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C. X. Nguyen, H. N. Phan, A. D. Lukianov, T. D. Pham, T. T. Nguyen, and H. T. Nguyen

Design of an Embedded Control System Based on the Quasi-Time Optimal Control Law When Limiting the Control Signal for The Ball and Beam System

C.X. Nguyen[1, a)], H.N. Phan[2, b)], A.D. Lukianov[3], T.D. Pham[4], T.T. Nguyen[1], H.T. Nguyen[2]

[1]*Department of Automation and Computing Techniques, Le Quy Don Technical University, Hanoi, Vietnam*
[2]*Department of Software Engineering, Le Quy Don Technical University, Hanoi, Vietnam*
[3]*Don State Technical University, Rostov on Don, Russia*
[4]*Department of Technology, equipment and aerospace, Le Quy Don Technical University, Hanoi, Vietnam*

[a)]Corresponding author: nguyenxuanchiem83@gmail.com
[b)]phanhai@mail.ru

Abstract: In the paper, it is presented about the design of the quasi-time optimal control law for Ball and Beam system when the control signal is limited. Impact signal limits are characteristic of real physical systems, so when designing embedded control systems this is an important factor to consider. In this design method, the transformation from the equations of object system to equations with limited control signals is made using extra variables. Proceed to synthesize the new control laws on the new system by the quasi-time optimal control. By the proposed synthesizing method, the control law ensures that the embedded control system works stably and the system works quickly in terms of time. Simulation and experimental results show the effectiveness of the proposed design.

INTRODUCTION

Controlling unstable systems is an important and interesting issue in controlling the system because the systems are not stable and dangerous when tested directly on real objects. Ball and Beam system is a typical system, used extensively in laboratories for automatic control. This system aims to simulate processes such as balance control in the spacecraft launch system; Stable horizontal control of aircraft during take-off and landing in air turbulence conditions [4-8]. Many studies from traditional PID control laws [11,12] to sustainable control methods [8], linear regression [4], nonlinear control [7], fuzzy logic control and Neuron network [5,6,13] has succeeded, but still has its weaknesses. The studies [11,12,13] have successfully built an embedded controller for Ball and Beam system based on PID control law.

The method of synthesizing the quasi-time optimal control law is presented in the works [1-3,9,14]. Its results have shown the advantages of this method. In the work [3], the authors synthesized the quasi-time optimal control law for Ball and Beam systems. The results of the method compared with the LQR method have shown its advantages. But in this project, the author group designed the control law not to include the control signal limit on real physical systems. This study focuses on a problem related to the quasi-time optimal signal control when blocked. The effectiveness of the proposed method is demonstrated through simulation results and implemented on real-time embedded systems.

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DYNAMICAL MODEL OF THE BALL AND BEAM SYSTEM

A. Experimental Setup

We consider a Ball and Beam system [4], which is schematically depicted in Figure. 1.

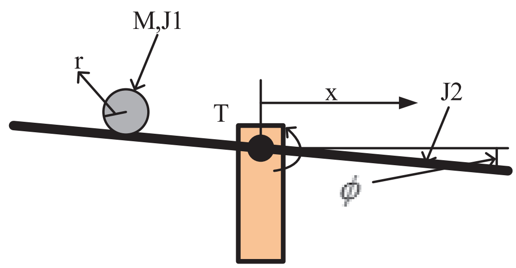


FIGURE 1. Ball and Beam system

It consists of a ball that is rolling on a beam without slipping. The beam rotates around its pivot point where a torque T can be applied. The ball position x is measured with respect to the pivot point and the beam angle ϕ is defined relative to the horizontal plane. The values of the mass m and radius r of the ball, its moment of inertia J1 as well as the moment of inertia J2 of the beam are given in Table 1 together with the gravity constant g and are the same as those used in [1,6]. They will be used for the simulations in Section IV.

TABLE 1. The parameter of Ball and Beam

Symbol	Discription	Value	Unit
M	ball mass	0.05	kg
r	ball radius	0.01	m
J_1	ball moment of inertia	$2x10^{-6}$	kg m^2
J_2	beam moment of inertia	0.02	kg m^2
g	gravity	9.81	m^2/s^2

A DC motor is used to generate the torque *T* on the beam. The ball position *x*(*t*) is measured with an ultrasonic sound distance sensor. The rotation angle is measured using an *incremental encoder*. Due to this, we do not measure the absolute rotation angle $\phi(t)$, but rather the relative angular difference with respect to the beginning of the measurement at $t = 0$, i.e. $\phi(t) - \phi_0$, where we have introduced the angular offset ϕ_0.

B. Equations of Motion

Following [4] and [13], the Lagrangian function

$$L = \frac{1}{2} J_2 \dot{\phi}^2 + \frac{1}{2} J_1 (\frac{\dot{x}}{r} + \dot{\phi})^2 + \frac{1}{2} m(\dot{x}^2 + x^2 \dot{\phi}^2) - mgx\sin(\phi) \quad (1)$$

L is defined as the difference between the kinetic energy and the potential energy is employed, yielding the following equations of motion

$$(J_1 + J_2 + mx^2)\ddot{\phi} + 2mx\dot{x}\dot{\phi} + mgx\cos(\phi) = T \quad (2)$$

$$(\frac{J_1}{r^2} + m)\ddot{x} - mx\dot{\phi}^2 + mg\sin(\phi) = 0 \quad (3)$$

C. State-Space Representation

Introducing the state vector $x = (x_1, x_2, x_3, x_4)^T = (x, \dot{x}, \varphi, \dot{\varphi})^T$ and the input $u = T \in R$ and $|T| < T_{max}$, we obtain the state-space representation $\dot{x} = f(x,u)$, $x(0) = x_0$ with

$$f(x,u) = \begin{pmatrix} x_2 \\ \dfrac{m x_1 x_4^2 - mg\sin(x_3)}{A} \\ x_4 \\ \dfrac{u - 2m x_1 x_2 x_4 - mg x_1 \cos(x_3)}{B + m x_1^2} \end{pmatrix} \tag{4}$$

where $A = J_1 / r^2 + m$ and $B = J_2 + J_1$ are employed for notational convenience.

APPROACH TO QUASI-TIME OPTIMAL CONTROL WHEN LIMITING THE SIGNAL CONTROL

With this approach, it is possible to assume the synthesizing the quasi-time optimal control law for the nonlinear system, in which this control law brings many advantages for the system, such as quasi-time optimal effects, asymptotic stability, and sustainable [1-3,9,14].

The idea of quasi-time optimal control with limited signal control is shown below. Suppose the system model in state-space is taken by Jordan in the form (5):

$$\begin{cases} \dot{x}_i = f_i\left(x_1, x_2, \ldots x_{i+1}\right),\ i = \overline{1, n-1}; \\ \dot{x}_n = f_n\left(x_1, x_2, \ldots x_n\right) + u. \end{cases} \tag{5}$$

where $f_i(\circ)$ is an analytic function, which means that the derivative exists according to all variables $x_1, x_2, \ldots x_{i+1}$, $\forall i < n \rightarrow \partial f_i / \partial x_{i+1} \neq 0$, and $u(t)$ is the limited control signal $|u| \leq u_{\max}$.

In order to synthesize the control law, we need to convert the equations (5) to equations (6) that are not limited to control signals by adding an extra variable.

$$\begin{cases} \dot{x}_i = f_i\ \ x_1, x_2, \ldots x_{i+1}\ \ ,\ i = \overline{1, n-1}; \\ \dot{x}_n = f_n\ \ x_1, x_2, \ldots x_n\ \ + k\dfrac{x_{n+1}}{\sqrt{x_{n+1}^2 + \rho^2}} u_{\max} \\ \dot{x}_{n+1} = \nu \end{cases} \tag{6}$$

where k is a positive constant; $0 < \rho << 1$ – fixed parameters;

The method of synthesizing the quasi-time optimal control law for the system (5) is the micro transformation that brings the system (6) to the virtual equations with the quasi-time optimal form of the k-order with the following form:

$$\begin{cases} \dot{y}_1 = -v_1^m h(y_1, \varepsilon_1) + y_2; \ldots; \dot{y}_k = v_k^m h(y_k, \varepsilon_k) + y_{k+1} \\ \dot{y}_{k+1} = -\dfrac{y_{k+1}}{\varepsilon_{k+1}} + y_{k+2}; \ldots; \dot{y}_{n-1} = -\dfrac{y_{n-1}}{\varepsilon_{n-1}} + y_n \\ \dot{y}_n = -y_n / \varepsilon_n \end{cases} \tag{7}$$

Solving the equations (7) with $y_1 = \phi(x_1)$, we get the quasi-time optimal control law according to the desired variable.

Synthesis the quasi-time optimal control law:

With model (4) when component $mx_1x_4^2$ has a very small angular speed of the Beam, we transform into Jordan form by removing the nonlinear component $mx_1x_4^2$ we get the state equation of the Ball and Beam with the following form:

$$f(x,u) = \begin{pmatrix} x_2 \\ \dfrac{-mg\sin(x_3)}{A} \\ x_4 \\ \dfrac{u - 2mx_1x_2x_4 - mgx_1\cos(x_3)}{B + mx_1^2} \end{pmatrix} \tag{8}$$

The system (8) is Jordan controlled form and still nonlinear system [3], but u are blocked. Using the extra variable to convert the system (8) into the system (9) is still in Jordan form, but the control signal is not blocked.

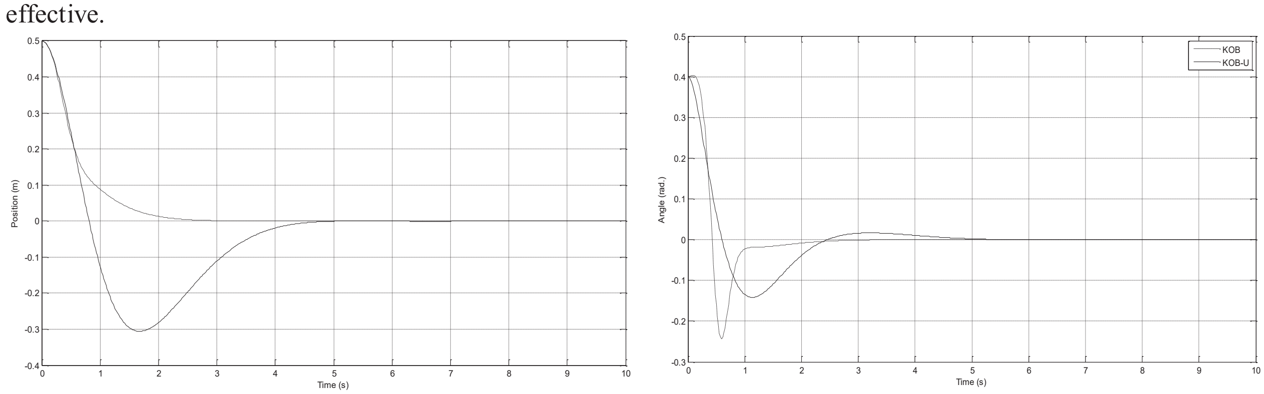
$$f(x,u) = \begin{pmatrix} x_2 \\ \dfrac{-mg\sin(x_3)}{A} \\ x_4 \\ \dfrac{u_{\max} k \dfrac{x_5}{\sqrt{x_5^2 + \rho^2}} - 2mx_1x_2x_4 - mgx_1\cos(x_3)}{B + mx_1^2} \\ v \end{pmatrix} \tag{9}$$

The selected virtual system takes the form (10), and then the Ball position will be guaranteed to return to preset position with the optimal time. The quasi-time optimal control law according to the distance u is found when solving (10) with $y_1 = x_1$. The formula of u is not given here because it is too long.

$$
\begin{cases}
\dot{y}_1 = -v\dfrac{y_1 - y_{sp}}{\sqrt{\left(y_1 - y_{sp}\right)^2 + \varepsilon_1{}^2}} + y_2 \\
\dot{y}_2 = -\dfrac{y_2}{\varepsilon_2} + y_3 \\
\dot{y}_3 = -\dfrac{y_3}{\varepsilon_3} + y_4 \\
\dot{y}_4 = -\dfrac{y_4}{\varepsilon_4} + y_5 \\
\dot{y}_5 = -\dfrac{y_5}{\varepsilon_5}
\end{cases}
\quad (10)
$$

SIMULATION AND EVALUATION OF THE QUASI-TIME OPTIMAL CONTROL LAW WHEN LIMITING THE CONTROL SIGNAL FOR THE BAR AND BEAM SYSTEM

By the method described above, the synthesized control law u provides stable control for the Ball and Beam system. Simulation results with the initial system value as follows: $x_{sp} = 0$, $x(0) = 0.5$, $\dot{x}(0) = 0$, $\phi(0) = 0.4$, $\dot{\varphi}(0) = 0$. The parameters of quasi-time optimal controller are $v = 1$, $\varepsilon_1 = 0.2, \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 0.3, \rho = 0.001$. Figure 2 shows the response to the position of the Ball with the two methods mentioned above. The position and angle response of the system when limited the control signal is slower than when no limit to the control signal, but the control signal of the system is much smaller than the unrestricted system in the same condition. Since then, when applying on a physical system, this method is more effective.



a) b)

FIGURE 2. a- The position response of Ball; b- The angle response of Beam

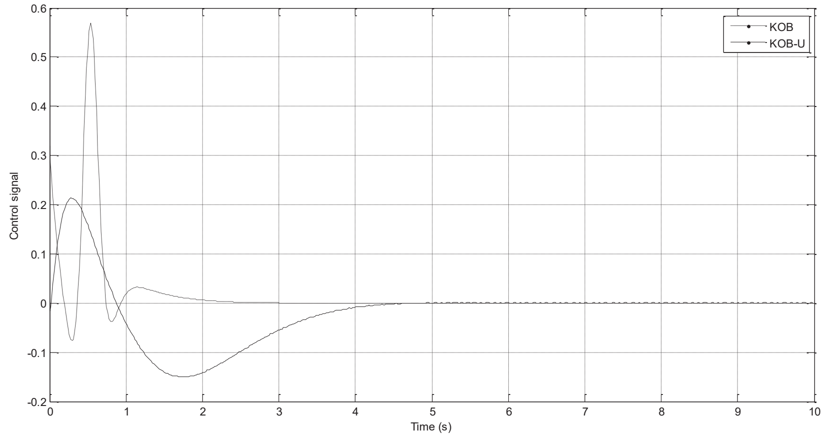


FIGURE 3. Control signal of the system

The results performed on real-time system:

Control law is applied on real-time embedded systems (Figure 4a) with a working length of 0.5 (m). The Ball is moved by the motor directly attached to the Beam. The angular measuring sensor of the Beam is an incremental encoder 500CPR attached to the motor shaft. The Ball position is determined by the Laser Radar sensor VL53L0X:

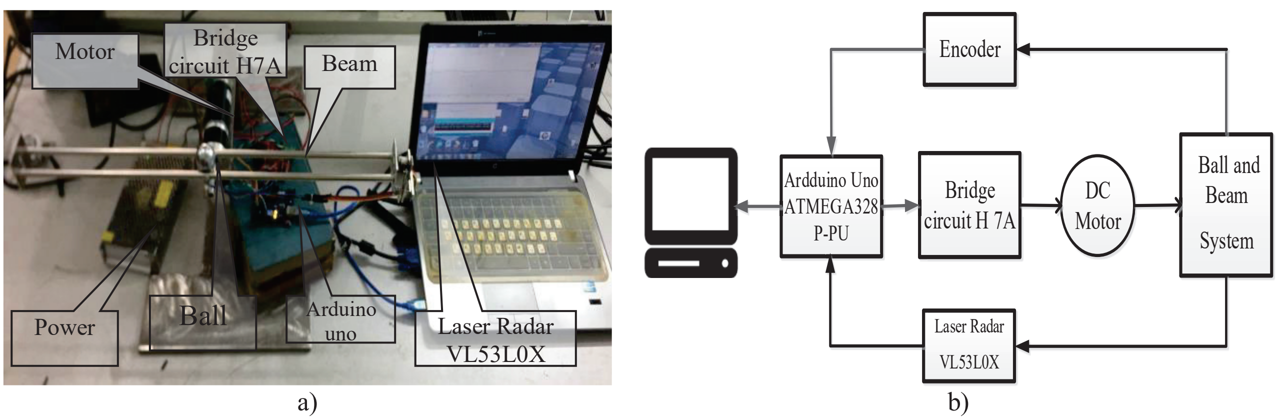
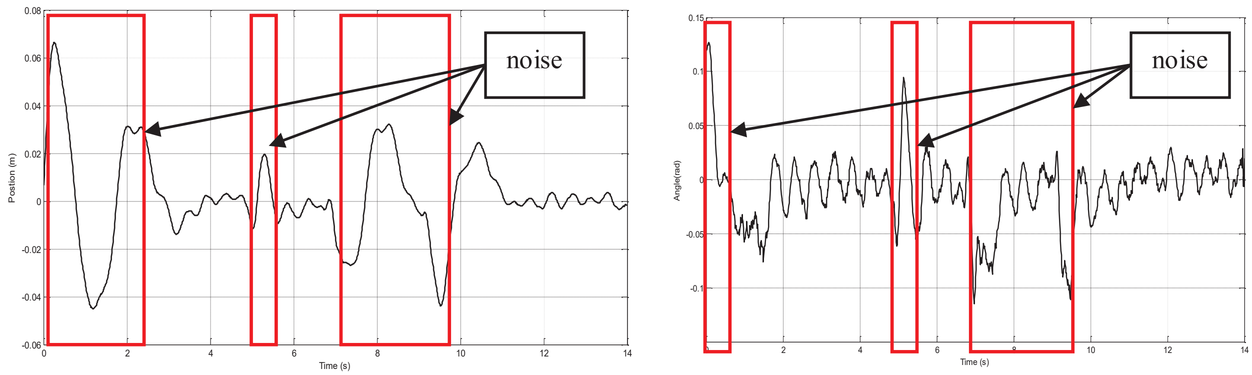


FIGURE 4. The real model of Ball and Beam system

Circuit block diagram of the controller is shown in Figure 4b. The ATmega16U2 microcontroller is used as a central processor working with a 16 kHz quartz frequency. Two encoders are connected to the microcontroller through two digital ports. DC motors are controlled via the H7A-Bridge Circuit and the microcontroller's PWM pins. Laser Radar Sensor VL53L0X connects to the central controller via I2C protocol. The power supply to the system is a DC source with voltage 12(V), current 7(A). The system connects to data collection software via RS232 serial port. The control interface is built on Visual Studio software to observe angle and position values by displaying numerical values and graphs on the computer screen.



a) b)

FIGURE 5. Angle response and position response of the real-time system

a- The angle response; b- The position response.

In Figures 5a and 5b, the angle and position of the Ball is shown by the quasi-time optimal controller when limiting the control signal. In the retangled area, the ball is deflected away from the balance position when there is an impact noise. From the response of the system, we find that although there exists a fluctuation due to the mechanical structure and the embedded program structure is not good, the ball is always at the desired position when there is an impact noise and only small fluctuates around Balance position. The position of the Ball and the angle of the Beam fluctuate around the desired position, which tends to decrease.

CONCLUSION

The results obtained when synthesizing control law on the Ball and Beam system when limiting the control signal. The proposed method has proved the advantage for complex technical systems with high nonlinearity when implemented on real-time embedded systems. The control quality has decreased when there is no limit to the control signal. Although control quality is reduced when there is no limit to the control signal, they ensure the real system operates more stable. Future studies will include adaptation of the control law parameters when there are changes in system parameters or external disturbance.

REFERENCES

1. Nguyen Xuan Chiem, Hai Nguyen Phan. Design controler of the quasitime optimization approach for stabilizing and trajectory tracking of inverted pendulum. MATEC Web of Conferences Volume 226, 2018 XIV International Scientific-Technical Conference “Dynamic of Technical Systems” (DTS-2018).
2. Hai N. Phan, Chiem X. Nguyen. Building embedded quasi-time-optimal controller for two-wheeled self-balancing robot. MATEC Web Conf.Volume 132, 2017XIII International Scientific-Technical Conference “Dynamic of Technical Systems” (DTS-2017)
3. Nguyen X.C. Phan N.H. Hoang D.L. Truong D.K. Kien L. M, Thuy X.P. Building quasi-time-optimal control laws for ball and beam system. 2019 3rd International Conference on Recent Advances in Signal Processing, Telecommunications & Computing (SigTelCom)
4. Philipp Rapp, Oliver Sawodny, and Cristina Tarın. “Stabilization of the Ball and Beam System by Dynamic Output Feedback using Incremental Measurements”. Control Conference (ECC), 2013 European.
5. Yeong-HwaChang, Chia-WenChang, Chin-WangTao, Hung-WeiLin, Jin-ShiuhTaur. “Fuzzy sliding-mode control for ball and beam system with fuzzy ant colony optimization”. Expert Systems with Applications Volume 39, Issue 3, 15 February 2012, Pages 3624-3633
6. MahmoodianHamid, LotfizadehDehkordiElaheh. “Designing a Fuzzy Type-2 Model-based Robust Controller for Ball and Beam System”. Procedia Computer Science Volume 105, 2017, Pages 125-130
7. J. Hauser, S. Sastry, and P. Kokotovic, “Nonlinear Control Via Approximate Input-Output Linearization: The Ball and Beam Example,” IEEE Transactions on Automatic Control, vol. 37, no. 3, pp. 392–398, 1992.

8. J. Huang and C.-F. Lin, "Robust Nonlinear Control of the Ball and Beam System," in Proceedings of the American Control Conference, Seattle, Washington, 1995
9. N.H. Fan, "Metody raspoznavanija upravljaemyh zhordanovyh form dinam.", Cand. tech. sciences. Diss., Rostov-on-Don, 193 (2008) [in Russian].
10. Priyank Jain, M. J. Nigam. "Real Time Control of Ball and Beam System with Model Reference Adaptive Control Strategy using MIT Rule," in 2013 IEEE International Conference on Computational Intelligence and Computing Research, Toronto, Dec 2013.
11. Mustafa Saad, Mohammad Khalallah, Design and Implementation of an Embedded Ball-beam Controller Using PID Algorithm. Universal Journal of Control and Automation 5(4): 63-70, 2017. DOI: 10.13189/ujca.2017.050402.
12. A. Taifour Ali*, Ahmed A. M., Almahdi H. A., Osama A. Taha, A. Naseraldeen A. Design and Implementation of Ball and Beam System Using PID Controller. Automatic Control and Information Sciences, 2017, Vol. 3, No. 1, 1-4 Available online at http://pubs.sciepub.com/acis/3/1/1 ©Science and Education Publishing DOI:10.12691/acis-3-1-1.
13. M. F. Rahmat, H. Wahid, and N. A. Wahab, "Application of intelligent controller in a ball and beam control system" International journal on smart sensing and intelligent systems, vol. 3, pp. 45-60, 2010.
14. Chiem Nguyen, Hai Phan , and Hung Nguyen, An energy saving method of stable control of inverted pendulum system when affected by external interference using auxiliary pendulum. E3S Web of Conferences 104, 01015 (2019) https://doi.org/10.1051/e3sconf/201910401015 EEESTS-2019.