Reflector Surface Shaping Method
for a Cassegrain Antenna

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Abstract—**Reflector surface shaping method for a dual reflector antenna has been developed to achieve an objective aperture distribution. The shaping method is composed of three differential equations. Two differential equations determine reflection laws on the main and sub reflectors. The third equation is relating to an objective aperture distribution. The third equation determines the main reflector coordinates. Other equation may be used to determine main reflector coordinates. In this paper, an equivalent parabola equation of a Cassegrain antenna is employed for the third differential equation. Through solving three differential equations by the developed MATLAB program, achievement of a dual reflector surfaces are ensured. The Obtained reflector surfaces become parabola and hyperbola at main and sub reflectors, respactively.**

Keywords—Dual reflector antenna, reflector shaping method, ray tracing method, equivalent parabola

I. INTRODUCTION

The reflector surface shaping of a dual reflector antenna was developed by V. Galindo et al [1],[2] to achieve an objective aperture distribution. Many types of aperture distributions such as taper and uniform distributions are achieved. The shaping method is based on the ray tracing method. The shaping method is composed of three differential equations. Two differential equations are based on reflection laws on the main and sub reflector surfaces. The third equation is relating to an aperture distribution. The third equation determines the main reflector coordinate. There may be other equation that determines the main reflector coordinate.

In this paper, for determination of the main reflector coordinate, an equivalent parabola equation of a Cassegrain antenna [3] is employed. In order to show adequateness of the new third equation, three differential equations are solved by developing MATLAB program [4]. The obtained reflector surfaces become parabola and hyperbola for main and sub reflectors, respectively. Coordinates of main and sub reflectors are compared with theoretical parabola and hyperbola equation results. Reflection laws on the main and sub reflectors are ensured. Equality of electrical path lengths from a feed to an aperture plane are ensured.

II. CONVENTIONAL REFLECTOR SHAPING METHOD

Antenna structure of a Cassegrain antenna is illustrated in Fig. 1 which consists of a feed, a parabolic main reflector and a hyperbolic sub reflector. The lines that start at the feed and enter the aperture are called rays. Reflector design method based on the following concepts are called ray taracing method. Rays on reflector surfaces satisfy reflection laws. Rays coming into apertures become parallel to the horizontal axis. Ray conditions are expressed by three crucial equations [4]:

1) Reflection law on the sub reflector
Relation of rays that enter at and get reflected from a reflector surface are expressed by the following equation:

$$\frac{dr}{d\theta} = r \tan \frac{\theta + \phi}{2} \qquad (1)$$

2) Reflection low on the main reflector
Relation of rays that enter at and get reflected from a reflector surface are expressed by the following equation:

$$\frac{dz}{d\theta} = \frac{dx}{d\theta} \tan \frac{\phi}{2} \qquad (2)$$

Here, rays are set parallel to the z axis after reflected on the main reflector as shown in Fig.1.

3) Energy conservation law
The relation of energy conservation within a small angle $d\theta$ and a small distance dx can be expressed by the next equation:

$$\frac{dx}{d\theta} = \frac{E_p^2(\theta) \int_0^{x_m} E_d^2(x)dx}{E_d^2(x)\int_0^{\theta_m} E_p^2(\theta)d\theta} \qquad (3)$$

4) Equality of electric pathe length
At all rays, electrical path lengths from the feed to aperture plane should be constant that is expressed by the next equation:

$$l(z)=r+z+(z-a+r\cos\theta)/\cos\phi=constant \qquad (4)$$

This condition coinsides with the situation of the parallel rays after reflected from the main reflector.

As for design parameters, the feed radiation pattern denoted by $E_p(\theta)$ is given by a simplified form shown by the next expression [4][5]:

$$E_p^2(\theta)_{given} = cos^q(\theta) \qquad (5)$$

The aperture distribution denoted by $E_d(x)$ is given by the next expression [4]:

$$E_d^2(x) = \left[1 - \left(\frac{x}{x_{max}}\right)^2\right]^p \tag{6}$$

As a result of the conventional shaping method, the three differential equations given by Eqs. (1) to (3) are solved based on design parameter given by Eqs. (4) to (6).
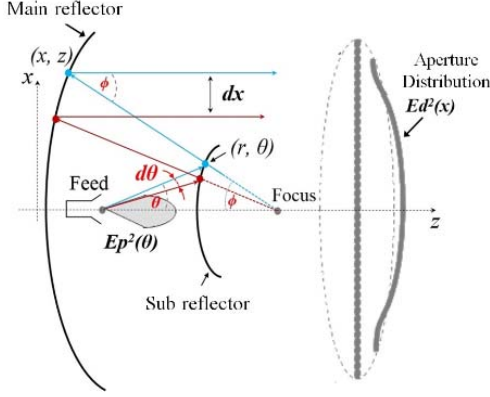

Fig. 1. Structure of a Cassegrain antenna

III. SHAPING METHOD BY EQUIVALENT PARABOLA

The Cassegrain antenna can be expressed by an equivalent parabola of a single reflector [5]. Relation of a Cassegrain antenna and an equivalent parabola is shown in Fig. 2
The equivalent parabola surface is expressed by the next equation.

$$x = 2 \sin\theta f_e / (1 + \cos\theta) \tag{7}$$

Where focal length (f_e) is given by the next equation:

$$f_e = \frac{(e+1)}{(e-1)} f \tag{8}$$

and the value of eccentricity (e) is given as follows:

$$e = \frac{\sin\frac{\phi+\theta}{2}}{\sin\frac{\phi-\theta}{2}} \tag{9}$$

The differential expression of Eq. (7) is given as follows:

$$\frac{dx}{d\theta} = 2 f_e \left(\frac{\sin\theta}{(\cos\theta+1)^2} + \frac{\cos\theta}{\cos\theta+1}\right) \tag{10}$$

As a result of this section, Eq. (10) can be used as a replacement of Eq. (3) in which determines relation of θ and x. The reflector shaping method composed of Eqs. (1), (2) and (10) is a new proposal.
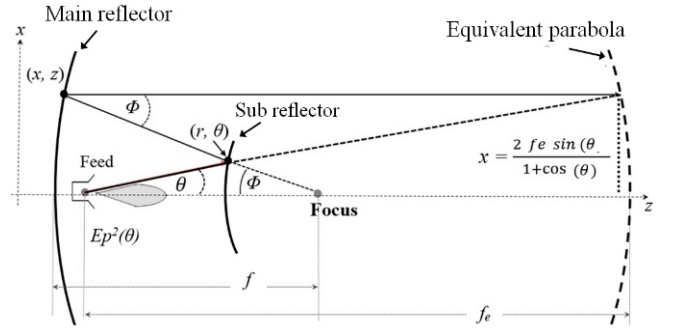

Fig. 2. Equivalent parabola for a Cassegrain antenna

IV. MATLAB PROGRAM

In order to ensure abilities of a proposed design method using Eqs. (1), (2) and (10), a MATLAB program is developed. The flow chart of a personal computer program is shown in Fig. 3. After setting initial points of reflectors, reflector surfaces are gradually determined by a small step of Δθ. Next, based on the obtained reflector coordinates, locus of rays is drawn.

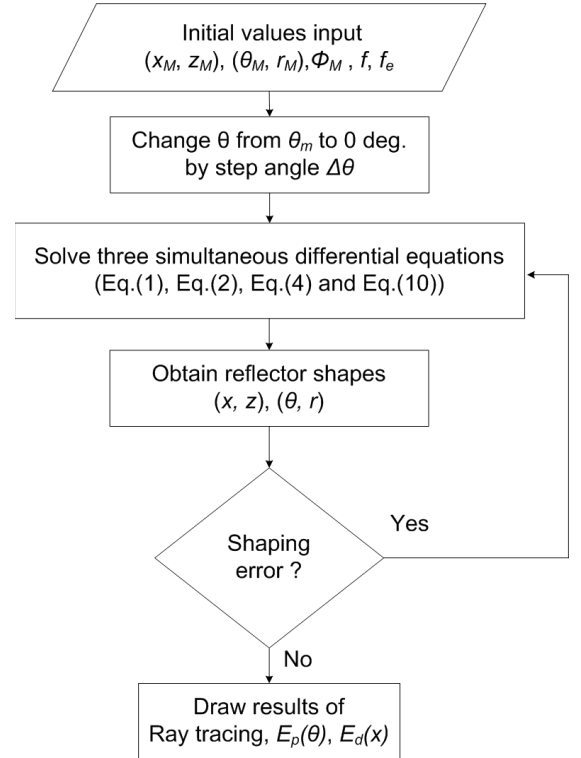

Fig. 3. Flow chart of MATLAB designing program

V. RESULTS OF REFLECTOR SHAPING

A. Design parameters

Antenna design parameters are summarized in Table I. The frequency of 28 GHz is selected, that will be used in the fifth

generation (5G) mobile system. Based on the definition of Fig.2 and Eq. (8), fe becomes 558mm.

TABLE I. INITIAL PARAMETERS

Parameters	Value	Unity
Frequency (*freq.*)	28	GHz
Diameter of main reflector (D_m)	300	mm
Diameter of main reflector (D_s)	36	mm
Main reflector focal length (f_m)	106	mm
Feed to focus length (*a*)	78	mm

ª mm = milimeter, GHz = Giga Hertz

The feed radiation pattern is shown in Fig.4. In this radiatin pattern, the edge levels of the sub reflector are designed to become -10dB and $q = 64$ is set in Eq. (5).
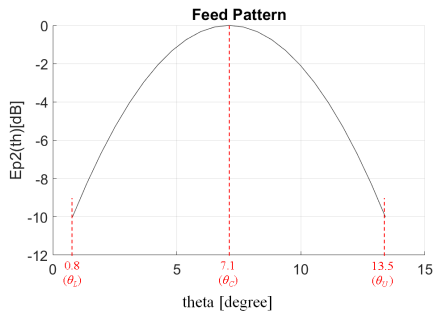
Fig. 4. Feed radiation pattern

B. Designed reflector surfaces

Designed reflector shapes and ray tracing results are shown in Fig. 5. Because the design step $\Delta\theta$ is set constant, ray separation on the aperture plane Δx becomes almost constant.
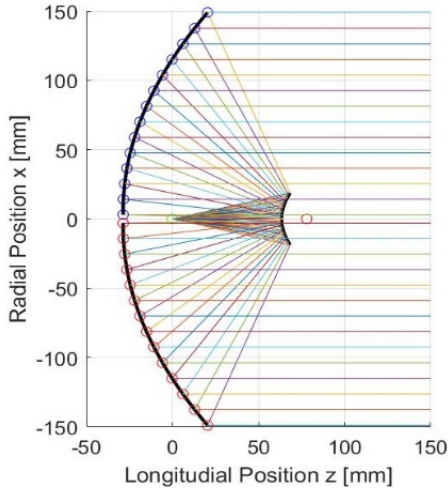

Fig. 5. Result of ray tracing

The aperture distribution $Ed^2(x)$ will be related to the horn radiation pattern $E_p^2(\theta)$ based on the electric energy conservation relations between rays. The $Ed^2(x)$ equation is given by the next expression.

$$E_d^2(x) = \frac{E_p^2(\theta)}{2 f_e \left(\frac{\sin\theta}{(\cos\theta+1)^2} + \frac{\cos\theta}{\cos\theta+1} \right)} \tag{11}$$

The aperture distribution results obtained by ray tracing results and Eq. (11) are shown in Fig. 6. Agreement of two results is very well.
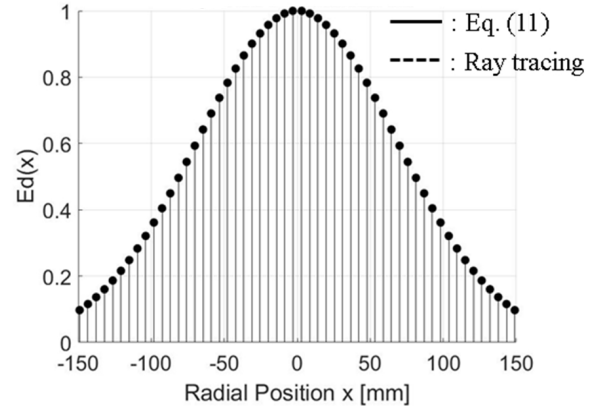

Fig. 6. Obtained aperture distribution

In order to investigate obtained reflector shapes, reflector coordinates were compared with the parabola and hyperbola data. The comparison of main reflector is shown in Table II. The comparison of main reflector is shown in Table III.

Here, the theoretical equation for parabola surface can be obtained by the following formula [5].

$$y^2 = 4 f_m x \tag{12}$$

Where, f_m symbolizes the focal length.
Meanwhile, the theoretical equation for hyperbola can be given by the next expression [5][6].

$$\frac{z_s^2}{a^2} - \frac{x_s^2}{c^2 - a^2} = 1 \tag{13}$$

Where, *a* and *c* values are related to the focus and vertex position of the hyperbola.

TABLE II. COMPARISON RESULTS OF MAIN REFLECTOR

Main Reflector				
Original Theoretically		Simulation		ΔM (X₀ - Xₛ)
Y_o	X_o	Y_s	X_s	
149,0	49,4	149,0	49,5	0,1
102,9	23,6	102,9	23,0	0,6
51,0	5,8	51,0	5,6	0,2
0,0	0,0	0,0	0,0	0,0

TABLE III. COMPARISON RESULTS OF SUB REFLECTOR

Sub Reflector				
Original Theoretically		Simulation		ΔS (X₀ₛ - Xₛₛ)
Y_{os}	X_{os}	Y_{ss}	X_{ss}	
18,3	89,0	18,3	89,5	0,5

12,6	86,4	12,6	86,7	0,3
5,8	84,4	5,8	84,3	0,1
0,0	0,0	0,0	0,0	0,0

By the results of Tables II and III, it can be concluded that the designed main and sub reflector become parabola and hyperbola, respectively.

C. Reflection law on reflector surfaces

In order to investigate accuracies of MATLAB program, the reflection law is examined on the designed main and sub reflectors.

The incident and reflected angles on the main and sub reflector surfaces are shown in Fig. 7. The incident and reflected rays are represented by i and r, respectively. n indicates the normal vector of the surface. The law of reflection can be expressed as follows [7].

$$i \cdot n = |i| \cdot |n| \cdot \cos \theta \tag{14}$$

$$r \cdot n = |i| \cdot |n| \cdot \cos \phi \tag{15}$$

where,

$$r = \frac{\overrightarrow{SM}}{|\overrightarrow{SM}|} \quad \text{and} \quad i = \frac{\overrightarrow{FS}}{|\overrightarrow{FS}|}$$

Vector SM is vector pointing from the main reflector surface to the sub reflector surface. The vector pointing from the sub reflector surface to the feed antenna is denoted by vector FS.
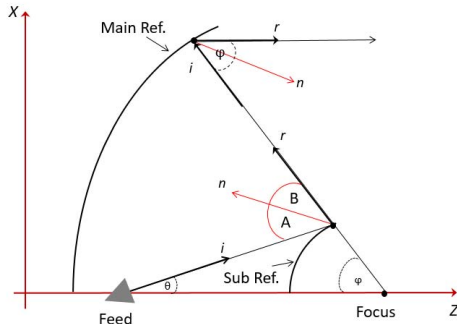

Fig. 7. Reflection at dual reflector surface

Fig. 8 shows the process to obtain the normal vector. The gradient value can be found using [8]:

$$m = \frac{X_1 - X_2}{Z_1 - Z_2} = \tan \theta \tag{16}$$

The gradient vector is orthogonal to the tangent line of any curve on the surface. The equation can be expressed as below.

$$-\frac{1}{m} = \cot \theta = \frac{X_f - X_2}{Z_f - Z_2} \tag{17}$$

The value of $|n|$ is always 1, hence the normal line of n can be defined as [8]:

$$n = \left(X_f - X_2\right)^2 + \left(Z_f - Z_2\right)^2 \tag{18}$$
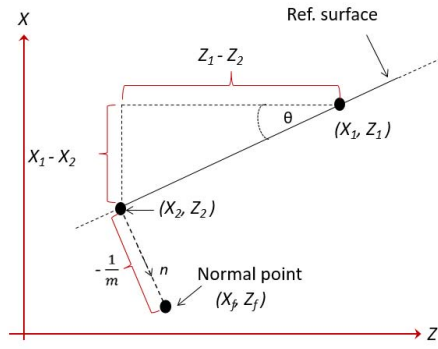

Fig. 8. Normal point on the reflector surface

Some points on the main reflector surfaces are shown in Figs. 9. Table IV provides comparison of theoretical calculation and MATLAB results. It is shown that the reflection laws are well accomplished on the main reflector.
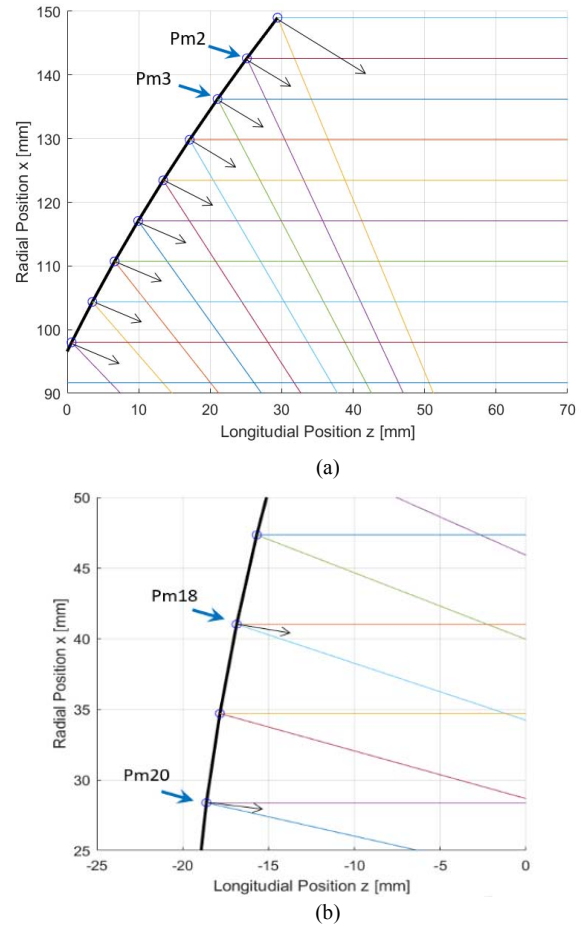

(a)

(b)

Fig. 9. Sample points on the main reflector surface

TABLE IV. COMPARISON RESULTS OF MAIN REFLECTOR POINT

Point (Pm$_n$)	Main Reflector Surface			
	MATLAB		Calculation	
	Angle 1 (degree)	Angle 2(degree)	Angle 1 (degree)	Angle 2 (degree)
Pm$_2$	32	32	31	32
Pm$_3$	30	30	29	29
Pm$_{20}$	10	10	10	9
Pm$_{18}$	5	6	6	6

Some points on the sub reflector surfaces are shown in Figs. 10. Table V provides comparison of theoretical calculation and MATLAB results. It is shown that the reflection laws are well accomplished on the sub reflector.
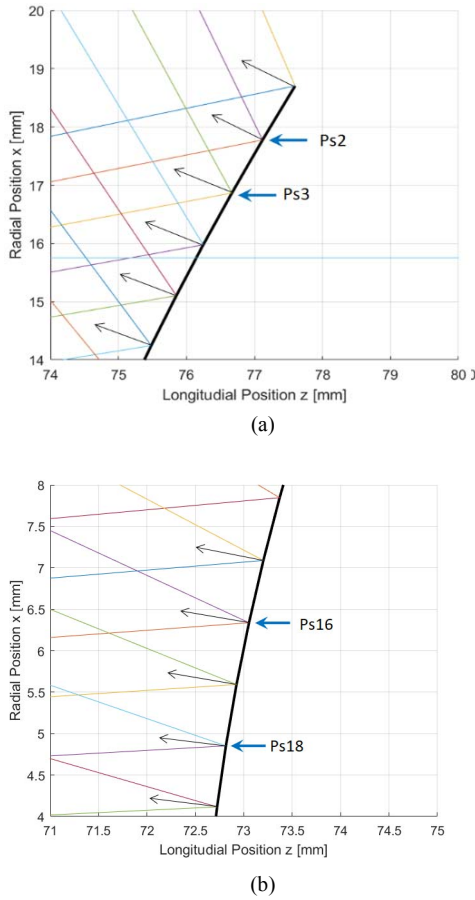

(a)

(b)

Fig. 10. Sample points on the sub reflector surface

TABLE V. COMPARISON RESULTS OF SUB REFLECTOR POINT

Point (Ps$_n$)	Sub Reflector Surface			
	MATLAB		Calculation	
	Angle 1 (degree)	Angle 2 (degree)	Angle 1 (degree)	Angle 2 (degree)
Ps$_2$	37	36	37	37
Ps$_3$	37	37	37	36
Pm$_{16}$	15	18	15	15
Pm$_{18}$	15	15	12	14

VI. CONCLUSION

New reflector shaping method for a Cassegrain antenna is proposed. The shaping method is composed of three differential equations. Two differential equations are based on reflection laws on the main and sub reflector surfaces. The third equation is relating to an aperture distribution. The third equation determines the main reflector coordinate. For the third equation, an equivalent parabolic reflector equation is applied. Through solving three differential equations by a developed MATLAB program, the main and sub reflector shapes can be successfully designed. Obtained main and sub reflectors become parabola and hyperbola, respectively. The usefulness of proposed design method is ensured by MATLAB program results.

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