

Radially Symmetric-Tangent Phase Mask to Obtain Invariant Imaging System to Defocus

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Abstract—In this paper, we propose a radially symmetric tangent phase mask to obtain the modulation transfer functions nearly symmetric through the in-focus plane and more invariant over a wide range of defocus. Imaging performance of the proposed phase mask is compared with the quartic phase mask and conventional imaging system based on the use of the evaluation functions, such as the modulation transfer function, the point spread function and the simulation images. The results demonstrated that the proposed phase mask has superior imaging performance in extending the depth of field.

Keywords— *Imaging systems, Computational imaging, Modulation transfer function, Depth of field, Point spread function*

I. INTRODUCTION

When the suitable phase mask is placed at the pupil plane, the depth of field of imaging system can be extended. This problem has been received much attention of researchers over the world because it can be used in many practical applications including aberration reducing [1], fluorescence microscopy [2], iris recognition [3]. Many phase masks to increase the depth of field have been introduced, such as the cubic phase mask [4], the tangent phase mask [5], the logarithmic phase mask [6], the square root phase mask [7], the exponential phase mask [8], the quartic phase mask (QPM) [9], the logarithmic asphere [10], the logarithmic axicon [11], diffraction hybrid lens [12] and so on. They can be divided in two types [13, 14]: asymmetrical phase masks including the cubic phase mask, the tangent phase mask, the logarithmic phase mask, the square root phase mask; radially symmetric phase masks consisting of the QPM, the logarithmic asphere, the logarithmic axicon, diffraction hybrid lens. For comparison between both the asymmetric phase masks and the radially symmetric phase masks under effect of defocus, the PSF of the asymmetric phase masks is more invariant over wide range of defocus than one of the radially symmetric phase masks. However, the PSF of the asymmetric phase mask widens over a wide area region and is asymmetric and hence, image artifacts and low signal to noise ratio (SNR) are two intrinsic problems due to the digital processing [15]. Whereas, the PSF of the radially phase masks is sharper and radially symmetric, and therefore, there are no image artifacts on the restored image while SNR maintaining.

The radially symmetric phase masks are generated by even functions and therefore, the modulation transfer functions (MTFs) of the radially symmetric phase masks at the positive and negative focus positions are not the same or the MTFs at the front and back focus plane are not the same when the defocused value is the same. This means that the defocused MTFs of the radially symmetric phase masks are not symmetric through the in-focus plane. This leads the defocused MTFs of the radially symmetric phase masks to be strong oscillation over a range of defocus which is from negative defocused value to positive defocused value. As a result, it is difficult to restore a real three-dimensional scene when only one filter kernel is used to deblur all recorded images. For more benefit in digital processing, in this paper, we proposed a radially symmetric-tangent phase mask to obtain the nearly symmetric defocused MTFs through the in-focus plane and to have the defocused MTFs more stable to defocus.

The organization of this paper is as follows. In Section 2, we propose a radially symmetric tangent phase mask to obtain the nearly symmetric MTF via the in-focus plane and carry out the optimization of phase masks. Section 3 shows a series of performance comparisons between proposed phase mask with the quartic phase mask and a traditional imaging system with the clear aperture. Finally, the conclusions are presented in Section 4.

II. RADIALLY SYMMETRIC-TANGENT PHASE MASK WITH OPTIMIZED PARAMETERS

In order to obtain the defocused MTFs nearly symmetric via the in-focus plane, in this paper, the proposed phase function of radially symmetric-tangent function is proposed as shown the following,

$$f(\rho, \theta) = \begin{cases} a\rho^4 \tan(\rho^2) & \text{if } \frac{m\pi}{2n} \leq \theta < \frac{(2m+1)\pi}{4n} \\ -a\rho^4 \tan(\rho^2) & \text{if } \frac{(2m+1)\pi}{4n} \leq \theta < \frac{(m+1)\pi}{2n} \end{cases}, (1)$$

where ρ is the normalized pupil radius and $0 \leq \rho \leq 1$; a is the mask parameter; θ is the angle ranging from 0 to 2π ; n is an integer; m is the integer ranging from 0 to $2n-1$.

In this paper, the modified quartic phase mask, as shown in [9], is used to compare imaging performance with the proposed phase mask and it can be presented by the following equation:

$$f(\rho) = a\rho^4 + b\rho^2, \quad (2)$$

where a and b are mask parameters.

Generally speaking, mask parameters of the phase masks should be firstly optimized before analysis and application. Several optimized methods to result in optimal mask parameters have been suggested, such as Fisher information, mean square error (MSE) of the MTF or the PSF and so on. These optimization models should satisfy two conditions: (1) imaging performance of wavefront coding system with the phase mask need to be invariant over a wide range of defocus; (2) the minimum acceptable magnitude of the MTF, which sure that the final image has high quality. MSE function of the MTF is a popular method which is chosen to evaluate stable level of the MTF to defocus. If the MSE of the MTF is equal to zero, all the defocused MTFs over the designed range of defocus are the same, showing that the modulation transfer function is absolutely invariant to defocus. According to Ref. 16, an optimization procedure based on MSE of the MTF to give mask parameters can be expressed by,

$$\begin{cases} \min \left[\sum_{\psi=-\psi_{max}}^{\psi_{max}} \sum_{u=-1}^1 |MTF(u, \psi) - MTF(u, \psi=0)|^2 \right] \\ \text{subject: } \begin{cases} \min(MTF) \geq TH_1 \\ \min(u_{cutoff}) \geq TH_2 \end{cases} \end{cases}, \quad (3)$$

where ψ is the defocus parameter and ψ_{max} is the maximum value of it. u is the spatial frequency. MTF is the modulation transfer function of wavefront coding imaging system. TH_1 is the minimum acceptable magnification of the modulation transfer function. u_{cutoff} is the cutoff frequency and TH_2 is the minimum acceptable value of the cutoff frequency.

To realize optimization of mask parameters, the starting parameters are $\psi_{max} = 10$, $TH_1 = 0.22$, $n = 6$ and $TH_2 = 0.4$. Form the optimization procedure as shown in Eq. (3) with the starting parameters above, the optimized mask parameters of the QPM are equal to $a = 28.35$, $b = -12.99$ and the optimal mask parameter of the proposed tangent phase mask is set to $a = 31.85$. The profiles of the QPM and tangent phase masks are shown in Fig. 2.

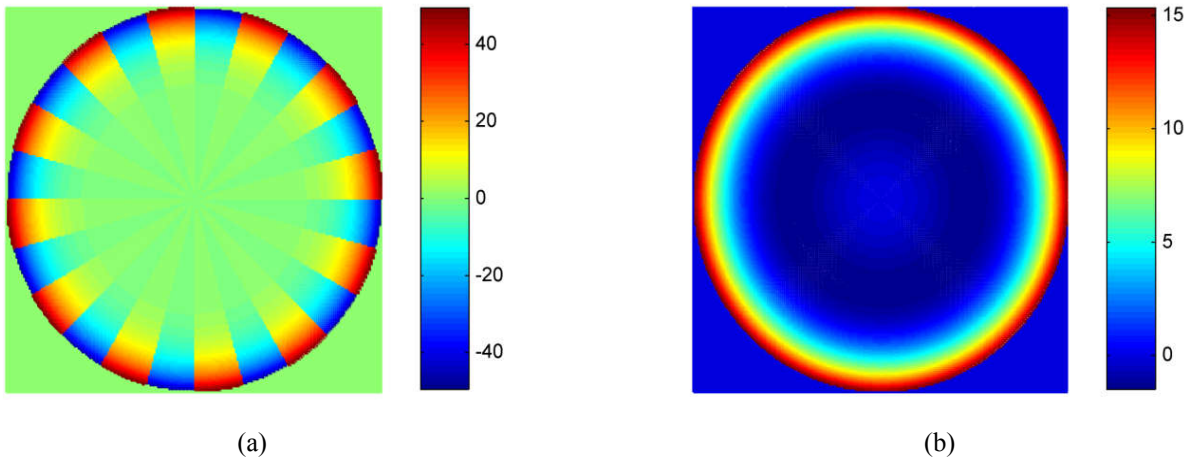


Fig. 1 Phase profiles of: (a) the tangent phase mask and, (b) the QPM.

III. IMAGING COMPARISON BETWEEN THE PHASE MASKS

With the above optimal mask parameters, the defocused MTF curves of the QPM and the tangent phase mask for five different values of defocus are shown in Fig. 2, where defocus parameter is set to $\psi = -10, -5, 0, 5$ and 10 . The corresponding defocused MTF curves of traditional imaging system are also indicated in Fig. 2. The defocused MTFs of the traditional imaging system are symmetric through the in-focus plane. The defocused MTFs of the traditional imaging system are sensitive to defocus. It is seen that the defocused MTF curves of the QPM and the tangent phase mask are less sensitive to defocus compared to the defocused MTF curves of traditional imaging system. The defocused MTFs of the tangent phase mask are nearly symmetric through the in-focus plane. The difference between the defocused MTFs of the before and after the in-focus plane is very small and it can be ignored. It can be seen that the defocused MTF curves of the QPM are more sensitive than that of the tangent phase mask. When only defocused MTF is employed as a deconvolution filter to

deblur all recorded images, the more invariant to defocus will has more benefit to obtain the actual status of scenery.

Based on the use of the above optimal phase mask parameters, the defocused PSFs of the QPM, the tangent phase mask and the traditional imaging system for four different values of defocus ($\psi = -10, -5, 5$ and 10) are shown in Fig. 3. As Fig. 3 indicates, the defocused PSFs of the traditional imaging system are symmetric via the in-focus plane and there is large difference between the defocused PSF at $\psi = -10$ or 10 and the PSF at $\psi = -5$ or 5 . The PSF at $\psi = -10$ or 10 is much wider than that at $\psi = -5$ or 5 . This means that the defocused PSFs of the traditional imaging system are sensitive to defocus. While, the defocused PSFs of both the QPM and the tangent phase mask are firmer. However, it is not difficult to see that the lobes of the PSFs of the tangent phase mask are more stable to defocus in comparison to the lobes of the PSFs of the QPM. This means that the proposed phase mask can be obtained better goal in extending depth-of-field.

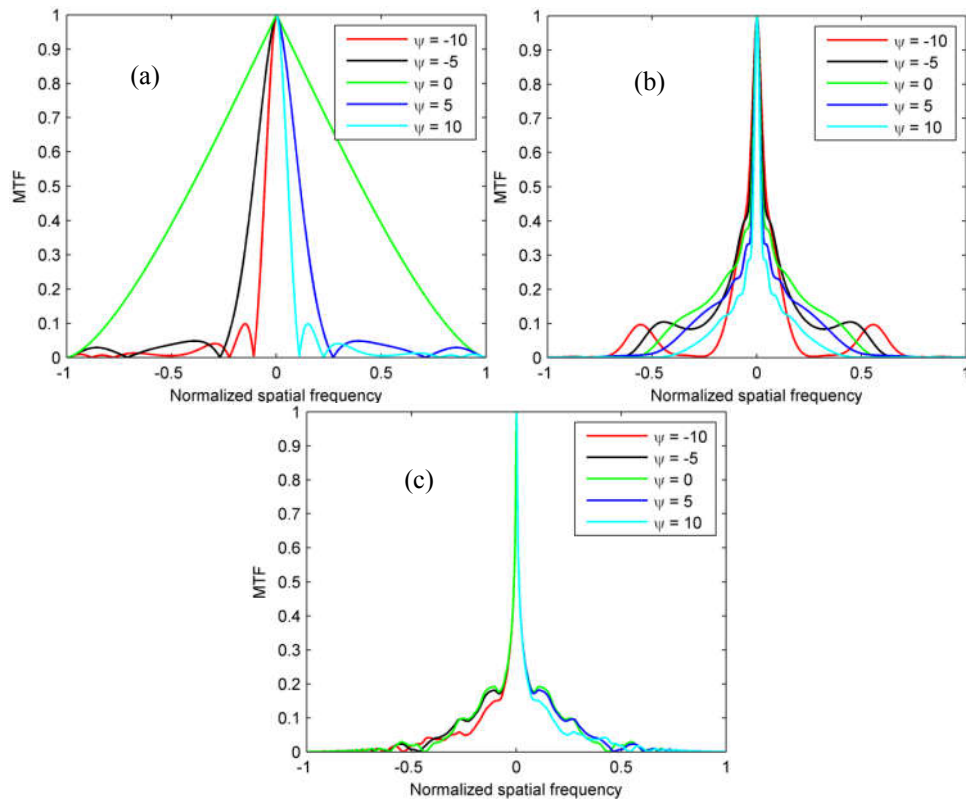


Fig. 2 The defocused MTF curves of: (a) traditional imaging system, (b) the QPM, and (c) the tangent phase mask. As shown in Fig. 2(c), the defocused MTFs of the tangent phase mask are nearly symmetric via the in-focus plane.

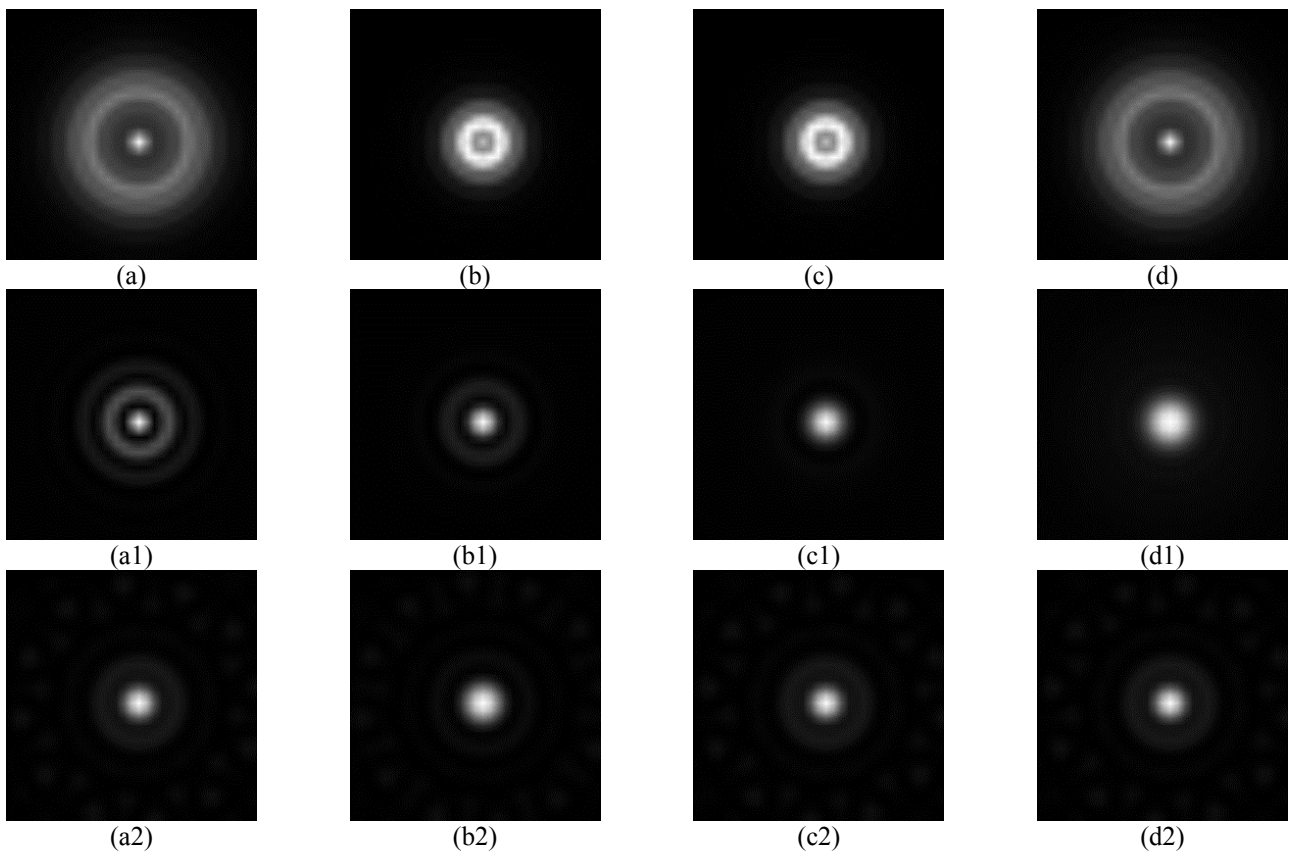


Fig. 3 The PSFs. Top to bottom: traditional imaging, the QPM, the tangent phase mask. The defocused value from left to right is equal to $\psi = -10$; $\psi = -5$; $\psi = 5$; $\psi = 10$. As Fig. 3 shows, the defocused PSFs of the tangent phase mask are more invariant over the range of defocus $[-10, 10]$.

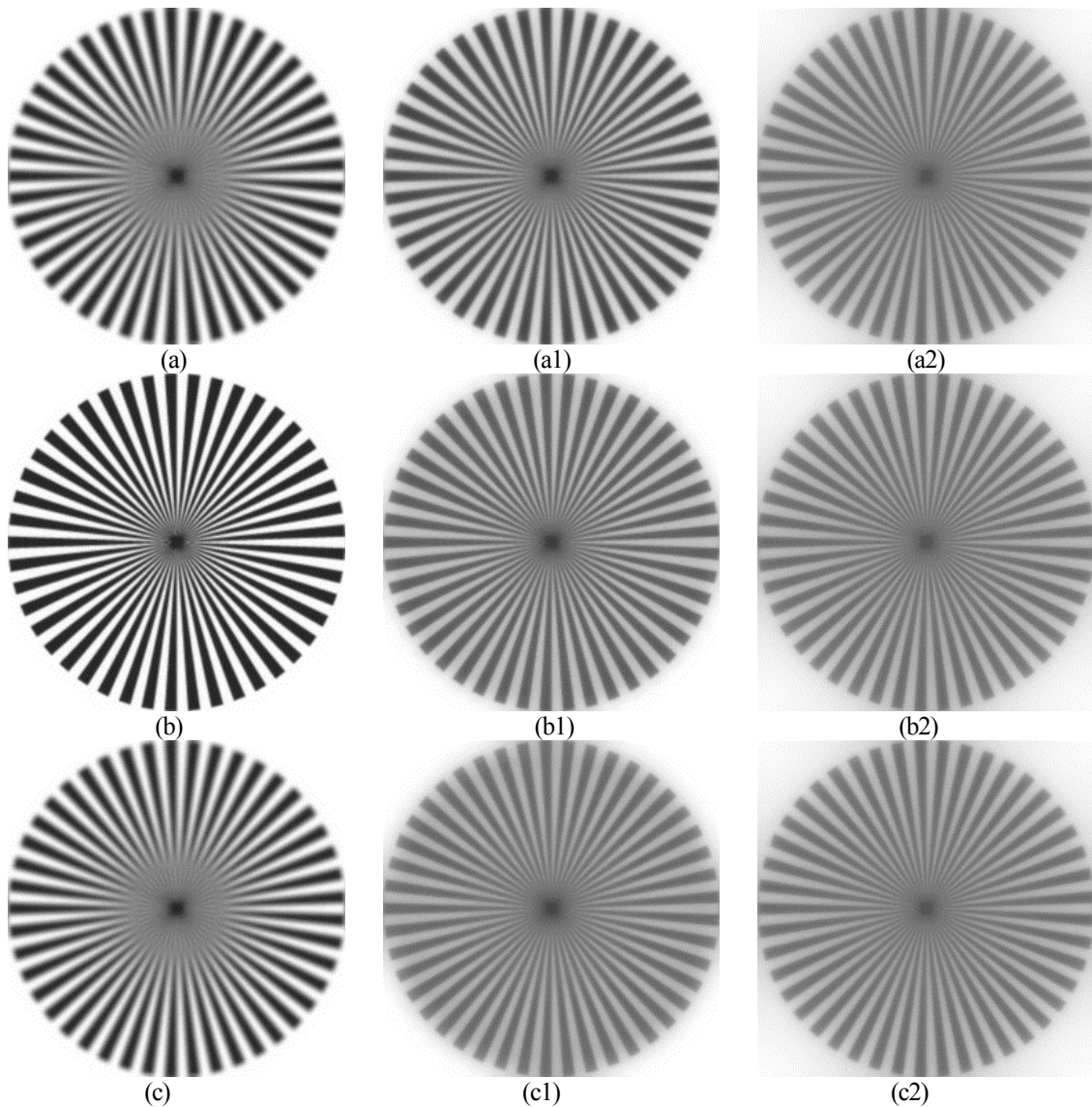


Fig. 4 Simulation images for spokes target. Left: traditional imaging system. Middle: the QPM. Right: the tangent phase mask. Rows: The defocused value from left to right is set $\psi = -10$; $\psi = 0$; $\psi = 10$.

Another important method to indicate the imaging performance of imaging system is to consider imaging with spokes target. The recorded images of the QPM and the tangent phase mask for three values of defocus are shown in Fig. 4. The corresponding recorded images of the traditional imaging system are also shown in Fig. 4. For a traditional imaging system, the spokes images are less invariant to defocus; for increasing defocus value, the edges of spokes image are more blurring. Additionally, as shown in Figs. 4(a) and 4(c), when the defocused value is set to $\psi = 10$ and -10 , the high information at high frequencies is lost. While, for the QPM and the tangent phase mask, the simulation images have less sensitivity to defocus and the edges of the recorded images are sharper. However, the recorded images of the tangent phase mask are more stable than one of the QPM. The QPM has different recorded images at different defocus values, for example, more blurred when defocus value is equal to $\psi = -10$, as shown in Fig. 4 (a1), and lower contrast when the defocus value is set to $\psi = 10$, as shown in Fig. 4(c1). It is not difficult to see that the recorded images of the tangent phase mask are nearly invariant to defocus, as shown in Figs. 4 (a2)-(c2).

IV. CONCLUSION

In this paper, a radially symmetric-tangent phase mask has been successfully designed, which is the modulation transfer functions nearly symmetric via the in-focus plane. Based on evaluation methods of the modulation transfer function, the point spread function and the simulation images, indicating that the proposed tangent phase mask can be used to achieve the significant improvement in imaging performance for extending the depth of field. The proposed phase mask can be applied in light sheet microscopy to extend the field of view. This 3D imaging method has strong development in the recent.

ACKNOWLEDGMENT

This work has been jointly supported by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under Grant number (103.03-2018.08).

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