State Estimation of Nonlinear Electromechanical System using Extended Kalman Filter

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***Abstract*— This paper presents an approach to estimate mechanical non-measurable coordinates of the state vector in servo tracking driver of the large radio telescope control system. The estimation algorithm of discrete extended Kalman filter based on the sequential recursive solution is developed for nonlinear systems. The proposed optimal state observer is investigated in the close-loop state feedback control system of large radio telescope. The results of computer simulation are carried out in MATLAB / Simulink environment.**

Keywords*— *servo control system; radio telescope; extended Kalman filter; optimal state observer

I. Introduction

Electromechanical structure part of the large radio telescopes (RT) servo driver is considered as complex nonlinear system. One of its features is limited rigidity of structure with large inertial loads. Due to large weight and complex mechanical configuration with limited rigidity, structure model of the large RT can be represented as a multi-mass system with elastic connections. These torsional vibrations of elastic property significantly influence the stability control system. In the other case, they can lead to decrease reliability and tracking accuracy servo system.

An effective approach for damping torsional oscillations in electric driver system with elastic properties is to apply advanced control structures with feedbacks from all state variables. In electric drives of the large RT, when synthesis control law bases on model predictive control (MPC) [1], information feedback from all coordinates of state variable vector should be known simultaneously: angular speeds of masses, torsional elastic torques and disturbance torques. However, direct measurements of these feedback signals are not always possible, because additional direct measurements of these variables are costly and difficultly realized in real condition. In these cases, the most appropriate method is using a special observer for estimation of those non-measurable variables. In this work, we introduce an optimal observer based on discrete extended Kalman filter (DEKF) to estimate non-measurable state variable in the large RT control system. DEKF can be applied for nonlinear electromechanical objects with good estimation accuracy.

II. Nonliear Mathematical Model of the Large Radio Telescopes Electric Drive Control System

In this section, the nonlinear mathematical model of electromechanical structure of the large RT electric drive control system in azimuthal moving is considered. Even though the real electromechanical structure part of the large RT is a complex multi-mass system, it can be approximated and simplified in a four-mass branched elastic model with concentrated parameters [2]. It should be noted that in the large RT electric drive, there are such nonlinearities as dry friction torques in moving parts, gaps (backlashes) in the kinematic connections between motors and rotating parts. These nonlinear factors lead to appear self-oscillations phenomenon in the transient process and increase static tracking errors, which are unacceptable in the precision electric drive control systems. In this work, damping due to the backlash in gear reducer is very small. So, it can be neglected without affecting tracking accuracy. Using the Lagrange equations second kind, nonlinear mathematical model of the large RT electric drive with taking into account nonlinear dry friction can be written in the form of first differential equations of moments acting on concentrated masses, as:

$$\begin{cases} \dot{\omega}_1 = J_1^{-1}\left(M_m - M_{21}\right); \\ \dot{M}_{21} = c_{21}\left(\omega_1 - \omega_2\right); \\ \dot{\omega}_2 = J_2^{-1}\left(M_{21} - M_{32} - M_{42} - M_{\tau 2}\right); \\ \dot{M}_{32} = c_{32}\left(\omega_2 - \omega_3\right); \\ \dot{\omega}_3 = J_3^{-1} M_{32}; \\ \dot{M}_{42} = c_{42}\left(\omega_2 - \omega_4\right); \\ \dot{\omega}_4 = J_4^{-1} M_{42}; \\ M_m = 2k_i^{-1} c_m u_p; \\ M_{f2} = M_{f0} \tanh(\sigma\omega_2), \end{cases} \tag{1}$$

where: the first mass (J_1) – electric motor; the second mass (J_2) – platform; the third mass (J_3) – parabolic main antenna;

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the fourth mass (J_4) – counterbalance, balancing of parabolic main antenna when it moves in elevation angle; J_1, J_2, J_3, J_4 – moment inertia of the motor, and moments inertia of platform, parabolic main antenna, counterbalance, respectively; $M_m, M_{21}, M_{32}, M_{42}$ – motor's torque and torsional torques between masses, respectively; M_{f2} – dry friction torque, acting on the second mass; M_{f0} – coefficient of static Coulomb friction, $M_{f0} = (0.1 \ldots 0.3)M_m$; σ – positive coefficient, characterizing function slope and can be freely selected; $\omega_1, \omega_2, \omega_3, \omega_4$ – angular speeds of each mass; c_{21}, c_{32}, c_{42} – stiffness factors between masses; k_i – transfer coefficient of electromagnetic torque control loop, which is a proportional regulator; c_m – constructive coefficient of electric motor ; u_p – optimal control signal at the output of MPC speed regulator.

The servo tracking electric drive control system of the large RT, in which consists of speed loop and position loop, is shown in Fig. 1.

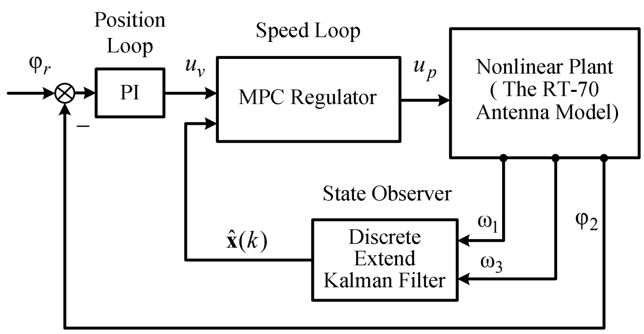

Fig. 1. Servo tracking control system of the large RT

In Fig. 1, inner loop is MPC speed regulator [1], which is closed with full state-feedback by using optimal observer based on discrete extended Kalman filter. In practical application like the large RT electric drive control system, it is difficult to directly measure the torsional torques between masses M_{21}, M_{32}, M_{42} and angular speeds of load side ω_2, ω_4. Thus, the state variables observer based on DEKF is applied to estimate these non-measurable mechanical signals. Bad effects of torsional oscillations and dry friction torque is eliminated in the speed control loop. Outer control loop is the position regulator with signal angular position feedback of the platform (φ_2). Position controller is proportional-integral (PI) regulator, which is synthesized by optimum module law. Output signal of position loop is speed reference signal ($u_v = \omega_r$).

III. State Estimation Based on DEKF

According to the Kalman filter theory, we consider solution to synthesize DEKF with sampling interval T_s. After the discretization with T_s sampling step, the nonlinear object (1) is described by the first differential equations in discrete domain, as:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{f}\left(\mathbf{x}(k), \mathbf{u}(k)\right) + \mathbf{w}(k); \\ \mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{v}(k), \end{cases} \tag{2}$$

where: $\mathbf{x}(k)$ – state column vector; $\mathbf{u}(k)$ – input column vector; $\mathbf{y}(k)$ – output column vector; $\mathbf{f}(\bullet), \mathbf{h}(\bullet)$ – nonlinear functions, that are continuously differentiable; k – current interval time index; $\mathbf{w}(k)$ – process noise; $\mathbf{v}(k)$ – measurement noise.

Both $\mathbf{w}(k)$, $\mathbf{v}(k)$ are Gaussian white noise with zero-mean random processes according to their diagonal covariance matrices $\mathbf{Q}_{\mathrm{H}}(k)$, $\mathbf{R}_{\mathrm{H}}(k)$, respectively. It is assumed that weight covariance matrices $\mathbf{Q}_{\mathrm{H}}(k)$, $\mathbf{R}_{\mathrm{H}}(k)$ are known and are considered mutually independent, i.e.

$$\begin{cases} \mathbf{w}(k) \sim N(0, \mathbf{Q}_{\mathrm{H}}(k)), \ \mathbf{Q}_{\mathrm{H}}(k) \geq 0; \\ \mathbf{v}(k) \sim N(0, \mathbf{R}_{\mathrm{H}}(k)), \ \mathbf{R}_{\mathrm{H}}(k) > 0. \end{cases} \tag{3}$$

Using the linearization method based on the extended Taylor series, the nonlinear object (2) can be written in the form of approximate linearized state-dependent state space model:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{w}(k); \\ \mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{v}(k), \end{cases} \tag{4}$$

where: $\mathbf{A}(k) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}^*, \mathbf{u}^*}$, $\mathbf{B}(k) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{\mathbf{x}^*, \mathbf{u}^*}$, $\mathbf{C}(k) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}^*, \mathbf{u}^*}$ – parameterized matrices with state-dependent parameters; $\mathbf{x}^*, \mathbf{u}^*$ – vectors of the equilibrium position.

The task of optimal state observer based on DEKF is to find estimation of state variable vector $\hat{\mathbf{x}}(k)$, that minimizes the root-mean-square error (RMS) objective function of mathematical expectation:

$$J = E\left\{\left[\mathbf{x}(k) - \hat{\mathbf{x}}(k)\right]\left[\mathbf{x}(k) - \hat{\mathbf{x}}(k)\right]^T\right\} \to \min. \tag{5}$$

DEKF algorithm is a set of mathematical equations that provides a sequential recursive solution for optimum processing of accidental digital data, minimizing functionality of quality (5). Process of the synthesis DEKF algorithm consists of two separate steps: *prediction* and *correction* [3, 4]. At the first step (prediction), the state and error covariance at the previous time are used to predict ahead the estimated state variables for the next step. In this step, estimated state variables are called by priori estimation, since they do not include information about the estimation at the current time. In the second step (correction), the current prior estimation is combined with the current measured states variables to improve and update the state estimate. This updated state variables are called posteriori estimation. The optimal estimated state variables can be obtained by using multi-step iteration of the following formulas:

Step 1 – prediction:

- predict the priori estimation of state variables ahead:

$$\hat{\mathbf{x}}(k\,|\,k-1)=\mathbf{A}(k)\hat{\mathbf{x}}(k-1\,|\,k-1)+\mathbf{B}(k)\mathbf{u}(k)\,; \qquad (6)$$

- predict the priori estimation of error covariance matrix:

$$\mathbf{P}^{-}(k)=\mathbf{A}(k)\mathbf{P}^{+}(k-1)\mathbf{A}^{T}(k)+\mathbf{Q}_{\mathrm{H}}(k)\ . \qquad (7)$$

Step 2 – correction:

- compute the gain vector of DEKF:

$$\mathbf{L}(k)=\mathbf{P}^{-}(k)\mathbf{C}^{T}(k)\left[\mathbf{C}(k)\mathbf{P}^{-}(k)\mathbf{C}^{T}(k)+\mathbf{R}_{\mathrm{H}}(k)\right]^{-1};\ (8)$$

- update the posteriori estimation of state variables:

$$\hat{\mathbf{x}}(k\,|\,k)=\hat{\mathbf{x}}(k\,|\,k-1)+\mathbf{L}(k)\left[\mathbf{y}(k)-\mathbf{C}(k)\hat{\mathbf{x}}(k\,|\,k-1)\right];\ (9)$$

- update the posteriori estimation of error covariance matrix:

$$\mathbf{P}(k)=\left[\mathbf{I}-\mathbf{L}(k)\mathbf{C}(k)\right]\mathbf{P}^{-}(k)\,; \qquad (10)$$

where $\mathbf{I}$ – diagonal unit matrix.

The above formulas are fair updated by state and covariance for receiving vector optimum coefficient of DEKF (8). In relation to the studied object (the large radio telescope RT-70), it is supposed that only directly measure the angular speeds of motor (the first mass) and main mirror (the third mass). Since, an optimal low-dimensional functional state observer based DEKF is used to reconstruct non-measurable state vector variables [2]. Control input signals of this state observer is angular speeds of motor ω_1 and output feedback signal is error estimate of angular speeds main mirror ω_3 (Fig. 1). In this case, the vector of estimated state variables in Eq. (6) has the form:

$$\hat{\mathbf{x}}(k)=\left[\hat{M}_{21}\quad \hat{\omega}_2\quad \hat{M}_{32}\quad \hat{\omega}_3\quad \hat{M}_{42}\quad \hat{\omega}_4\right]^{T},$$

where: $\hat{\omega}_2,\hat{\omega}_3,\hat{\omega}_4$ – estimated angular speeds of the second, the third and the fourth masses, respectively; $\hat{M}_{21},\hat{M}_{32},\hat{M}_{42}$ – estimated torsional torques between masses, respectively.

From Eq. (6) to Eq. (10), discrete vectors and matrices for DEKF can be written as:

$$\mathbf{B}(k)=\left[T_s c_{21}\quad 0\quad 0\quad 0\quad 0\quad 0\right]^{T};$$

$$\mathbf{C}(k)=\left[0\quad 0\quad 0\quad 1\quad 0\quad 0\right];\ \mathbf{u}(k)=\omega_1\,;\ \mathbf{y}(k)=\omega_3\,;$$

$$\mathbf{Q}_{\text{н}}=\mathbf{diag}(q_1,q_2,q_3,q_4,q_5,q_6)\,;\ \mathbf{R}_{\text{н}}=\left[r_1\right];$$

$$\mathbf{L}(k)=\left[l_1\quad l_2\quad l_3\quad l_4\quad l_5\quad l_6\right]^{T};$$

$$\mathbf{A}(k)=\begin{bmatrix} 1 & -T_s c_{21} & 0 & 0 & 0 & 0\\ T_s J_2^{-1} & a_{22} & -T_s J_2^{-1} & 0 & -T_s J_2^{-1} & 0\\ 0 & T_s c_{32} & 1 & -T_s c_{32} & 0 & 0\\ 0 & 0 & -T_s J_3^{-1} & 1 & 0 & 0\\ 0 & T_s c_{42} & 0 & 0 & 1 & -T_s c_{42}\\ 0 & 0 & 0 & 0 & -T_s J_4^{-1} & 1 \end{bmatrix};$$

$$a_{22}=1-T_s J_2^{-1} M_{f0}\sigma(1\text{-tanh}^2(\sigma\omega_2))\ .$$

We obtain estimated state variables, written following as:

$$\begin{cases} \hat{M}_{21}(k\,|\,k)=\hat{M}_{21}(k\,|\,k-1)+l_1(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1));\\ \hat{\omega}_2(k\,|\,k)=\hat{\omega}_2(k\,|\,k-1)+l_2(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1));\\ \hat{M}_{32}(k\,|\,k)=\hat{M}_{32}(k\,|\,k-1)+l_3(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1));\\ \hat{\omega}_3(k\,|\,k)=\hat{\omega}_3(k\,|\,k-1)+l_4(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1));\\ \hat{M}_{42}(k\,|\,k)=\hat{M}_{42}(k\,|\,k-1)+l_5(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1));\\ \hat{\omega}_4(k\,|\,k)=\hat{\omega}_4(k\,|\,k-1)+l_6(\omega_3(k\,|\,k)-\hat{\omega}_3(k\,|\,k-1)). \end{cases} \qquad (11)$$

IV. Simulation Results

The simulation of state variables observer based DEKF is carried out in the MATLAB/Simulink environment. Its parameters are presented in Table I. Electromechanical parameters of the RT-70 antenna electric drive were detail presented in [1].

In this paper, study of estimation quality is investigated in the closed-loop control structure at very low speed servo tracking, corresponding to mode of motion “slow tracking” of the RT-70 antenna (Fig. 1). The state variables need be estimated as angular speeds of the second, the third and the fourth masses ($\hat{\omega}_2,\hat{\omega}_3,\hat{\omega}_4$) and torsional torques between masses ($\hat{M}_{21},\hat{M}_{32},\hat{M}_{42}$).

TABLE I. DEKF Observer Parameters

Parameter	Symbol	Value
Sampling instant, sec	T_s	0.001
Weight covariance matrix of state variables	$\mathbf{Q}_{\text{н}}(k)$	$diag[0.05]_{6\times 6}$
Weight covariance matrix of measurement	$\mathbf{R}_{\text{н}}(k)$	2.5

The simulation results of estimating state variables using optimal observer based on DEKF are illustrated in Fig. 2, 3, 4, where: $1-M_{21}$; $1'-\hat{M}_{21}$; $2-\omega_2$; $2'-\hat{\omega}_2$; $3-M_{32}$; $3'-\hat{M}_{32}$; $4-\omega_3$; $5-M_{42}$; $5'-\hat{M}_{42}$; $6-\omega_4$; $6'-\hat{\omega}_4$; $7-\omega_r$.

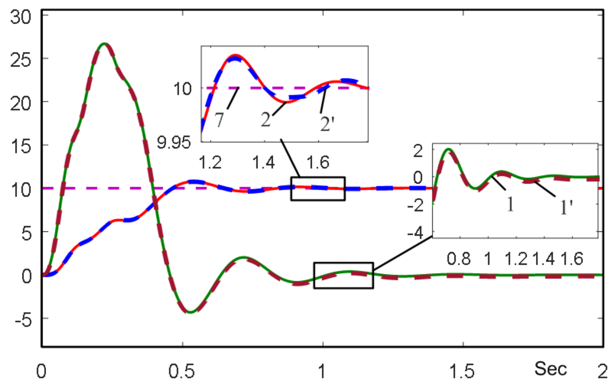

Fig. 2. Simulated transients of the real and estimated state variables of the second mass

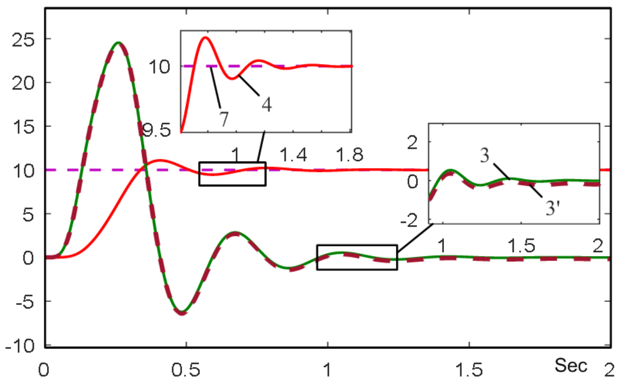

Fig. 3. Simulated transients of the real and estimated state variables of the third mass

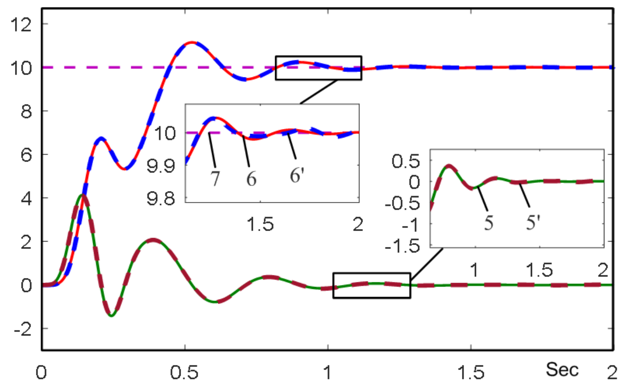

Fig. 4. Simulated transients of the real and estimated state variables of the fourth mass

V. Conclusion

In this paper, a methodology of discrete extended Kalman filter was investigated in application for nonlinear control system. The electrical drive system of the large RT with elastic connections and nonlinear dry friction torque was considered. The simulation results indicated that proposed optimal observer based on DEKF can be applied to retrieve non-measurable mechanical state variables in electromechanical control system.

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