The Effect of The Nozzle Ultimate Section Diameter on Interior Ballistics of HV-76 Trial Gun

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Abstract—The paper presents a system of differential equations to solve the interior ballistics of HV-76 trial gun [1], which uses the principle of high - low pressure weapon. The paper has researched the effect of the nozzle ultimate section diameter on the highest value of pressure in the high-pressure chamber as well as in the low-pressure chamber, and the muzzle velocity of the projectile fired from HV-76 trial gun to determine parameters of electronic fuse [7]. The results obtained of the paper were used as a scientific basis which helps to determine the reasonable parameter of the nozzle ultimate section diameter for design and manufacture of HV-76 trial gun as well as other types of high - low pressure weapon.

Keywords-shaped charge; nozzle; HV-76; high pressure chamber; low pressure chamber; muzzle velocity; fuze

I. INTRODUCTION

In order to improve the efficiency of destroying aerial targets, anti-aircraft artillery systems in the world today have been developing technologies which automatically transfer parameters to the fuze at the time of ammunition leaving the barrel. Ammunition will explode at a suitable distance from the target, using fragments and shock waves to destroy the target. This is a technology, a combination of existing antiaircraft fire systems with the development of radar, microchip, and computer technologies. The HV-76 trial gun [1] was used to test and evaluate the transmission of parameters of electronic fuze at the velocity of 250 m/s to 350 m/s. Previous research results [2] were obtained the interior ballistics to find out the dependence of the gas pressure on the velocity of the electronic fuze fired from the HV-76 test barrel.

However, results of previous study have not evaluated the effect the nozzle ultimate section diameter on the interior ballistics of the gun. The study of this effect in this paper is focused to select the suitable value of the nozzle ultimate section diameter for the design and manufacture of the HV-76 trial gun as well as the improvement, design, and manufacture new weapons based on the principle of high low pressure weapons.

II. DIFFERENTIAL EQUATIONS SYSTEM FOR SOLVING THE INTERIOR BALLISTICS OF HV-76 TRIAL GUN

Previous studies [1][2] have developed the model of the HV-76 trial gun, using the principle of high – low pressure weapon as shown in Fig. 1. The firing of the HV-76 trial gun begins when the firing pin hits the primer. The propellant is burning inside the weapon and increases the pressure in the high-pressure chamber reaches the pressure value enough to penetrate the nozzle and the nozzle is opened. The pressure gas is moving through the nozzle from the high pressure chamber to the low pressure chamber, increasing the pressure in this chamber. When the gas pressure in this chamber reaches the pressure required to move the projectile, the projectile starts to move in the barrel bore. The above analysis indicates that the pressure directly applied to the gun barrel was very small (dozens of times smaller than the value of the gas pressure in the high-pressure chamber). The main disadvantage of this system is the limited velocity of the projectile.



Figure 1. The trial gun HV-76

1. Propellant; 2. High pressure chamber; 3. Nozzle; 4. Coupling part; 5. Low pressure chamber; 6. Projectile with fuze; 7. Barrel hore.

- A. Assumptions of System of Differential Equations [3,6]:
 - The propellants only burn in the high-pressure chamber and the product composition burned does not change during the duration of the firing.
 - Propellants are burned according to assumptions of burn rules.
 - The burn rate is as follows: $u = u_1 p$. •
 - The flow of the gases is moving in one direction, and it is an adiabatic process.

- The projectile moves under the effect of the average pressure in the low pressure chamber.
- Ignore the heat loss in high pressure chamber and barrel.
- Ignore the gap between the projectile and the barrel, the projectile only moves forward.

B. System of Differential Equations for Interior Ballistics of HV -76 [3,5,6]

The firing process of high – low pressure gun includes the following periods:

- The burn process and creating product in the high pressure chamber.
- The process of ejecting gases from the high-pressure chamber to the low-pressure chamber through the nozzle.
- Energy exchange process between the high pressure and the low pressure chamber.
- The process of changing the gaseous state in the high-pressure chamber.
- Energy exchange process in the low pressure chamber.
- The process of changing the gaseous state in the low pressure chamber.
- The process of the projectile movement.

Examine the firing process at the time t, the system of differential equations is built as shown below:

• Systems equations of burn rate and gases:

$$\frac{dz}{dt} = \begin{cases} \frac{p_{c}}{I_{k}} & (0 < t \le t_{k}) \\ 0 & (t > t_{k}) \end{cases}$$
(1)
$$\frac{d\psi}{dt} = \begin{cases} \chi (1 + 2\lambda z + 3\mu z^{2}) \frac{dz}{dt} & (0 < t \le t_{k}) \\ 0 & (t > t_{k}) \end{cases}$$
(2)

where: p_c - pressure in the high pressure chamber; I_k impulse of the gas pressure during the time of the burning propellant; ψ - relative amount of propellant burned; χ , λ , μ - the geometric characteristics of propellant; z - relative thickness burned; t_k - at the time, when the propellant burned out.

• The equation of ejecting of gas through the nozzle:

when
$$p_{\rm c} \le p_{\rm mlp} : \frac{\mathrm{d}\eta}{\mathrm{d}t} = 0;$$
 (3)

- when $p_c > p_{mlp}$: depending on the value of the gas pressure in the high pressure chamber (p_c) and the pressure of the gas in the low pressure chamber (p_t), the gas product can be flowed from the high pressure chamber into the low pressure chamber or vice versa. From the assumption that propellants only burn in the high-pressure chamber, and the gas flow will only moves from the high-pressure chamber into the low-pressure chamber. The equation of determining the gas flow through the nozzle from high pressure chamber into the low pressure chamber has the form:

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \begin{cases} \frac{\varphi_2 \cdot K_0(k) \cdot F_{\mathrm{th}} \cdot p_{\mathrm{c}}}{\omega \sqrt{RT_{\mathrm{c}}}} & \text{when } \frac{p_{\mathrm{c}}}{p_{\mathrm{t}}} \ge \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \\ \frac{\varphi_2 \cdot F_{\mathrm{th}} \cdot p_{\mathrm{c}}}{\omega \sqrt{RT_{\mathrm{c}}}} \sqrt{\frac{2k}{k-1}} \left[\left(\frac{p_{\mathrm{t}}}{p_{\mathrm{c}}}\right)^{\frac{2}{k}} - \left(\frac{p_{\mathrm{t}}}{p_{\mathrm{c}}}\right)^{\frac{k}{k}+1}} \right] & \text{when } 1 \le \frac{p_{\mathrm{c}}}{p_{\mathrm{t}}} < \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \end{cases}$$
(4)

Where: $K_0(k) = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2k}{k+1}}$; η - gas ejecting

through the nozzle; k - exponent of gases adiabatic expansion; $F_{\rm th}$ - nozzle ultimate section area; ω - propellants mass; φ_2 - loss factor of gas flow; $T_{\rm c}$ - temperature of gas product in the high pressure chamber; $p_{\rm mlp}$ – penetration pressure of the nozzle.

• Equation of state of gas in the high pressure chamber:

Assuming that after time *t*, the relative amount of propellant burned is ψ , the relative gas ejecting through the nozzle is η . Then the amount of gas retained in the high pressure chamber at time *t* is $\omega(\psi - \eta)$. In the high pressure chamber using the equation of state of van der Waals:

$$p_{\rm c}.W_{\rm kh,c} = \omega (\psi - \eta) R T_{\rm c} \Rightarrow p_{\rm c} = \frac{\omega (\psi - \eta) R T_{\rm c}}{W_{\rm kh,c}}$$
 (5)

where: $W_{\rm kh,c}$ - volume of the high pressure chamber, $W_{\rm kh,c} = W_{\rm c,0} - \frac{\omega}{\delta}(1-\psi) - \alpha\omega(\psi-\eta)$; $W_{\rm c,0}$ - initial volume of high pressure chamber; α - covolume of powder gases; δ - powder density; R - gas constant.

• The energy equation of the high pressure chamber:

The energy equation of the high pressure chamber bases on the basis of thermodynamics [5]: $dQ = dU + \sum dL$. After time *dt*, relative burned propellant $d\psi$ creates the energy, $dQ = \omega d\psi C_y T_1$.

At the time t, the amount of gas retained in the high pressure chamber $\omega(\psi - \eta)$ has internal energy U : $U = \omega(\psi - \eta)C_{\rm v}T_{\rm c}$.

Where C_v - specific heat of gases at constant volume and $C_v = \frac{R}{k-1}$.

The variableness of the internal energy of the gases in the high pressure chamber after time *dt*:

$$dU = d[\omega(\psi - \eta)C_{v}T_{c}] = \omega C_{v}[(d\psi - d\eta)T_{c} + (\psi - \eta)dT_{c}]$$
$$= \omega \frac{R}{k-1}[(d\psi - d\eta)T_{c} + (\psi - \eta)dT_{c}].$$

After time dt, the gas outflow has the following energy equation:

$$dL = \omega d\eta C_{\rm p} T_{\rm c} = \frac{kR}{k-1} \omega d\eta T_{\rm c}$$

The heat transfer between gases and its surroundings can be ignore, and the equation of energy is:

$$\frac{\mathrm{d}T_{\mathrm{c}}}{\mathrm{d}t} = \frac{1}{\psi - \eta} \left[T_1 \frac{\mathrm{d}\psi}{\mathrm{d}t} - \left(\frac{\mathrm{d}\psi}{\mathrm{d}t} - \frac{\mathrm{d}\eta}{\mathrm{d}t}\right) T_{\mathrm{c}} - kT_{\mathrm{c}} \frac{\mathrm{d}\eta}{\mathrm{d}t} \right]. \tag{6}$$

• Equation of state of gas in the low pressure chamber:

At the time *t*, the projectile was moved in the low pressure chamber a distance I, amount of gases in the low pressure chamber is $\omega\eta$, the area of the low pressure chamber cross section is S_n , the volume of the low pressure chamber will be: $W_t = W_{t,0} + S_n l$.

Equation of state of gas for the low pressure chamber is:

$$p_{\rm t}.W_{\rm kh,t} = \omega \eta R T_{\rm t} \Rightarrow p_{\rm t} \frac{\omega \eta R T_{\rm t}}{W_{\rm kh,t}}$$
 (7)

where, $W_{\rm kh,t}$ - volume of gases in the low pressure chamber, $W_{\rm kh,t} = W_{\rm t,0} - \alpha \omega \eta + S_{\rm n} l$; $T_{\rm t}$ - gas temperature in the low pressure chamber; $W_{\rm t,0}$ - initial volume of the low pressure chamber before the firing.

• Equation of the projectile movement:

System of equations of the projectile movement is shown in (8) and (9):

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \begin{cases} 0 & (0 \le t \le t_{\mathrm{o}}) \\ v & (t > t_{\mathrm{o}}) \end{cases}$$
(8)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \begin{cases} 0 & (0 \le t \le t_{\mathrm{o}}) \\ \frac{S_{\mathrm{n}} p_{\mathrm{t}}}{\varphi m} & (t > t_{\mathrm{o}}) \end{cases}$$
(9)

where: φ - fictitious factor, and [5]: $\varphi = \varphi_1 \left(1 + \frac{1}{3} \frac{\omega \eta}{\varphi_1 m} - \frac{1}{3} \frac{u_a \cos \theta_0 + v}{a_0} \right); v, m$ - velocity, mas of

projectile; φ_1 - factor of kinetic energy; u_a - gaseous velocity at the nozzle; θ_0 - the tilt angle between the axis of nozzle and the axis of barrel bore; a_0 - speed of sound, $a_0 = \sqrt{kRT_1}$; p_0 - initial pressure; t_0 - at the time, the projectile begins to move, $p_t = p_0$.

• The energy equation of the low pressure chamber:

After time dt, amount of gas ejecting from the high pressure chamber to the low pressure chamber $\omega d\eta$ has energy: $\omega d\eta C_p T_c = \omega d\eta \frac{kR}{k-1}T_c$. This energy is used for the variableness of the internal energy and creates work.

The variableness of the internal energy in the low

pressure chamber is:

$$\mathbf{d} \left[\omega \eta C_{\mathbf{v}} T_t \right] = \omega C_{\mathbf{v}} (T_t \mathbf{d} \eta + \eta \mathbf{d} T_t) = \frac{R}{k-1} \omega (T_t \mathbf{d} \eta + \eta \mathbf{d} T_t)$$

The energy equation in the low pressure chamber is shown:

$$\frac{\mathrm{d}T_t}{\mathrm{d}t} = \frac{k-1}{\omega R\eta} \left[\omega \frac{R}{k-1} \frac{\mathrm{d}\eta}{\mathrm{d}t} (kT_\mathrm{c} - T_\mathrm{t}) - \varphi mv \frac{\mathrm{d}v}{\mathrm{d}t} \right]. \tag{10}$$

Combine equations (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), the system of differential equations for interior ballistics of HV-76 trial gun is built. Initial conditions of this system of equations discussed above at the time t = t0 =0 is: t = 0; v = 0; $T_c = T_{c,0}$; $T_t = T_{bd,c}$; l = 0; $p_{c,0} = p_{moi}$; $p_{t,0} = p_0$; $\eta = \eta_0$; $\Psi = \Psi_0$; $z = z_0$,

and:
$$\eta_0 = \frac{p_0 \cdot W_{t,0}}{RT_t \cdot \omega + \alpha \cdot \omega \cdot p_0}$$
,
 $\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_{c,0}} + \alpha - \frac{1}{\delta}}$, with density $\Delta = \frac{\omega}{W_{c,0}}$

 $z = z_0$, with z_0 is the solution of the equation: $\psi_o = \chi(z_o + \lambda z_o^2 + \mu z_o^3)$.

The system of differential equations of interior ballistics of the HV-76 trial gun can be fully solved by numerical method with Runge-Kutta algorithm. The results of this solution will be: the pressure in the high-pressure chamber, the pressure in the low-pressure chamber and the velocity of the fuze. Tab. I shows initial data for solving interior ballistics of HV-76 trial gun. In this case, the nozzle ultimate section diameter is used the value of $d_{th} = 8.17e^{-3}$ m.

Data	Unit	Value	
S_n	m ²	7.065.e ⁻⁴	
d _{th}	m	8.17e ⁻³	
F _{th}	m ²	5.24 e ⁻⁵	
la	m	0.7	
Wc,0	m ³	3.3e ⁻⁵	
W _{t.0}	m ³	1e ⁻⁵	
т	kg	0.15	
ω	kg	0.016	
p _{mlp}	MPa	12.936	
<i>p</i> ₀	MPa	0.985	
Ømoi	kg	0.0025	
T_1	К	2900	
f	J/kg	950000	
f moi	J/kg	180000	
α	m³/kg	0.001	
δ	kg/m ³	1600	
I_k	MPa s	340e ⁴	
χ	-	1.06	
λ	-	-0.0566	
μ	-	0	
k	-	1.2	
φ_1	-	1.2	

 TABLE I.
 INITIAL DATA FOR SOLVING INTERIOR BALLISTICS OF HV-76 TRIAL GUN

The pressure – trajectory curve and the velocity – trajectory curve of the fuze are shown in Fig. 2 and the Fig. 3 shows the pressure – time graph and the velocity – time curve of the fuze. Tab. II shows values of interior ballistics of HV-76 trial gun at several important positions.



Figure 2. Interior ballistics output - trajectory curve of the fuze

1. The pressure – trajectory curve in the high pressure chamber; 2. the pressure – trajectory curve in the low pressure chamber; 3. the velocity of the fuze.



Figure 3. Interior ballistics output - time curve of the fuze

TABLE II. VALUES OF THE TIME, TRAJECTORY, PRESSURE, TIME, AND THE VELOCITY OF PROJECTILE AT SEVERAL IMPORTANT POSITIONS

	<i>t</i> (10 ⁻³ s)	р _{стах} (MPa)	<i>p</i> tmax (MPa)	v (m/s)
Maximum value of <i>p</i> c	4.262	218.58	22.06	184.06
Maximum value of <i>p</i> t	4.366	199.16	23.16	193.24
propellant burned out	4.325	214.64	20.76	191.27
Projectile at the barrel muzzle	5.922	65.68	14.84	288.34

The pressure in the high-pressure chamber increases rapidly and reaches the maximum value at t = 0.004262 s, then this pressure value decreases rapidly because the nozzle is opened and the flow of gas is moving from the high-pressure chamber to the low-pressure chamber.

III. THE EFFECT OF THE NOZZLE ULTIMATE SECTION DIAMETER TO THE RESULTS OF INTERIOR BALLISTICS OF HV-76 TRIAL GUN

The maximum value of pressure in the high pressure chamber, in the low pressure chamber, and the muzzle velocity of the projectile are important parameters that are used for the weapon design. Therefore it is necessary to select a reasonable the nozzle ultimate section diameter so that the pressure in the high-pressure chamber and in the low pressure chamber are small and the velocity of the projectile reaches the greatest value.

From the selected parameters of the high-pressure chamber, the low pressure chamber, the type of the projectile and the propellant, the interior ballistics of HV-76 trial gun is solved with different values of d_{th} . The obtained results are the value of maximum pressure in the high pressure chamber, in low pressure chamber and the velocity of the projectile at the muzzle of the barrel when the nozzle ultimate section diameter changes. Based on these results, it is possible to select a nozzle ultimate section diameter. During the previous design [1], it was found that the nozzle

ultimate section diameter was 8.17e⁻³ m. Calculated with the parameters are given in Tab. I and the value of $d_{\rm th}$ changes from $d_{\rm th,min} = 4e^{-3}$ m to $d_{\rm th,max} = 9.8e^{-3}$ m, the received results are shown in Fig. 4, Tab. III.



Figure 4. Outputs of interior ballistics - d_{th} gragh

TABLE III.DATA OF THE PRESSURE AND VELOCITY DEPENDED ON
VALUES OF d_{th}

<i>d</i> _{th} .10 ⁻³ [m]	p _{cmax} [MPa]	P _{tmax} [MPa]	v _{max} [m/s]
4	501.25	23.21	260.92
5	432.6	25.46	278.23
6	364.46	25.95	288.68
6.2	324.14	25.85	290.69
7	300.06	25.54	291.97
7.2	280.14	25.36	291.64
7.6	268.76	25.06	289.12
8	228.26	23.14	289.10
8.5	198.15	21.68	284.96
9	168.59	20.27	280.14
9.8	124.95	17.12	268.11

When increasing the value of the nozzle ultimate section diameter, the maximum value of the pressure in the high pressure chamber p_{cmax} decreases rapidly and the maximum value of pressure in the low pressure chamber increases.

At the value of $d_{\rm th} = 5.8 {\rm e}^{-3}$ m, the pressure $p_{\rm tmax}$ reaches the maximum value (25.98MPa), and then the $p_{\rm tmax}$ decreases when the nozzle ultimate section diameter is greater than $5.8 {\rm e}^{-3}$ m.

The maximum velocity v_{max} of the projectile increases rapidly about from $d_{\text{th}} = 6.2e^{-3}$ m to $d_{\text{th}} = 7.6e^{-3}$ m and reaches the maximum value of velocity $v_{\text{max}} = 291.97$ m/s at the value of $d_{\text{th}} = 7.0e^{-3}$ m. When the value d_{th} is about from $6.2e^{-3}$ m to $7.6e^{-3}$, the value of v_{max} varies insignificantly.

When d_{th} is less than 7.0e⁻³ m, the maximum value of pressure in the high pressure chamber p_{cmax} is greater than

300 MPa (this value exceeds the the maximum pressure p_{cmax} , which is allowable when designing, [p_{cmax}]= 292 MPa [1]).

In addition to the maximum projectile velocity at the muzzle of the barrel, the material of high pressure chamber and low pressure chamber is selected so that their strength is sufficient and suitable for production technology.

The structure of the HV-76 trial gun has the size of the lower pressure chamber much larger than the high-pressure chamber size, thus the smallest value of the p_{tmax} that can be selected in the low pressure chamber is the preferred option. The optimal nozzle ultimate section diameter is selected so that the velocity of the projectile is great and the pressure in the low pressure chamber is small. The high pressure chamber can be made with current materials and technology. According to results in Tab. III and Fig. 4, the value of optimal nozzle ultimate section diameter is $d_{\rm th} = 7.2e^{-3}$ m. With the selected value of nozzle section diameter, the velocity of projectile at the muzzle of the barrel reaches the value of 291.64 m/s and the maximum value of the pressure in the high pressure chamber is less than 292 MPa. The maximum value of pressure in the high pressure chamber is 280.14 MPa and the maximum value of pressure in the low pressure chamber is 25.36 MPa. All results of the interior ballistics for HV-76 trial gun, using $d_{\text{th}} = 7.2e^{-3}$ m, are shown in Fig. 5. The gas pressure in the low pressure chamber is 10 times smaller than pressure in the high pressure chamber.



Figure 5. Interior ballistics output - time curve of the fuze

1. The pressure – time curve in the high pressure chamber; 2. the pressure – time curve in the low pressure chamber; 3. the velocity of the fuze.

IV. EXPERIMENT

Based on the calculation results of interior ballistics, the HV-76 trail gun was designed and manufactured. The experiment at the Luong Son Shooting range in Viet Nam was produced and directed by Military Technical Academy Viet Nam to get the movement of the projectile as well as the value of the gas pressure in the high pressure chamber and the low pressure chamber.

Fig. 6 - Fig. 8 show the nozzle (with 3 different section diameters), the fuse of projectile, and the experiment setup.



Figure 6. The nozzle



Figure 7. Fuzes



Figure 8. Experiment setup at shooting range Lurong Son

To confirm the computed results of the model using the velocity of the projectile at the muzzle of the barrel, that is shown in Tab. IV. From the theoretical calculation results (Tab. III) and experimental results (Tab. IV) indicated that the calculation model of interior ballistics for HV-76 trial gun is quite suitable, results of calculations are in agreement with the experimental results very well.

TABLE IV. EXPERIMENT RUSULTS

No.	The nozzle ultimate section diameter	number of shots	Average velocity <i>v</i> _{TB}
1	6.2e ⁻³ m	10	285.17 m/s
2	7.2e ⁻³ m	10	286.06 m/s
3	7.6e ⁻³ m	10	284.17 m/s

V. CONCLUSION

The paper has established the system of differential equations to solve the interior ballistics for the HV-76 trial gun which is based on the principle of high - low pressure weapons.

Solving this system, the effect of the nozzle ultimate section diameter to the maximum value of the pressure in the high pressure chamber, the low pressure chamber and the velocity of the projectile at the muzzle of the barrel is investigated. From results of model, the trial gun was designed and manufactured.

All results of calculations are in agreement very well with results of the experiment. This model as well as the trial gun HV-76 can be used for design and manufacture new weapons to work on the principle of high – low pressure gun.

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