Note that in the first case, the level of RSI is intentionally set smaller than the noise variance to bypass the RSI effect and explore the system performance under the
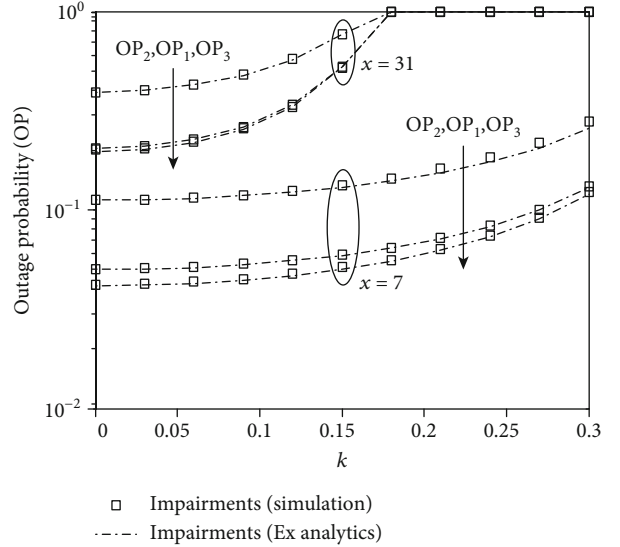

FIGURE 5: OP for the three cases versus the distortion factor k. SNR $= 25$ dB, $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB, $x = 31$ and 7, and $\Omega_3 = 0.01$.

impact of only HI. As can be seen from the figure when the threshold is low ($x = 3$), the performance loss due to hardware impairments is very small and can be neglected. However, when the threshold becomes larger ($x = 31$), this loss becomes significant and cannot be overlooked. This means that the effect of hardware impairments on the system quality is more significant in a high-rate system than in a low-rate one. Therefore, employed devices in the high-rate systems should be fabricated with high precision in order to avoid unnecessary performance loss. It is also noticed from the figure that when the RSI varies with the transmit power
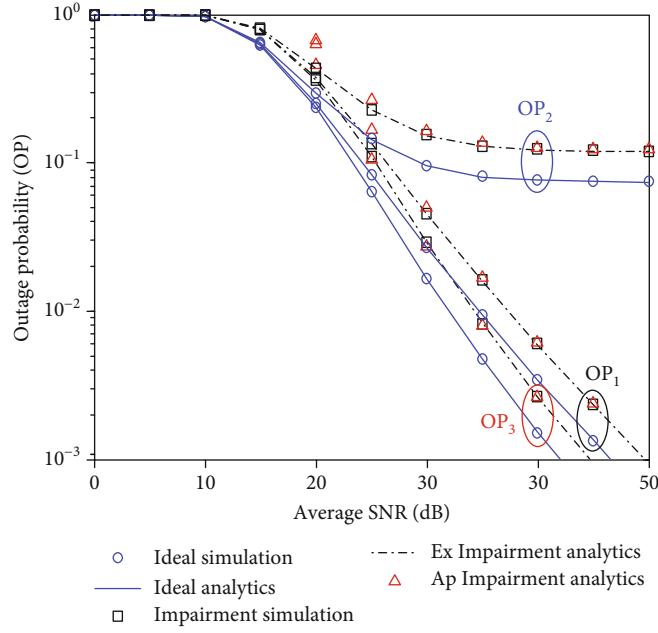
FIGURE 6: OP performance for the case with the optimal transmit power. $k_{S_1R} = k_{RS_2} = 0.1$, $x = 31$, $\tilde{\Omega}_R = -30$ dB, and $\Omega_3 = 0.01$ for OP_2 and $\Omega_3 = 0.1$ for OP_3.

of the relay node, the OP performance is significantly affected especially at the high SNR region.

Figure 3 shows the achieved OP for the case with the direct link but without cooperation (Case 2) using analytical equations (24), (25), and (33) with $\sigma_{RSI}^2/\sigma_R^2 = 5$ dB and the distortion factors $k_{S_1R} = k_{RS_2} = 0.1$ and $\Omega_1 = \Omega_2 = 1$. Since cooperation was not used, the signal from the source node to the destination node was regarded as interference. Two specific power levels of interference, namely, $\Omega_3 = 0.01$ and $\Omega_3 = 0.1$, were used for investigation. Compared with the previous case, the outage performance of the system is seriously degraded, especially when the interference level is high. Note that for this level of interference, the OP performance will not increase when the RSI decreases, i.e., $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB. Moreover, the system suffers from an irreducible outage floor at the high SNR region. When it is possible, cooperation can be used with an appropriate combining scheme such as Case 3 to alleviate the effect of this interference and improve the system performance.

Figure 4 shows the OP for the case using cooperation (Case 3) with $\Omega_3 = 0.1$. Other parameters for numerical calculation are the same as in Figure 2. We can see from the figure that although cooperation via the direct link is used, the ideal system achieves only diversity order 1 as in the case without cooperation in Figure 2 since DF was used for relaying. However, the system with HI benefits significantly from the cooperation which allows for performance improvement. Compared with those shown in Figures 2 and 3, the system achieves the best performance thanks to cooperation.

Figure 5 illustrates the impact of the distortion factor k on the outage performance of the system for the above-mentioned three cases, i.e., without the direct link (OP_1),
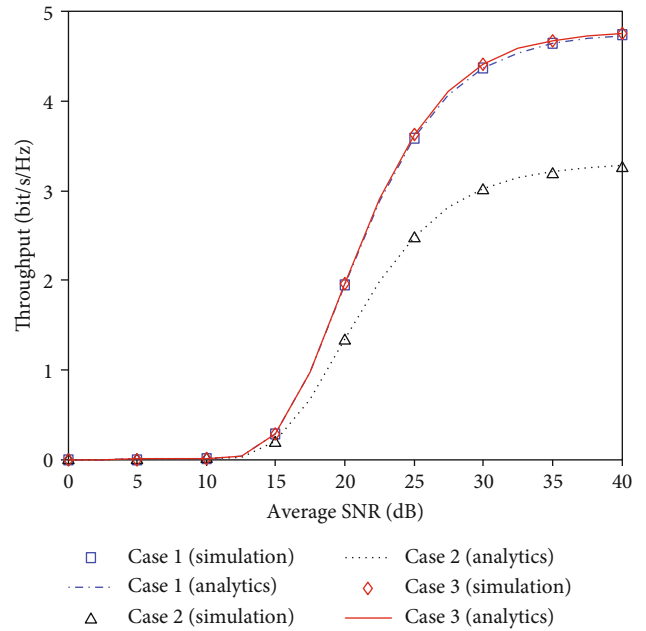

FIGURE 7: System throughput for the three cases using the optimal power values. $\Omega_1 = \Omega_2 = 1$, $\sigma_R^2 = \sigma_2^2 = 1$. $k_{S_1R} = k_{RS_2} = 0.1$, $\Omega_3 = 0.01$, and $\tilde{\Omega}_R = -30$ dB.

with the direct link but without cooperation (OP_2), and with the direct link and with cooperation (OP_3). The average SNR at the relay node and the destination node were both set to 25 dB, $\sigma_{RSI}^2/\sigma_R^2 = -5$ dB, and $\Omega_3 = 0.01$. Two values of x were used for investigation, namely, $x = 31$ and $x = 7$. As can be seen from the figure, the impact of hardware impairments for all the three cases is trivial for small distortion factor x
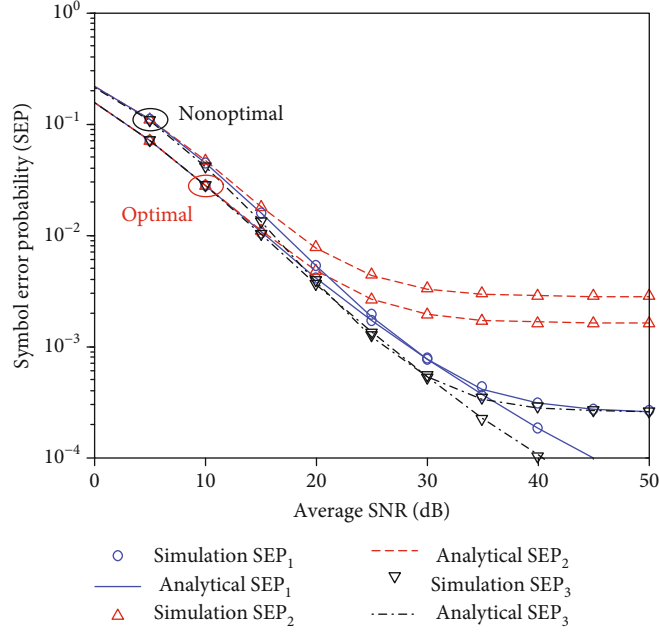
FIGURE 8: Effect of hardware impairment on the SEP performance with and without the optimal power values. $k_{S_1R} = k_{RS_2} = 0.1$. $\Omega_3 = 0.01$ and $\tilde{\Omega}_R = -30$ dB.

but becomes more significant as it increases. The system with a higher transmission rate is more influenced by hardware impairments than that with a lower one. For $x = 31$, the system soon experiences link outage with probability one at $k = 0.17$. The system with a lower rate ($x = 7$), however, can resist OP less than 0.1 within the acceptable hardware impairment range. In order to reduce OP, the optimal power needs to be used to reduce the impact of the RSI. On the other hand, reducing transmit power can also help to decrease the distortion factor k because hardware impairments depend on the amplifier operation.

Figure 6 shows the OP versus the average SNR at the relay node when optimal power values are used. In this figure, the transmit power at the FD node was set using equations (41) for OP_1 and OP_3 and (43) for OP_2. The system was set up with $x = 31$ and $\tilde{\Omega}_R = -30$ dB while the remaining parameters are used as shown in Figure 2. The OP_2 curve was obtained with $\Omega_3 = 0.01$ and the OP_3 curve with $\Omega_3 = 0.1$. Comparing Figures 2–4 for the case in which the RSI is varied by the transmit power of the FD node, we can see clearly that the optimal power values can increase the outage performance of the system with hardware impairments significantly. Although the outage floor still exists for Case 2 at SNR > 30 dB, it is completely avoided for Cases 1 and 3.

Figure 7 plots the throughput of the systems as a function of the average SNR at the relay node for the three cases when $\mathcal{R} = 5$ bits/s/Hz, $\Omega_3 = 0.01$, and $\tilde{\Omega}_R = -30$ dB. As can be seen in the figure, for high-rate systems, the throughput reaches the target rate $\mathcal{R}$ at SNR > 40 dB for Cases 1 and 3. For Case 2, due to the effect of interference, the throughput cannot reach the target rate even when using self-interference cancellation.

The SEP of the system using BPSK modulation and the same assumptions used in Figure 2 is analyzed as shown in Figure 8. Other parameters were as follows: $\Omega_3 = 0.01$ and $\tilde{\Omega}_R = -30$ dB. The analytical curves were obtained using equation (37) by using MATLAB. Denoted by SEP_1, SEP_2, and SEP_3 are the SEPs of the system for Cases 1, 2, and 3, respectively. The SEPs are shown as a function of the average SNR at the relay and the destination node with and without the optimal power. It is clear that the derived optimal power values help to increase the SEP performance of the system, especially for Cases 1 and 3.

6. Conclusion

In this paper, we analyzed the system performance of the IBFD DF relay systems with hardware impairments and imperfect self-interference cancellation. We derived exact and approximate expressions of OP, as well as its asymptotic values at high signal power. In order to alleviate the effect of RSI, we have proposed an optimal power calculation method based on the minimum OP for the case without the direct link and with the direct link but without cooperation. For the case of a direct link with cooperation, the proposed scheme still works but yields suboptimal solutions. Various performance evaluations have been conducted to investigate the effect of hardware impairments and the RSI on the system behavior. The evaluated results show that the high-rate system is more vulnerable to the hardware impairments than the low-rate one. However, using the proposed optimal power value can reduce the effect of hardware impairments and imperfect SIC. Moreover, for the IBFD DF relay networks, it is recommended that cooperative communication

should be used for the case with the direct link in order to avoid the irreducible outage floor. Also, although our analysis and optimal power calculation can be extended to the case of multiple-relay networks, the impact of superposed hardware impairments and RSI together with the disappearance of the direct link makes it more difficult for these networks to cope with the early performance saturation issue. Thus, more effective RSI cancellation solutions and high-quality hardware devices should be employed when considering the IBFD communication for the multiple-relay networks.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Disclosure

The work by Ba Cao Nguyen, Xuan Nam Tran, and Thi Thu Hang Nguyen was performed as part of the employment in Le Quy Don Technical University, and that by Ba Cao Nguyen and Dinh Tan Tran as part of the employment in Telecommunications University.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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