Impact of Imperfect CSI on Capacity of ISI MIMO Systems Based on Joint Precoding and Equalization Designs

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***Abstract*—It is clear that a channel state information (CSI) influences performance of communication systems. Therefore, researching on impact of the CSI on the system performance and developing new techniques for the state of the art communication systems under different CSI conditions attract the great attention of researchers. In this paper, we analyze the performance of system which shares redundancy and combines designs of precoder and equalizer. Furthermore, the proposed system is evaluated based on imperfect CSI at the receiver and calculation results have proved the strong impact of channel estimation error on the system capacity.**

***Index Terms*—Channel state information, Minimum mean square error, Precoder, Equalizer, Redundancy, ISI MIMO system, Block transmission, Leading zero, Trailing zero**

I. Introduction

In multi-path propagation environment, block transmission systems may meet with requirements of advanced communications because of their high data speeds, relatively low cost and high-quality services. However, due to existence of inter symbol interference (ISI), guard intervals in form of the cyclic prefix (CP) or zero padding (ZP) are added to eliminate the ISI, leading to a decrease in spectral efficiency, especially for channels with long impulse response [1], [2].

To avoid a drawback of the spectral efficiency and improve the performance of systems, there have been many solutions that have received the great attention from researchers around the world. For instance, in order to cancel the ISI, a redundancy is utilized instead of guard intervals during the transmission [3]–[5]. In addition, a number of solutions were proposed based on precoding techniques or combinations of precoder and equalizer for multiple-input multiple-output (MIMO) channels [6]–[11].

It can be well observed that all of the above mentioned solutions have assumed that the channel state information (CSI) can be perfectly obtained. However, in practical scenarios, it is absolutely hard to aim the perfect CSI owing to a channel estimation error, feedback delay and finite rate channel quantization. In order to investigate and discuss about the CSI matter, there were some papers that develop precoder or joint precoder and equalizer designs according to the CSI at the transmitter, receiver or transceiver.

Firstly, in [12]–[16], the authors proposed and analyzed many approaches with the impact of imperfect CSI at the transmitter. For example, in [12], the authors demonstrated the impact of imperfect CSI on the MIMO system using Dirty Paper Coding and beamforming techniques. In [14], [15], the authors described the calculation of mutual information for multi-path channel by precoding technique under MMSE criterion. Secondly, in case the CSI is imperfect at the receiver because of the channel estimation error [17], [18], the authors focused on analyzing the effect of the CSI on the capacity of MIMO channels. Finally, when the imperfect CSI happens at both transmitter and receiver, combinations of the linear precoder and decoder to minimize total mean-square error (MSE) of transceiver for the MIMO systems is investigated in [19], [20]. As a further advance, the authors considered the impact of the CSI on resource allocation of base station used techniques as transmit antennas selection, beamforming, power allocation and time division protocol [21].

In this paper, we concentrate on combining designs of precoder and equalizer, which utilize the redundancy, for the ISI MIMO systems. In our proposed design, a combination of precoder and equalizer is performed based on sharing redundancy for both transmitter and receiver. The redundancy is generally understood as the length of the guard intervals as in [5]. After that, the impact of the imperfect CSI at the receiver on the capacity of these systems is evaluated and discussed.

The rest of the paper is organized as follows: In Section 2, we introduce the system model with imperfect CSI, whereas in Section 3, we demonstrate combining designs of the precoder and equalizer for the ISI MIMO channels and then analyze the impact of the imperfect CSI on the system capacity. The simulation results are given in Section 4, and conclusions are presented in Section 5. The notations used in this paper are as follows: boldface font is used for vector and matrix; A set of complex numbers

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is denoted by symbol $\mathbb{C}$; $(\cdot)^*$ and $(\cdot)^T$ are the Hermitian transpose and transpose operation, respectively.

II. System Model

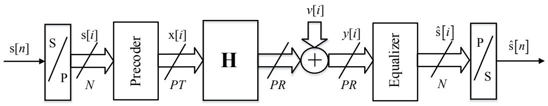

Fig. 1. The system model of precoding and equalization for ISI MIMO channel.

In this paper, we consider the system model of block transmission based on joint precoding and equalization design for ISI MIMO channels illustrated in Fig.1. The ISI MIMO channel with T transmit and R receive antennas is assumed to be stationary, frequency selective fading and has finite impulse response (FIR) with order of L. The channel impulse response (CIR) is given by matrices $\mathbf{H}[0], \mathbf{H}[1], ..., \mathbf{H}[L]$, where $\mathbf{H}[l] \in \mathbb{C}^{T\times R}, (l = 0, ..., L)$.

A single stream of input symbols $s[n]$ is sent to the serial-to-parallel converter, and then converted in to vectors $s[i]$ with the size of $N \times 1$. After that, the precoder generates symbol vectors $x[i]$ with P vectors in which each has the size of $T \times 1$, and then transmitted through the ISI MIMO channel. At the receiver site, the serial-to-parallel converter forms symbol vectors $y[i]$ with the size $PR \times 1$ from P received symbol vectors, and then the vectors $y[i]$ are sent to the equalizer. The equalizer regenerates symbol vectors $\hat{s}[i]$ with the size of $N \times 1$.

$$\mathbf{s}[i] = [s[iN], s[iN{+}1], \dots, s[iN{+}N{-}1]]^T \quad (1)$$
$$\mathbf{x}[i] = [x[iPT], x[iPT{+}1], \dots, x[iPT{+}PT{-}1]]^T \quad (2)$$
$$\mathbf{y}[i] = [y[iPR], y[iPR{+}1], \dots, y[iPR{+}PR{-}1]]^T \quad (3)$$
$$\hat{\mathbf{s}}[i] = [\hat{s}[iN], \hat{s}[iN{+}1], \dots, \hat{s}[iN{+}N{-}1]]^T \quad (4)$$
$$\mathbf{v}[i] = [v[iPR], v[iPR{+}1], \dots, v[iPR{+}PR{-}1]]^T \quad (5)$$

We assume that $\mathbf{v}[i] \backsim CN(0, 1)$ represents the additive white Gaussian noise (AWGN) samples vector with the size of $PR \times 1$.

III. Performance analysis

In this work, we consider multi-path MIMO channels with a dispersing structure. Moreover, we adopt a narrow-band channel model according to the extended Saleh-Valenzuela model [22]. Based on conventional ideas in [3], [4], when $P \geq L$, the symbol vectors $\hat{\mathbf{s}}[i]$ are given by

$$\hat{\mathbf{s}}[i] = \mathbf{G}\bar{\mathbf{H}}_0\mathbf{F}\mathbf{s}[i] + \mathbf{G}\bar{\mathbf{H}}_1\mathbf{F}\mathbf{s}[i-1] + \mathbf{G}\mathbf{v}[i] \quad (6)$$

where $\mathbf{F} \in \mathbb{C}^{PT\times N}$ and $\mathbf{G} \in \mathbb{C}^{N\times PR}$ respectively express the precoder, the equalizer, and are expressed by equations (7) and (8).

Furthermore, $\mathbf{H}_0$ and $\mathbf{H}_1$ are used to define the lower and upper Toeplitz matrices with the same size of $PR \times PT$ and they can be illustrated by

$$\bar{\mathbf{H}}_0 = \begin{bmatrix} \bar{\mathbf{H}}[0] & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \bar{\mathbf{H}}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \bar{\mathbf{H}}[L] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \bar{\mathbf{H}}[L] & \cdots & \bar{\mathbf{H}}[0] \end{bmatrix}, \quad (9)$$

$$\bar{\mathbf{H}}_1 = \begin{bmatrix} \mathbf{0} & \cdots & \bar{\mathbf{H}}[L] & \cdots & \bar{\mathbf{H}}[1] \\ \vdots & \ddots & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \cdots & \ddots & \cdots & \bar{\mathbf{H}}[L] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}. \quad (10)$$

The second term in the right side of the equation (6), $\mathbf{G}\mathbf{H}_1\mathbf{F}\mathbf{s}[i-1]$, plays the role of the ISI effect in the ISI MIMO system model as in Fig. 1. In order to cancel the term of ISI, there are two basic approaches called as trailing zero (TZ) and leading zero (LZ) [3], [4].

In fact, the TZ and LZ methods show almost the same performance. In this paper, we focus on the TZ method. In this method, while the equalizer is unchanged ($\mathbf{G}_{TZ} = \mathbf{G}$), the last LT rows of precoder $\mathbf{F}$ are set to zero and then given by following equation.

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_0[0] & \mathbf{F}_1[0] & \cdots & \mathbf{F}_{N-1}[0] \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_0[M{-}1] & \mathbf{F}_1[M{-}1] & \cdots & \mathbf{F}_{N-1}[M{-}1] \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (11)$$

where the $\mathbf{F}_{TZ} \in \mathbb{C}^{(P-L)T\times N}$ is defined as equation (12).

After that, the precoder $\mathbf{F}_{TZ}$ and the equalizer $\mathbf{G}_{TZ}$ are jointly designed by either the zero-forcing (ZF) or the minimum mean square error (MMSE) criteria in order to improve the system performance.

As a result, when the ISI is eliminated completely meaning the term $\mathbf{G}\mathbf{H}_1\mathbf{F}\mathbf{s}[i-1]$ equals to zero, the equation (6) can be rewritten as follows.

$$\hat{\mathbf{s}}[i] = \mathbf{G}_{TZ}\bar{\mathbf{H}}\mathbf{F}_{TZ}\mathbf{s}[i] + \mathbf{G}_{TZ}\mathbf{v}[i]. \quad (13)$$

where $\bar{\mathbf{H}}$ consists of the first $(P-L)T$ columns of $\bar{\mathbf{H}}_0$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_0[0] & \mathbf{F}_1[0] & \cdots & \mathbf{F}_{N-1}[0] \\ \mathbf{F}_0[1] & \mathbf{F}_1[1] & \cdots & \mathbf{F}_{N-1}[1] \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_0[PT-1] & \mathbf{F}_1[PT-1] & \cdots & \mathbf{F}_{N-1}[PT-1] \end{bmatrix}, \quad (7)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_0[0] & \mathbf{G}_1[0] & \cdots & \mathbf{G}_{(PR-1)}[0] \\ \mathbf{G}_0[1] & \mathbf{G}_1[1] & \cdots & \mathbf{G}_{(PR-1)}[1] \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{G}_0[N-1] & \mathbf{G}_1[N-1] & \cdots & \mathbf{G}_{(PR-1)}[N-1] \end{bmatrix} \quad (8)$$

$$\mathbf{F}_{TZ} = \begin{bmatrix} \mathbf{F}_0[0] & \mathbf{F}_1[0] & \cdots & \mathbf{F}_{(N-1)}[0] \\ \mathbf{F}_0[1] & \mathbf{F}_1[1] & \cdots & \mathbf{F}_{(N-1)}[1] \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_0[M-1] & \mathbf{F}_1[M-1] & \cdots & \mathbf{F}_{(N-1)}[M-1] \end{bmatrix} \quad (12)$$

and is given by

$$\bar{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{H}}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \bar{\mathbf{H}}[\mathrm{L}] & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \bar{\mathbf{H}}[0] \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \bar{\mathbf{H}}[\mathrm{L}] \end{bmatrix} \quad (14)$$

It is clear that the use of guard intervals or redundancy of system can help to eliminate the ISI interference in the frequency selective channels. However, the last LT columns of the $\bar{\mathbf{H}}_0$ matrix is lost, hence a part of channel energy is lost. To overcome this issue, we proposed a joint precoding and equalization design based on the MMSE criterion [9]. The key idea of the proposed design is that the redundancy is shared for both transmitter and receiver, consequently, this can reduce the loss of channel energy and improve the performance of system. The precoder and equalizer are designed as follows.

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{opt} \\ \mathbf{0}_{KT\times N} \end{bmatrix}, \quad (15)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{N\times(L-K)R} & \mathbf{G}_{opt} \end{bmatrix}, \quad (16)$$

where $\mathbf{F}_{opt} \in \mathbb{C}^{(P-K)T\times N}$, $\mathbf{G}_{opt} \in \mathbb{C}^{N\times(P-L+K)R}$, and $K = L/2$.

When the ISI is completely cancelled, the equation (6) can be rewritten as

$$\hat{\mathbf{s}}[i] = \mathbf{G}_{opt}\hat{\mathbf{H}}\mathbf{F}_{opt}\mathbf{s}[i] + \mathbf{G}_{opt}\mathbf{v}'[i] \quad (17)$$

where the $\hat{\mathbf{H}}$ is defined as

$$\hat{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{H}}[L-K] & \cdots & \bar{\mathbf{H}}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & \ddots & \ddots & \vdots \\ \bar{\mathbf{H}}[L] & & & & \ddots & \mathbf{0} \\ \mathbf{0} & \ddots & & & & \bar{\mathbf{H}}[0] \\ \vdots & \ddots & \ddots & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \bar{\mathbf{H}}[L] & \cdots & \bar{\mathbf{H}}[K] \end{bmatrix}. \quad (18)$$

and $\mathbf{v}'[i]$ is the block of noise samples with the length of $(P-L+K)R$.

In this paper, we assume that the transmitter has the perfect CSI while the receiver has the imperfect one. Consequently, the channel matrix estimated by the receiver can be modeled as [18].

$$\tilde{\mathbf{H}} = \rho\hat{\mathbf{H}} + \sqrt{1-\rho^2}\hat{\mathbf{H}}_{\mathrm{e}} \quad (19)$$

where $\hat{\mathbf{H}}$ is the true channel matrix and $\hat{\mathbf{H}}_{\mathrm{e}}$ denotes the error matrix with a complex Gaussian distribution, $\hat{\mathbf{H}}_{\mathrm{e}} \in CN(0,1)$. These maxtrices have the size of $(P-L+K)R \times (P-K)T$, and ρ illustrates the accuracy of channel estimation, $0 \leq \rho \leq 1$. Moreover, the transmit power is assumed to be constrained to p_0. The optimal precoder $\mathbf{F}_0$ and equalizer $\mathbf{G}_0$ matrices are designed under the MMSE criterion and expressed by following equations [9].

$$\mathbf{F}_{opt} = \tilde{\mathbf{V}}\tilde{\mathbf{\Phi}}_{\mathbf{f}}, \quad (20)$$

$$\mathbf{G}_{opt} = \tilde{\mathbf{\Phi}}_{\mathbf{g}}\tilde{\mathbf{V}}^*\left(\rho\hat{\mathbf{H}} + \sqrt{1-\rho^2}\hat{\mathbf{H}}_{\mathrm{e}}\right)^*\mathbf{R}^{-1}_{\mathbf{v}'\mathbf{v}'}, \quad (21)$$

where $\tilde{\mathbf{V}}$, $\tilde{\mathbf{\Lambda}}$ are matrices derived by the eigenvalue decompositions (EVD) algorithm.

$$\left(\rho\hat{\mathbf{H}} + \sqrt{1-\rho^2}\hat{\mathbf{H}}_{\mathrm{e}}\right)^* \mathbf{R}_{\mathbf{v}'\mathbf{v}'}^{-1} \times \left(\rho\hat{\mathbf{H}} + \sqrt{1-\rho^2}\hat{\mathbf{H}}_{\mathrm{e}}\right) = \tilde{\mathbf{V}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{V}}^* \tag{22}$$

and the $\tilde{\mathbf{\Phi}}_{\mathbf{f}}$ and $\tilde{\mathbf{\Phi}}_{\mathbf{g}}$ are given by

$$\left|\tilde{\phi}_{\mathrm{f},ii}\right|^2 = \left[\frac{p_0 + \sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1}\right)}{\sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1/2}\right)}\tilde{\lambda}_{ii}^{-1/2} - \tilde{\lambda}_{ii}^{-1}\right], \tag{23}$$

$$\left|\tilde{\phi}_{\mathrm{g},ii}\right|^2 = \left\{\frac{p_0 + \sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1}\right)}{\sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1/2}\right)}\tilde{\lambda}_{ii}^{-1/2} - \left[\frac{\sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1/2}\right)}{p_0 + \sum_{j=1}^{k}\left(\tilde{\lambda}_{jj}^{-1}\right)}\tilde{\lambda}_{ii}^{-1/2}\right]^2 \tilde{\lambda}_{ii}^{-1}\right\}\tilde{\lambda}_{ii}^{-1}, \tag{24}$$

where $\tilde{\lambda}_{ii}$ is the main diagonal element of $\tilde{\mathbf{\Lambda}}$. As a result, signal to interference plus noise ratios (SINRs) of subchannel and system capacity can be calculated by following equations.

$$\mathrm{SINR}_i = \frac{\rho^2\left|\tilde{\phi}_{\mathrm{f},ii}\right|^2\hat{\lambda}_{ii}}{(2\rho\sqrt{1-\rho^2} + (1-\rho^2))\left|\tilde{\phi}_{\mathrm{f},ii}\right|^2\hat{\lambda}_{ii} + 1} \tag{25}$$

$$C = \sum_{i=1}^{i=N}\log_2\left(1 + \frac{\rho^2\left|\tilde{\phi}_{\mathrm{f},ii}\right|^2\hat{\lambda}_{ii}}{(2\rho\sqrt{1-\rho^2} + (1-\rho^2))\left|\tilde{\phi}_{\mathrm{f},ii}\right|^2\hat{\lambda}_{ii} + 1}\right) \tag{26}$$

where $N \leq rank(KR, KT)$, and $\hat{\lambda}_{ii}$ is the main diagonal of $\hat{\mathbf{\Lambda}}$ that can be derived from following equation.

$$\hat{\mathbf{H}}^*\mathbf{R}_{\mathbf{v}'\mathbf{v}'}^{-1}\hat{\mathbf{H}} = \hat{\mathbf{V}}\hat{\mathbf{\Lambda}}\hat{\mathbf{V}}^* \tag{27}$$

IV. Simulation Results

In order to evaluate the proposed design under the imperfect CSI condition, we develop the simulation scenario in case the ISI MIMO channel has three transmit antennas and three receive ones. The CIR is generated from the Saleh-Valenzuela indoor channel model as in [22]. The system capacity is evaluated according to the channel estimation accuracy ρ, FIR order L and transmission block size P. The system capacity of proposed design is theoretically calculated and compared with that of TZ designs.

First of all, the capacity of system with the proposed design is slightly higher than that of system with the TZ design in all analyzed cases as can be seen from Figs. 2, 3 and 4. Second, when the CSI condition becomes worse, the system capacity decreases. For instance, the system capacity decreases sharply when the CSI condition moves from perfect to imperfect. Moreover, when the channel estimation accuracy ρ reduces slightly, the system capacity decreases significantly in both designs. For detail, at the SNR equal to 25 dB in the Fig. 2, when the channel estimation accuracy ρ reduces from 0.98 to 0.9, the system capacity decreases from about 85 to 50 bit/s/Hz. Finally, the system capacity increases when the transmission block size rises, in contrast, it decreases when the FIR order rises in both designs as shown in Figs. 2, 3 and 4.

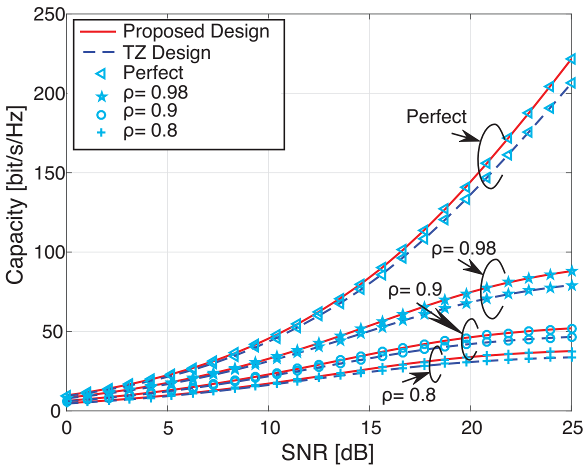

Fig. 2. System capacity of the proposed and TZ designs in case of L = 12 and P = 30

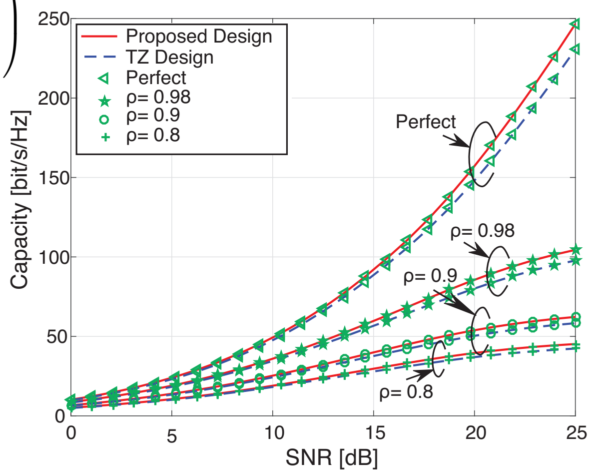

Fig. 3. System capacity of the proposed and TZ designs in case of L = 12 and P = 34

V. Conclusion

In this paper, we focus on analyzing and evaluating jointly precoder and equalizer designs based on the MMSE criterion in the imperfect CSI scenario at the receiver side. The simulation results show that the proposed design takes advantage of redundancy and produces better system capacity than that of the conventional design. In addition, the system capacity reduces remarkably when the CSI

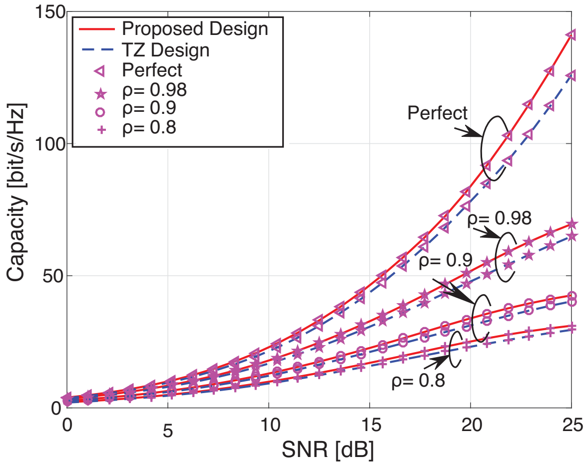

Fig. 4. System capacity of the proposed and TZ designs in case of $L = 14$ and $P = 30$

condition is imperfect. In the future, we hope to evaluate the proposed system performance in other imperfect CSI scenarios. Moreover, the system performance will be analyzed in different aspects as capacity, bit error rate and throughput.

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