A New Imaging Geometry Model for Determining Phase Distribution in Multi-receiver Synthetic Aperture Sonar

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***Abstract*— This paper proposes a novel imaging geometry model for the determination of phase distribution and beam pattern in multi-receiver synthetic aperture sonar (SAS). With the proposed model, the phase errors can be completely compensated based on the accuracy calculation of the propagation time and the Doppler frequency when reconstructing SAS image. Compared with the existing models, the proposed model shows that it can improve the imaging performance by using the back projection algorithm (BPA) due to avoiding approximations. Simulations are used to validate the proposed model and to evaluate the SAS imaging performance.**

Keywords*— *synthetic aperture sonar, multi-receiver, phase distribution, back projection

I. INTRODUCTION

Synthetic aperture sonar (SAS) [1] systems have been widely used in ocean discovery, wreck search and geological exploration due to high resolution when detecting underwater targets. Compared with mono-static synthetic aperture sonars, multi-receiver synthetic aperture sonars [1 – 2] provide higher resolution and mapping rates. However, the utilization of multi-receiver technique often increases the complexity of imaging processing as well as challenges in combining both the magnitude and phase of the echoes.

In order to efficiently combine the echoes of the receiver array at subsequent pings, the phase distribution in each receiver at each ping is determined according to the target coordinate, transmitter position, receiver position and velocity of the platform consisting of the SAS. With some conventional models, the approximations of the signal propagation time and the Doppler frequency are exploited such as the phase center approximation (PCA) [2 – 3] and the stop-and-hop (S&H) assumption [2] when reconstructing SAS imagery. The PCA considers a bi-static transmitter/ receiver pair as a single co-located transducer, whereas the S&H assumption considers that the platform stops, a sonar measurement is employed, and then the platform is moved to its next position on the system synthetic aperture [2]. With the two methods, it is difficult to compensate the phase errors completely when integrating the echoes of receivers at pings.

In some recent literatures [4 – 6], the authors have been trying to decrease the errors by more accurately calculating the delay time from the transmitter to the target and then back to the receivers at each ping. After that, the back projection algorithm (BPA) [4] can be used in order to determine the phase distribution and the beam pattern of the SAS array. However, when determining the delay time, the authors [4 – 6] neglected the motion of the transmitter during the signal transmission and the change of acoustic velocity from the transmitter to the target as well as from the target to receivers due to effects of SAS platform motion. With restricted conditions, the Doppler frequency term is uncompensated when the BPA is utilized. The errors of the delay time and the Doppler frequency may lead to degradation of the SAS imaging performance.

In this study, we propose a new imaging geometry model, which can calculate the delay time and the Doppler frequency accurately when the motion of the transmitter and the change of frequency in each receiver are taken into account. With the proposed model, the errors of the delay time and the Doppler frequency are completely compensated by using the BPA. Consequently, the phase distribution of all receivers and the beam pattern are mathematically analyzed when the main beam needs to be steered to desirable positions in the azimuth dimension. Due to the accuracy compensation of the phase terms, the presented model improves the SAS imaging performance compared with the conventional model. The validity and the performance of the proposed model are evaluated by simulation results.

II. MATHEMATICAL MODEL OF IMAGING GEOMETRY

A. Conventional Model

The conventional model of the two-dimensional (2D) imaging geometry for multi-receiver SAS is shown in Fig. 1(a). The receiver array of the SAS consists of N uniformly spaced receivers by distance d. The baseline distance between the transmitter and the ith receiver is denoted by d_i.

The sonar platform velocity and the acoustic velocity in water are denoted by v and c, respectively. With an ideal point target located at coordinate $(r, 0)$, the slow time in the azimuth dimension is t, and the transmitter is located at coordinate $(x_T, 0)$ $(x_T = vt)$.

Considering the forward distance during the signal reception, the signal propagation time from the transmitter to the target and then back to the ith receiver is $\tau_i(t)$ (or τ_i) represented as in [2]

$$\tau_i = \frac{v(vt + d_i) + c\sqrt{(vt)^2 + r^2}}{c^2 - v^2} + \frac{\sqrt{\left[v(vt + d_i) + c\sqrt{(vt)^2 + r^2}\right]^2 + (c^2 - v^2)(2vtd_i + d_i^2)}}{c^2 - v^2} \quad (1)$$

With the determination of the signal propagation time, after demodulation, the echo of the ith receiver is given by

$$ss_i(\tau,t) = p(\tau-\tau_i)\omega_a(t)\exp(-j2\pi f_c\tau_i)\sigma_i(t) \tag{2}$$

where $p(\tau)$ denotes the pulse envelope of transmitted signal, τ represents the past time in the range dimension, f_c is the carrier frequency. $\omega_a(t)$ expresses the combined beam pattern of the transmitter and the receiver, which is usually ignored in order to mainly concentrate on processing the phase coherently. $\sigma_i(t)$ is the reflection coefficient in the direction from the target to the ith receiver. In order to simplify the calculation, the target has a similar reflection coefficient in all directions.

B. Proposed Model

Although the conventional imaging geometry model considers motion of sonar platform during signal reception, motion of the transmitter during transmission and the change of acoustic velocity from the transmitter to the target as well as from the target to receivers due to effects of the motion platform are suppressed. The suppression of some conditions leads to challenges in determination the Doppler frequency of the echoes. As a result, the imaging performance can be degraded when the platform moves with a high velocity.

In order to completely investigate the effect of the motion on the Doppler frequency of the echoes and the propagation time, we propose a new imaging geometry model expressed as Fig. 1(b). Based on this model, as explained later, the Doppler frequency is more explicitly depicted and the delay time is more accurately determined compared with the conventional model. Therefore, the phase distribution and the beam pattern of the SAS array can be completely presented by the proposed imaging geometry model.

The proposed model also uses some terms which are similar to the terms in the above conventional model and introduces some new symbols as below. Besides, some conditions simplifying the calculation in the proposed model are chosen as in the above conventional model.

Considering a point M is located at coordinate (x, r), in order to propagate signal from the transmitter to point M, the angle between the total velocity vector $\overrightarrow{v_{\Sigma T}}$ and the platform velocity vector $\vec{v}$ is α_1 expressed as

$$\alpha_1 = \arccos\left(\frac{x-vt}{\sqrt{(x-vt)^2+r^2}}\right) \tag{3}$$

The total velocity vector $\overrightarrow{v_{\Sigma T}}$ is determined as

$$v_{\Sigma T} = v\cos\alpha_1 + \sqrt{c^2 - v^2\sin^2\alpha_1} \tag{4}$$

The propagation time for the signal to travel from the transmitter to point M is τ_1 represented as

$$\tau_1 = \frac{\sqrt{(x-vt)^2+r^2}}{v_{\Sigma T}} = \frac{\sqrt{(x-vt)^2+r^2}}{v\cos\alpha_1 + \sqrt{c^2-v^2\sin^2\alpha_1}} \tag{5}$$

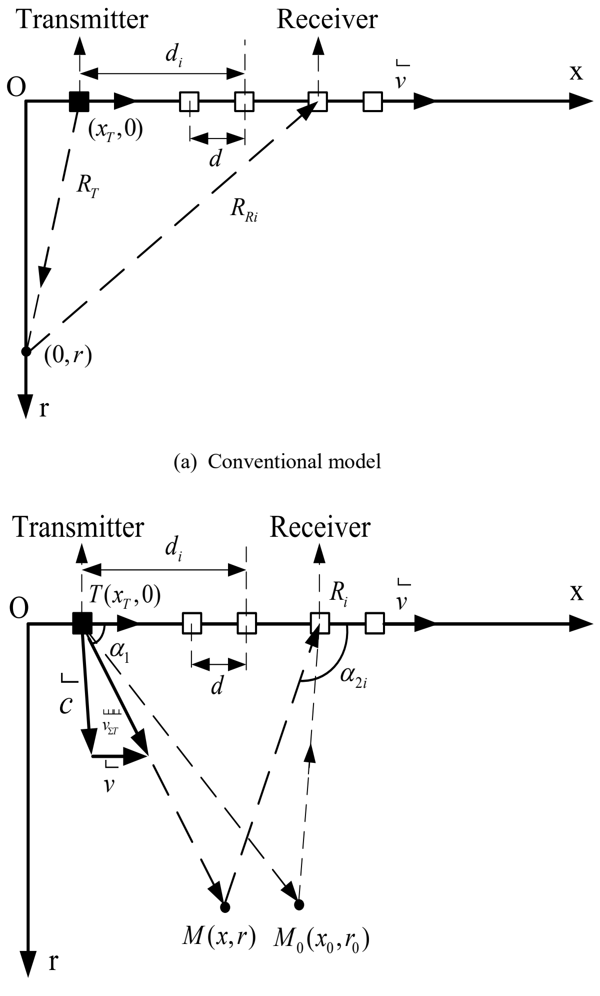
After receiving the signal at point M, the scatter is employed to all directions and the ith receiver obtains the scattering signal at the direction from M to R_i determined by angle α_{2i} as in Fig. 1(b).

The delay time for the echoes to travel from point M to the ith receiver is τ_{2i} satisfying the following equation

$$\begin{aligned} c\tau_{2i}^2 &= (x-vt)^2 + r^2 + (d_i + v\tau_1 + v\tau_{2i})^2 \\ &\quad - 2\sqrt{(x-vt)^2+r^2}\,(d_i + v\tau_1 + v\tau_{2i})\cos\alpha_1 \end{aligned} \tag{6}$$

After solving this quadratic equation, the delay time τ_{2i} can be expressed as

$$\tau_{2i} = \frac{v\left(\sqrt{(x-vt)^2+r^2} - (d_i + v\tau_1)\cos\alpha_1\right) + \sqrt{\Delta_i}}{c^2 - v^2} \tag{7}$$



(b) Proposed model

Fig. 1. Imaging geometry of the multi-receiver SAS

where Δ_i is the discriminant of the quadratic equation derived from (6), which is represented as

$$\begin{aligned}\Delta_i &= v^2\left(\sqrt{(x-vt)^2+r^2}-(d_i+v\tau_1)\cos\alpha_1\right)^2\\ &+\left(c^2-v^2\right)\left(\begin{aligned}&(x-vt)^2+r^2+(d_i+v\tau_1)^2\\ &-2\sqrt{(x-vt)^2+r^2}\,(d_i+v\tau_1)\cos\alpha_1\end{aligned}\right)\end{aligned} \tag{8}$$

With this proposed imaging geometry model, the delay time is split into two terms: the first term τ_1 denotes the delay time for the signal to propagate from the transmitter to point M, the second term τ_{2i} expresses the propagation time for the echoes to travel from point M to the ith receiver. In the each duration, the Doppler frequency is explicitly determined.

After ignoring the beam pattern of the transmitter and the receiver and assuming that the target has a similar reflection coefficient in all directions, the echo signal of the ith receiver after demodulation is given by

$$ss_i(\tau,t) = p(\tau-\tau_1-\tau_{2i})\exp\left(-j2\pi f_1\tau_1 - j2\pi f_{2i}\tau_{2i}\right) \tag{9}$$

where f_1 and f_{2i} are received frequency at point M and at the ith receiver, respectively, which are determined the formulas as below

$$f_1 = f_c\frac{c}{c-v\cos\alpha_1} \tag{10}$$

$$f_{2i} = f_1\frac{c+v\cos\alpha_{2i}}{c} = f_c\frac{c+v\cos\alpha_{2i}}{c-v\cos\alpha_1} \tag{11}$$

Based on the corrected expressions of the Doppler frequency and the delay time, the phase distribution and the beam pattern of the multi-receiver SAS array at the subsequent pings are mathematically determined when the main beam needs to be steered to any desired position.

III. Determination of the Phase Distribution and the Beam Pattern by Using the BPA

Because the BPA can provide high imaging performance, it can be exploited in order to directly calculate the phase distribution and the beam pattern of the SAS array at the subsequent pings. Before combining the echoes of the receivers at subsequent pings, the BPA performs the range compression, the range cell migration correction (RCMC) [4] when using the digital sampled signals. The RCMC can be employed by using the interpolation, such as that based on the Kernel function [4]. Generally, the SAS imaging performance depends on the accuracy of the interpolation [4]. In order to ignore the interpolation errors as well as focus on the determination of the phase distribution and the beam pattern, the continuous signals at each ping are used when coherently integrating the echoes.

With the conventional model, the phase distribution of the SAS array is determined by the utilization of the BPA, which is expressed as [4]:

$$\psi_i(t,x_0,r_0) = 2\pi f\tau_i \tag{12}$$

Expression (12) does not take into account the Doppler frequency. The suppression of the Doppler frequency may decrease the imaging performance when the platform moves with a high velocity.

With the proposed model, the phase distribution and the beam pattern of the multi-receiver SAS array are determined by the BPA when the Doppler frequency is taken into account. The phase distribution is determined as below when steering the main beam to the target located at the point $M_0(x_0, r_0)$:

$$\psi_i(t,x_0,r_0) = 2\pi\left(f_{01}\tau_{01} + f_{02i}\tau_{02i}\right) \tag{13}$$

where τ_{01}, τ_{02i}, f_{01}, f_{02} are determined similarly as τ_1, τ_{2i}, f_1, f_2 in (5), (7), (10) and (11), but x and r in these formulas are replaced by x_0 and r_0, respectively.

After obtaining the phase distribution, the beam pattern of the SAS array is derived after combining the echoes compensated the phase in the receivers at the pings. It is determined as

$$ff(x,r,x_0,r_0) = \sum_t\sum_{i=1}^{N} ss_i(\tau,t)\exp\left(j\psi_i(t,x_0,r_0)\right) \tag{14}$$

Equation (14) takes into account the Doppler frequency when synthesizing the beam pattern of the SAS array after each pulse repetition interval. Due to the ability of the accurate calculation of the delay time and the Doppler frequency, the proposed model can provide a higher imaging performance compared with the conventional model.

The beam pattern of the SAS array at the subsequent pings in (14) is expressed as the function of the two variables (x, r) in the azimuth dimension and the range dimension. When the range variable r is fixed, the beam pattern at coordinate (x_0, r_0) only depends on the azimuth variable x, which is called azimuth slices.

IV. Simulation Results

In order to highlight the validity of the proposed model, we consider an example of the multi-receiver SAS with the parameters expressed in Table I.

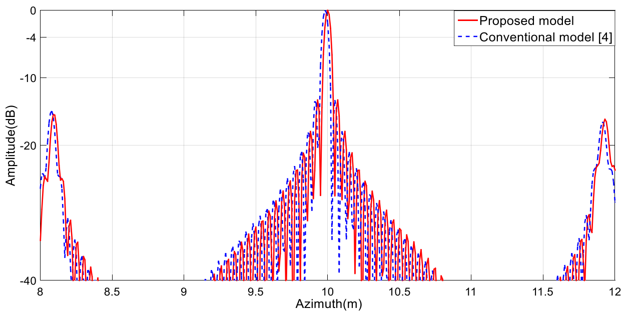
Assuming that the sound speed in sea water is 1500 m/s, and two cases are considered with two ideal point targets located at coordinates (50 m, 10 m) and (150 m, 10 m). The simulated results of the beam patterns in each case at platform velocities 0.5 m/s and 2.0 m/s are shown as Fig. 2 and Fig. 3, respectively. In each figure, there are a solid curve resulting from the BPA based on the proposed model and a dashed curve resulting from and the BPA based on the conventional model for comparison.

It can be seen that in this example, the main lobe is steered to the desired azimuth when using the BPA based on the proposed model. With the BPA based on the conventional model, the azimuth coordinates deviate from the ideal values. When the platform velocity is 0.5 m/s, the deviation is about 0.02 m with the close target (Fig. 2(a)) and is increased to 0.05 m with the far target (Fig. 2(b)). For platform velocity 2.0 m/s, the deviations of the target at the close range and the far range are 0.07 m and 0.2 m, respectively. Generally, the deviation increases with the platform velocity and range. The error occurred because the BPA based on the conventional model ignores the Doppler frequency and the change of acoustic velocity from the transmitter to the target. With distributed targets, the error can degrade the imaging performance. Therefore, the proposed model can compensate the phase completely for the multi-receiver SAS due to approximations.

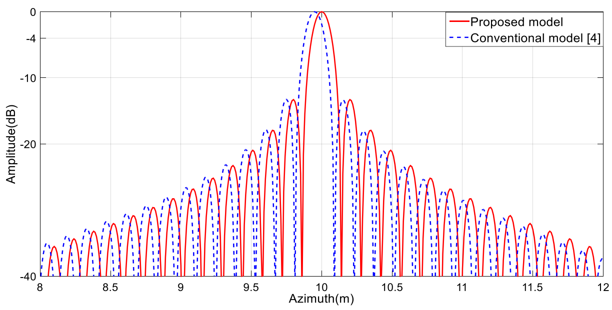
Fig. 3 shows that the peak sidelode level ratio (PSLR) [4] obtained by the BPA based on the proposed model decreases more than 1 dB compared with the PSLR obtained by the BPA based on the conventional model. Therefore, the proposed model can improve the SAS imaging performance when the platform moves with a high velocity.

TABLE I. THE SAS SYSTEM PARAMETERS

Parameters	Value	Units
Carrier frequency	100	kHz
Platform velocity	0.5; 2	m/s
Transmitter length in azimuth	0.04	m
Distance between two adjacent receivers (d)	0.02	m
Number of receivers (N)	32	
Pulse repetition interval	0.4	s
Number of pings	40	

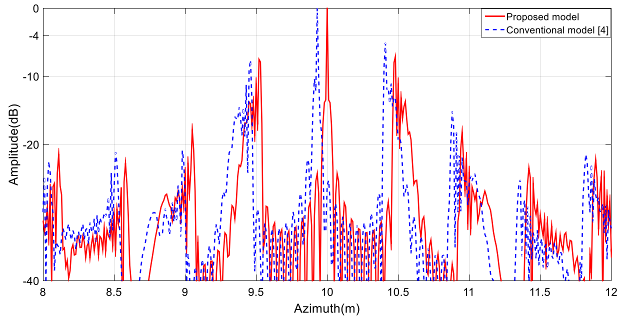


(a) Range coordinate 50 m

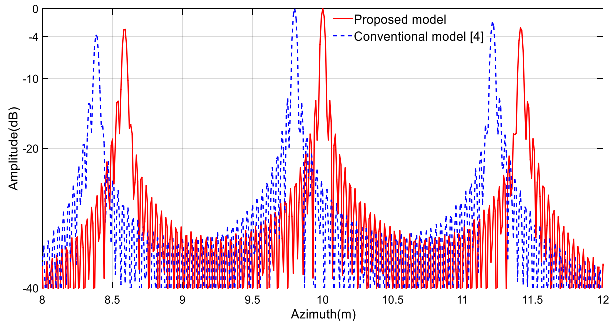


(b) Range coordinate 150 m

Fig. 2. Azimuth slice of point target with platform velocity 0.5m/s



(a) Range coordinate 50 m



(b) Range coordinate 150 m

Fig. 3. Azimuth slice of point target with platform velocity 2.0 m/s

V. CONCLUSION

In this study, with a proposal of a new multi-receiver SAS imaging geometry model, the phase distribution and the beam pattern have been accurately determining by using the BPA. Due to the accurate calculation of delay time and Doppler frequency of the proposed model, the compensation of the phase errors have more completely implemented and the SAS imaging performance has been improved compared with the conventional models. The validity and merits of our proposal have been verified by the simulation results.

REFERENCES

[1] N. Kolev, Sonar Systems, InTech, Croatia, 2011, pp.3–25.

[2] X. Zhang, J. Tang, and H. Zhong, "Multireceiver Correction for the Chirp Scaling Algorithm in Synthetic Aperture Sonar," IEEE Journal of Oceanic Engineering, vol. 39, pp.472 – 481, July 2014.

[3] A. Bellettini and M. A. Pinto, "Theoretical Accuracy of Synthetic Aperture Sonar Micronavigation Using a Displaced Phase-Center Antenna," IEEE Journal Of Oceanic Engineering, vol. 27, No. 4, pp. 780–789, October 2002.

[4] X. Zhang, P. Yang, C. Tan, and W. Ying, "BP algorithm for the multireceiver SAS," IET Radar Sonar Navig., vol. 13 Iss. 5, pp. 830-838, February 2019.

[5] X. Zhang, H. Huang, W. Ying, H. Wang, and J. Xiao, "An Indirect Range-Doppler Algorithm for Multireceiver Synthetic Aperture Sonar Based on Lagrange Inversion Theorem," IEEE Transactions On Geoscience And Remote Sensing, vol. 55, Issue: 6, pp. 3572 – 3587, March 2017.

[6] X. Zhang, C. Tan, and W. Ying, "An Imaging Algorithm for Multireceiver Synthetic Aperture Sonar," Remote Sens, 2019, 11 (6), 672.