Modeling and synthesizing quasi-time control laws for two degrees of freedom robotic arm

Cite as: AIP Conference Proceedings **2188**, 030009 (2019); https://doi.org/10.1063/1.5138402
Published Online: 17 December 2019

C. X. Nguyen, H. N. Phan, L. D. Hoang, H. T. Nguyen, KH. D. Truong, and N. H. V. Nguyen

Modeling and Synthesizing Quasi-Time Control Laws for Two Degrees of Freedom Robotic Arm

C.X. Nguyen[1, a)], H.N. Phan[2, b)], L.D. Hoang[1], H.T. Nguyen[1], KH.D. Truong[1], N.H.V. Nguyen[1]

[1]*Department of Automation and Computing Techniques, Le Quy Don Technical University, Hanoi, Vietnam*
[2]*Department of Software Engineering, Le Quy Don Technical University, Hanoi, Vietnam*

[a)]Corresponding author: nguyenxuanchiem83@gmail.com
[b)]phanhai@mail.ru

Abstract: In the paper, the model and method of synthesizing quasi-time optimal control laws for Two Degree of Freedom (2-DOF) Robotic Arms are presented. 2-DOF Robotic Arms is a non-linear MIMO System with large nonlinear characteristics and uncertain parameters. In this design method, it is necessary to convert the MIMO system to the SISO system, in which each SISO system is guaranteed in Jordan form, followed by using the conversion system in the form of quasi-time optimal equation. By this method, the control law ensures a stable global Robotic Arm. In addition, the control law also ensures that the Robotic Arm System has a quick response of time, but also ensures a stable and robust system when changing parameters and impact noise. The simulation results on the 2-DOF Robotic Arm show the effectiveness of the proposed method.

INTRODUCTION

Trajectory Tracking Control of the Robotic Arm is a very interesting field due to its complex dynamic model and its changing model parameters. The dynamic analysis of the robot model is to study the relationship of torque at the joints made up of the actuators and the position of the robotic arm. Non-linear dynamics and joint relationships make it difficult to control accurately and sustainably. Therefore, designing a controller with traditional control methods depends on the dynamics of the robot model is difficult. Many methods of synthesizing control laws have been implemented for this object. Control methods according to errors based on traditional control rules such as PID are presented in [4], [5], [18], [19]. The downside of these controllers is that it is difficult to find controller parameters and poor control quality when the input varies in large ranges. In [6-8], the authors use LQR and MPC controllers but are based on linear models, which lead to a rapid reduction in control quality when the system goes away from the working point. Sliding Mode Control is presented in [9-10], but current characting at high frequency leads to breaking or damaging the physical structure. In [11, 12] presented techniques to reduce or eliminate this phenomenon, but the quality of control still has limitations. The method of using Fuzzy Logic and Neural Network is presented in [13-15]. The construction of Fuzzy Model, Neural Network Model, and implementation time need to be solved by this method. In addition, Evolutionary Algorithms (EA) have appeared as a method of designing or optimizing the parameters of traditional controllers for 2-DOF Robotic Arms presented in [16- 17].

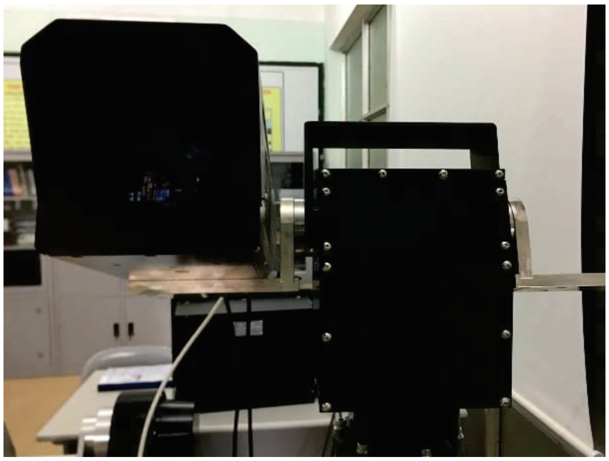
This study focuses on modeling Robotic Arm and synthesizing them to the quasi-time optimal controller for the MIMO system. Two controllers are synthesized nonlinear state feedback of SISO system based on Jordan formula. The SISO system is based on the principle of MIMO model separation to ensure that the system of subsystems form Jordan. The Controllers are synthesized effective rapid impact without relying on linearized equations and ensure a stable MIMO system is the contribution of the project.

XV International Scientific-Technical Conference "Dynamic of Technical Systems" (DTS-2019)
AIP Conf. Proc. 2188, 030009-1–030009-7; https://doi.org/10.1063/1.5138402
Published by AIP Publishing. 978-0-7354-1935-3/$30.00

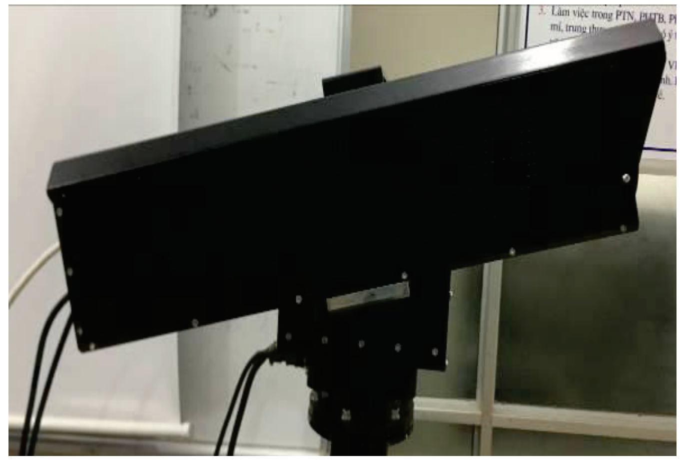
A DYNAMIC MODEL OF THE ROBOTIC SYSTEM

A. Experimental Setup

The model of 2-DOF Robotic Arm was built at the Control Systems Laboratory, Department of Automation and Computing Techniques, Le Quy Don Technical University shown in Figure 1.



a)



b)

FIGURE 1. Experimental 2-dof robotic arm mounted camera

We assume the mass of each link set at the top of link. The above model is described as a 2-DOF Robotic Arm shown on Figure 2 diagram.

2_DOF Robotic Arm mounted camera consists of two links with motors. The first link can rotate around the z axis by DC1 motor. The second link around in the plane is finished with the z axis by the DC2 motor and the center is the end of the first link. The purpose of the system is to maintain the camera following a given or unknown trajectory. The mass of links m_1, m_2, length of links l_1 and l_2 are given in Table 1. These values will be used in simulation and evaluation.

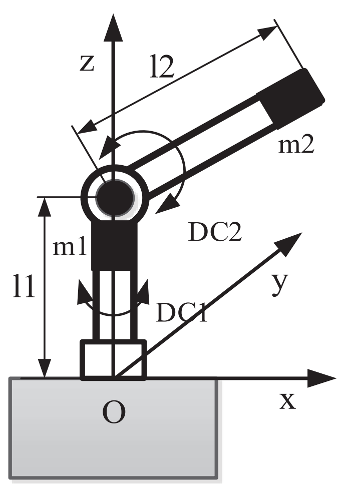


FIGURE 2. Kinematic Model of 2-DOF Robotic Arm mounted camera

The dynamics of 2-DOF Robotic Arm mounted camera were built on the Lagrange-Euler formula to solve problems related to dynamic models. Figure 2 shows the diagram of 2-DOF Robotic Arm with links 1 and 2, torque for links 1 and 2 respectively τ_1, τ_2. This moment is generated by two motors, DC1 and DC2, respectively.

TABLE 1. Parameters of Robotic Arm

Symbol	Description	Value	Unit
m_1	Mass of link 1	30	kg
m_2	Mass of link 2	20	kg
l_1	Length of link 1	0.3	m
l_2	Length of link 2	0.4	m
g	Gravity	9.81	m^2/s^2

B. Equations of Motion

Referring to Figure 2 the kinematical relations for each mass can be written easily:

$$\begin{cases} x_1 = 0 \\ y_1 = 0 \\ z_1 = l_1 \end{cases} \quad and \quad \begin{cases} x_2 = l_2 \sin(\theta_2)\cos(\theta_1) \\ y_2 = l_2 \sin(\theta_2)\sin(\theta_1) \\ z_2 = l_1 + l_2\cos(\theta_2) \end{cases} \tag{1}$$

The potential and kinetic energies of the system are given by [18,19] :

$$\begin{aligned} V &= m_1 g l_1 + m_2 g(l_1 + l_2\cos(\theta_2)) \\ T &= \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2) \end{aligned} \tag{2}$$

By applying Lagrange's equation [19]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = \tau_j \tag{3}$$

we get the following system of equations, after some manipulation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{4}$$

Where the state variable q, τ is defined by: $q=[\theta_1\ \theta_2]^T$, $\tau=[\tau_1\ \tau_2]^T$ and M represent the inertia matrix (symmetric positive definite) , C is the Coriolis and centrifugal matrix, gravity vector $g=[g_{12}\ g_{21}]^T$ which is given by:

$$M = \begin{bmatrix} m_2 l_2^2 \sin^2(\theta_2) & 0 \\ 0 & m_2 l_2^2 \end{bmatrix}, \quad C = \begin{bmatrix} m_2 l_2^2 \sin(2\theta_2)\dot{\theta}_2 & 0 \\ -m_2 l_2^2 \sin(2\theta_2)\dot{\theta}_1 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ -m_2 g l_2 \sin(\theta_2) \end{bmatrix}$$

Finally, $\tau=[\tau_1\ \tau_2]^T$ is torque exerted by actuators (servo motors) at each joint.

C. State-Space Representation

Introducing the state vector $x = (x_1, x_2, x_3, x_4)^T = (\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)^T$ and the input $\tau = \begin{pmatrix}\tau_1 & \tau_2\end{pmatrix}$ we obtain the state-space representation $\dot{x} = f(x,\tau)$, $x(0) = x_0$ with

$$f(x,\tau) = \begin{pmatrix} x_2 \\ -2\cot(x_3)x_4x_2 + \dfrac{\tau_1}{m_2 l_2^2} \\ x_4 \\ \sin(2x_3)x_2^{\,2} + \dfrac{g}{l_2}\sin(x_3) + \dfrac{\tau_2}{m_2 l_2^2} \end{pmatrix} \tag{5}$$

SYNTHESIS METHOD OF CONTROL FOR MULTIPLE INPUT NONLINEAR SYSTEMS

The control method for multiple input systems is built on the principle of separating multiple input systems into subsystems. Subsystems have one input and are structured based on the Controlable Jordan Form (CJF). The form of control Jordan [3], [20] order n of the dynamics control is a form of mathematical model as follows:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) \\ \dot{x}_2 = f_2(x_1, x_2, x_3) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, \dots, x_n) + g(x_1, x_2, \dots, x_n)u \end{cases} \tag{6}$$

with the differentiable functions f_i according to $x_1, x_2, \dots, x_{i+1}$, and $\partial f_i / \partial x_{i+1}$ # 0, for all $i < n$, $| g | > 0$. f_i functions have arbitrary nonlinearity.

For Jordan's control form, the control synthesis brings the state variables from an initial value to the balanced point that is presented in [20].

Suppose there are multiple input systems that control the order n with m control signals (m n)

$$\dot{x}_i = f_1(x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n), \quad i = \overline{1, n} \tag{7}$$

Jordanian subsystem order n_p of system (12) with the first state variable $x_{p,1}$ of $\{ x_1, x_2, \dots, x_n \}$ and the control signal u_j as subsystem of the form:

$$\begin{cases} \dot{x}_{p,1} = f_{p,1}(x_{p,1}, x_{p,2}) \\ \dot{x}_{p,2} = f_{p,2}(x_{p,1}, x_{p,2}, x_{p,3}) \\ \dots \\ \dot{x}_{p,n_p} = f_{p,n_p}(x_{p,1}, x_{p,2}, \dots, x_{n_p}, u_j) \end{cases} \tag{8}$$

with condition

$$\begin{cases} u_j^i \notin \arg\left(\dfrac{d^{n_p - 1} x_{p,1}}{dt^{n_p - 1}} \right), \quad i = 0, 1, \dots, n-2 \\ \dfrac{\partial}{\partial u_j} \left(\dfrac{d^{n_p} x_{p,1}}{dt^{n_p}} \right) \neq 0 \end{cases}$$

then the subsystem (8) is a subsystem with the global Jordanian form with the first state variable $x_{p,1}$.

The global Jordan and Jordan subsystems of a multi-input system are considered to have one input, and the synthesis of controls for them is as the control synthesis for dynamic systems with one input which is the form of Jordanian control (6).

From the above analysis, the synthesis method controls for multiple input systems (7) based on the principle of separating the system (13) into Jordanian subsystems with the following basic steps:

1. Study the possibility and lists all the options separated inputs dynamic system (7) to form the subsystems Jordan (global Jordan);
2. Select a separating option, synthesize the control as a parameter for each subsystem in this separation by applying the micro method;
3. Based on the controls for subsystems found as parameters, find the control vector for the system (7) in the form of clear analysis.

The Jordan subsystems have the advantage that finding the control vector is a process of solving linear equations.

The idea of quasi-time optimal control is shown as follows. Suppose the system state model returned to Jordan form has the form:

$$\begin{cases} \dot{x}_i = f_i(x_1, x_2, \ldots x_{i+1}), \; i = \overline{1, n-1}; \\ \dot{x}_n = f_n(x_1, x_2, \ldots x_n) + u. \end{cases} \tag{9}$$

where $f_i(\circ)$ is the analytic function, which means that the derivative is followed by all the variables $x_1, x_2, \ldots x_{i+1}$, $\forall i < n \rightarrow \partial f_i / \partial x_{i+1} \neq 0$, and $u(t)$ is the control signal.

The method of synthesizing the quasi-time optimal law for the system (9) is to use the micro transformation to bring the system (9) to the system of virtual equations of the form (10) [1-3]:

$$\begin{cases} \dot{y}_i = v_i \dfrac{y_i}{\sqrt{y_i^2 + \varepsilon_i^2}} + g_i(y_1, y_2, \ldots, y_{i-1}, y_{i+1}, \ldots . y_{i+1}, \varepsilon_i), \; i = \overline{1, n-1}; \\ \dot{y}_n = \dfrac{y_n}{\sqrt{y_n^2 + \varepsilon_n^2}} \end{cases} \tag{10}$$

where $h(y, \varepsilon) = \dfrac{y}{\sqrt{y^2 + \varepsilon^2}} u_n$ is quasi-time optimal function, $g_i(\circ)$ - linear function, $\varepsilon_i; i = \overline{1, n};$ – quasi-time optimal parameter.

Solving the equations with $y_1 = \phi(x_1)$, we get the quasi-time optimal control law according to the desired variable.

SYNTHESIS CONTROL LAW FOR 2-DOF ROBOTIC ARM MOUNTED CAMERA

Quasi-time optimal control law:

From the model (5), it is necessary to separate the Jordanian system into two subsystems as follows:

Subsystem 1

$$f(x_1, \tau_1) = \begin{pmatrix} x_{1,1} \\ -2\cot(x_{2,1}) x_{2,2} x_{1,2} + \dfrac{\tau_1}{m_2 l_2^2} \end{pmatrix}$$

Subsystem 2

$$f(x_2, \tau_2) = \begin{pmatrix} x_{2,1} \\ \sin(2x_{2,1}) x_{1,2}^{\;2} + \dfrac{g}{l_2} \sin(x_{2,1}) + \dfrac{\tau_2}{m_2 l_2^2} \end{pmatrix}$$

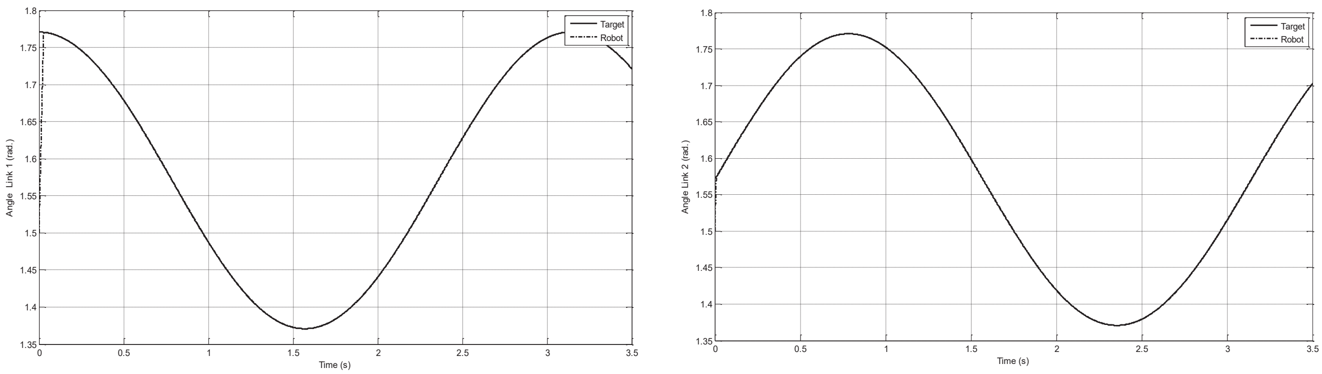
The subsystem is Jordan-controlled form, virtual system for each subsystem to be selected in the form (9), and the angles of joints will be ensured to be placed in the preset position with the optimal approach. The law of quasi-time optimal control u_1, u_2 is found when solving (11) with $y_1 = x_1$ and $y_1 = x_2$. The formula of u_1, u_2 is not given here because it is too long.

$$\begin{cases} \dot{y}_1 = -v \dfrac{y_1 - y_{sp}}{\sqrt{(y_1 - y_{sp})^2 + \varepsilon_1^2}} + y_2 \\ \dot{y}_2 = -\dfrac{y_2}{\sqrt{y_2^2 + \varepsilon_2^2}} \end{cases} \tag{11}$$

SIMULATION AND EVALUATION OF THE RESULTS OF THE QUASI-TIME OPTIMAL CONTROL FOR A 2-DOF ROBOTIC ARM MOUNTED CAMERA

By the method presented above, the control law is synthesized u_1, u_2 gives global stability control for Robotic Arm system. The simulation results of the system with the initial value of the system are as follows:

$\theta_{1sp} = \dfrac{\pi}{2} + \sin(2t)$ *(rad)*, $\dot{\theta}_{2sp} = \dfrac{\pi}{2} + \sin(2t + \dfrac{\pi}{2})$ *(rad)*, $\theta_1(0) = 1.5$ *(rad)*, $\dot{\theta}_1(0) = 0$ *(rad/s)*, $\theta_2(0) = 1.5$ *(rad)*, $\dot{\theta}_2(0) = 0$ *(rad/s)*. Value Parameter of the quasi-time optimal controller $\nu = 10$, $\varepsilon_1 = 0.01$; $\varepsilon_2 = 0.01$. Figure 3a, b shows the response to the position of the first and second link of the Robot Arm. The results show that the system is fast tracking and does not overshoot and oscillation occurs, the error is almost zero. On Figure 4 shows the tracking trajectory of the 2-DOF Robotic Arm, it is clear that the robot has been tracking the desired trajectory with small errors and without oscillation.



a) b)

FIGURE 3. The angle response: a- the angle of the first link of the Robotic Arm; b-the angle of the second link of the Robotic Arm

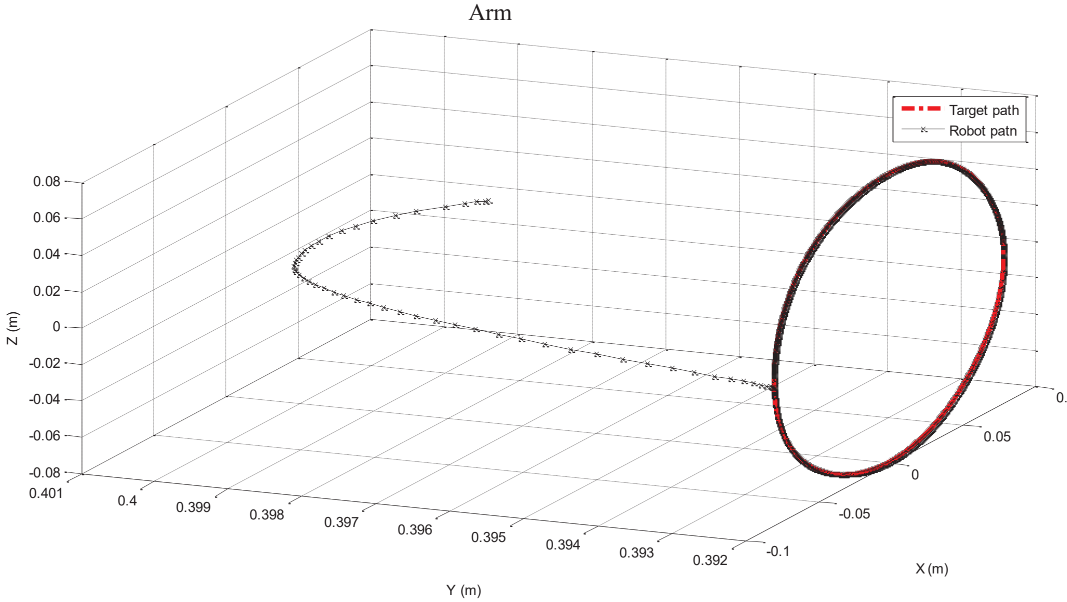


FIGURE 4. The trajectory tracking response of the Robotic Arm

CONCLUSION

The results obtained when synthesizing the law to control the 2-DOF Robotic Arm have proved effective for the MIMO technical system with high nonlinearity. The result of the 2-DOF Robotic Arm response in good quality has no overshoot and oscillation. Future studies will add to the adaptation of control laws when executing on embedded systems with restricted control signals, as well as when a MIMO nonlinear system is not in Jordan form.

REFERENCES

1. Nguyen Xuan Chiem, Hai Nguyen Phan. Design controler of the quasi-time optimization approach for stabilizing and trajectory tracking of inverted pendulum. MATEC Web of Conferences Volume 226, 2018 XIV International Scientific-Technical Conference “Dynamic of Technical Systems” (DTS-2018).
2. Hai N. Phan, Chiem X. Nguyen. Building embedded quasi-time-optimal controller for two-wheeled self-balancing robot. MATEC Web Conf.Volume 132, 2017XIII International Scientific-Technical Conference “Dynamic of Technical Systems” (DTS-2017)
3. Nguyen X.C. Phan N.H. Hoang D.L. Truong D.K. Kien L. M, Thuy X.P. Building quasi-time-optimal control laws for ball and beam system. 2019 3rd International Conference on Recent Advances in Signal Processing, Telecommunications & Computing (SigTelCom)
4. N. Elkhateeb and R. Badr, “Employing Artificial Bee Colony With Dynamic Inertia Weight for Optimal Tuning of PID Controller,” 5th International Conference on Modeling, Identification and Control, pp. 42–46, Cairo, Egypt, 2013
5. J. P. Perez, R. Soto, A. Flores, F. Rodriguez, and J. L. Meza, “Trajectory Tracking Error Using PID Control Law for Two-Link Robot Manipulator via Adaptive Neural Networks,” Procedia Technology, vol. 3, pp. 139–146, 2012. https://doi.org/10.1016/j.protcy.2012.03.015.
6. El-Hadi Guechi, Samir Bouzoualegh, Youcef Zennir and Saso Blzic. MPC Control and LQ Optimal Control of A Two-Link Robot Arm: A Comparative Study. Journal Machines, Volume 6, Issue 3. https://doi.org/10.3390/machines6030037
7. Kai, C.-Y.; Huang, A.-C. Adaptive LQ control of robot manipulators. In Proceedings of the 9th IEEE Conference on Industrial Electronics and Applications, Hangzhou, China, 9–11 June 2014.
8. Bauer, P.; Bokor, J. Development and performance evaluation of an infinite horizon LQ optimal tracker. Eur. J. Control 2018, 39, 8–20.
9. G. H. Xu, Z. H. Xiao, Y. Guo, and X. B. Xiang, “Trajectory tracking for underwater manipulator using sliding mode control,” IEEE International Conference on Robotics and Biomimetics (ROBIO), pp. 2127–2132, Sanya, China, Dec. 2007
10. P. R. Ouyang, J. Tang, W.H. Yue, and S. Jayasinghe, “Adaptive PD plus Sliding Mode Control for Robotic Manipulator,” Proc. of IEEE International Conference on Advanced Intelligent Mechatronics (AIM), Banff, AB, Canada, July 2016.
11. D.l Vyas, J. Ohri and A. Patel, “Comparison of conventional & fuzzy based sliding mode PID controller for robot manipulator,” International Conference on Individual and Collective Behaviors in Robotics (ICBR), Sousse, Tunisia, Dec. 2013
12. Umme Zakia, Mehrdad Moallem and Carlo Menon. PID-SMC controller for a 2-DOF planar robot. 2019 International Conference on Electrical, Computer and Communication Engineering (ECCE). https://www.researchgate.net/publication/332586279.
13. Duka, A.-V. Neural Network based Inverse Kinematics Solution for Trajectory Tracking of a Robotic Arm. Procedia Technol. 2014, 12, 20–27.
14. Chaudhary, H.; Panwar, V.; Prasad, R.; Sukavanam, N. Adaptive neuro fuzzy based hybrid force/position control for an industrial robot manipulator. J. Intell. Manuf. 2016, 27, 1299–1308.
15. Wuxi Shi, Mu Zhang, Wencheng Guo, Lijin Guo. Stable adaptive fuzzy control for MIMO nonlinear systems. Journal "Computers and Mathematics with Applications" 62 (2011) 2843–2853.
16. P. V. Savsani and R. L. Jhala, “Optimal Motion Planning For a Robot Arm by Using Artificial Bee Colony (ABC) Algorithm,” International Journal of Modern Engineering Research (IJMER), vol. 2, no. 6, pp. 4434–4438, 2012.
17. Q. Ma and X. Lei, “Dynamic Path Planning of Mobile Robots Based on ABC Algorithm,” Lecture Notes in Computer Science, pp. 267–274, 2010. https://doi.org/10.1007/978-3-642-16527-6_34.
18. Nasr A. Elkhateeb, Ragia I. Badr. Novel PID Tracking Controller for 2DOF Robotic Manipulator System Based on Artificial Bee Colony Algorithm. Journal Electrical, Control and Communication Engineering, 2017, vol. 13, pp. 55–62.
19. Amin A. Mohammed, Ahmed Eltayeb. Dynamics and Control of a Two-link Manipulator using PID and Sliding Mode Control. International Conference on Computer, Control, Electrical, and Electronics Engineering (ICCCEEE) 2018.
20. Gaiduk A.R.: Synthesis of nonlinear systems based on the Jordan controlled form. Automation and Telemechanics, 2006. № 7