Limit Impact Velocity of Frangible Bullet

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Abstract—**The article deals with the determination of a limit impact velocity of a frangible bullet. Limit impact velocity is the lower limit impact velocity of the bullet on the hard target, at which the bullet begins to shatter into the fragments. The limit impact velocity of the bullet is one of the criteria to evaluate the frangibility of the frangible bullet. In order to determine the limit impact velocity, the theoretical method or the experimental method or combination of theoretical and experimental method can be used. This article presents the different procedures for determining the limit impact velocity on the basis of the purely theoretical models, the combined theoretical-experimental models based on the frangibility factor and also on the basis of simulation.**

Keywords-small arms ammunition; frangible bullet; limit impact velocity; frangibility; frangibility factor; DEM; Ansys Autodyn

I. INTRODUCTION

Frangible ammunition for small arms is one of main trends in the last decades in the field of ammunition [1, 2]. When using cartridge with frangible projectile in the training of shooting and even in the police service, the risk of injury to shooter or non-participants by the ricochet of the bullet is significantly reduced. Terminal ballistics of the frangible bullet is very specific, especially due to the capability of the bullet to disintegrate into different fragments upon impact on a hard target. For some frangible bullets, it is typical that the disintegration into fragments occurs not only hitting the target with the high velocity but also with the relatively low velocity (less than 100 $m.s^{-1}$) [3].

The impact velocity of the bullet on the standard hard target, at which the frangible bullet begins to disintegrate, is called the limit impact velocity. This velocity is one of the important functional characteristics of the frangible bullet. The value of the limit impact velocity is the indicator of the frangibility of the bullet – capability to disintegrate into fragments. The frangibility of the bullet increases with decreasing of the value of the limit impact velocity in the same other conditions. The limit velocity as well as the frangibility of the bullet depends on many factors, especially the mechanical properties of the bullet's material, the shape of the bullet (FP, RN, HP), the construction of the bullet (presence or absence the jacket), the manufacturing technology of the bullet (the process of densification from raw materials, using binder, sintering), the impact conditions of the bullet upon on the target (impact velocity, angle of arrival), the type of the target resp. obstacle, etc.

In order to determine the limit impact velocity, the theoretical method or the experimental method or combination of theoretical and experimental method can be used. The theoretical methods are based on either on the analytical relations or simulation using a suitable software (e.g. Ansys, Abaqus). However, some important input parameters for theoretical solution need to experimentally determine, often under the different conditions from the real, whereby the accuracy of the limit velocity determination is reduced. The experimental methods of determining the limit velocity associated with the real shooting are more objective, but the propelling of the bullet from a barrel with a very low initial velocity (several tens of m/s) is difficult (extremely low weight of the propellant charge and even charge density, a large dispersion of internal ballistic parameters and quite frequent jamming of the bullet in the bore barrel).

This article presents the different procedures for determining the limit impact velocity on the basis of the purely theoretical models, the combined theoretical-experimental models based on the frangibility factor and also on the basis of simulation.

II. THEORETICAL METHODS FOR DETERMINING THE LIMIT VELOCITY OF THE FRANGIBLE BULLET

In literature [3, 4] a calculation model of limit velocity of frangible bullet is presented, which is based on the deformation work of the axial loaded bullet upon impact on the target. Furthermore, the original model is proposed to determine the limit velocity of the frangible bullet for different targets, published in [5].

To determine the limit impact velocity for different targets in this paper, an elastic model for the bullet and a modified elastic model for target are applied. Elastic model is based on the use of the spring (Fig. 1). In this model, the behavior of the bullet is described by an ideal elastic model. The bullet in the real conditions elastically deforms only at low load to the elastic limit. When exceeding the elastic limit of the bullet's material, the bullet plastically deforms, the bullet is cracked when exceeding the ultimate strength of the material, usually with multiple fragile fractures.

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To model the interaction of the bullet with the target, the method DEM (Discrete Element Method) in the dimension 1-D [4, 6] can be used, according to which the bullet can be substituted by the chain of springs. To simplify the mathematical model, in this paper the bullet is substituted by one spring with the equivalent stiffness k_s and the weight m_q. The target is also substitute by a modified elastic model, according to which the resistance force exponentially depends on the deformation (Fig. 1).

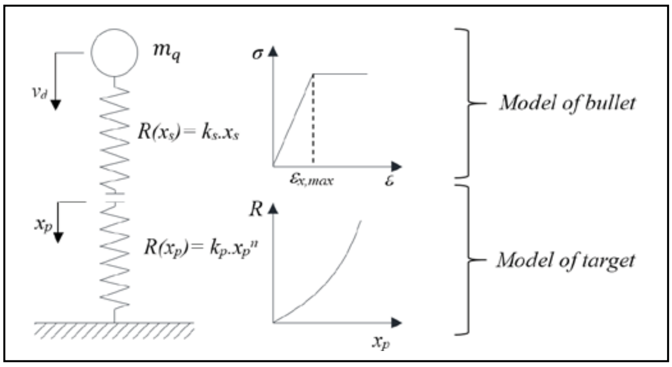

Figure 1. Scheme of loaded model when hitting of the bullet on the target.

If we neglect the rotating motion of the bullet and the loss of energy during interaction of the bullet and target, then the energy conservation law of the system, which contains two springs, can be applied at the moment of maximal deformation of the system elements, i.e. the bullet and the target (velocity of the bullet is zero) by following equation:

$$\int_0^{x_p} R(x_p)dx_p + \int_0^{x_s} R(x_s)dx_s = \frac{1}{2}m_q v_{mez}^2 \ , \qquad (1)$$

where: m_q – equivalant mass of the bullet [kg], v_{mez} – limit impact velocity of the bullet [m.s^{-1}], x_s – deformation of the bullet [m], x_p – compression deformation of the obstacle / target [m], $R(x_p)$, $R(x_s)$ – resistance force of the springs at deformatiion x_p and x_s [N], X_p, X_s – deformation of the springs at the moment of reaching the ultimate strength of the bullet [m].

Assume that the spring corresponding to the modified elastic model of the target has force characteristic given by the equation:

$$R(x_p) = k_p . x_p^n \ , \qquad (2)$$

where: k_p – pseudo stiffness of the spring [N.m^{-n}], n – index of stiffness [1]. In the case of $n = 1$, the target / obstacle behaves like a real spring.

The limit velocity can be determined from the following equation:

$$\int_0^{x_p} k_p x_p^n dx_p + \int_0^{x_s} k_s x_s dx_s = \frac{1}{2}m_q v_{mez}^2 \ . \qquad (3)$$

After integration, we have:

$$\frac{k_p}{n+1}X_p^{n+1} + \frac{1}{2}k_s X_s^2 = \frac{1}{2}m_q v_{mez}^2 \ . \qquad (4)$$

Assume that the resistance force of the spring is equal at the point of contact of the two springs, thus:

$$k_s X_s = k_p X_p^n \qquad (5)$$

$$\Leftrightarrow X_p = \left(\frac{k_s}{k_p}X_s\right)^{\frac{1}{n}} . \qquad (6)$$

Substituting (6) into (4) we obtain:

$$\frac{k_p}{n+1}\left(\frac{k_s}{k_p}X_s\right)^{\frac{n+1}{n}} + \frac{1}{2}k_s X_s^2 = \frac{1}{2}m_q v_{mez}^2 \ , \qquad (7)$$

where the relation for the limit velocity of the bullet can be derived:

$$v_{mez} = \sqrt{\frac{2}{m_q}\left(\frac{k_p}{n+1}\left(\frac{k_s}{k_p}X_s\right)^{\frac{n+1}{n}} + \frac{1}{2}k_s X_s^2\right)} \ . \qquad (8)$$

According to the elastic model of the bullet, we have:

$$X_s = \varepsilon_{x,\max} l_q \ , \qquad (9)$$

where: l_q – length of the bullet [m], $\varepsilon_{x,max}$ – maximum uniaxial relative deformation of the bullet in compression [1].

To determine the equivalent stiffness of the bullet, the model with the cylindrical solid (substitution of the bullet) and equivalent spring, which are placed on the pad. The solid has a mass of m_q, an elastic modulus in compression of E_c, a height of l_q and a diameter of d. The pad moves against to the solid in the direction of solid's logitudinal axis with the acceleration of a (Fig. 2).

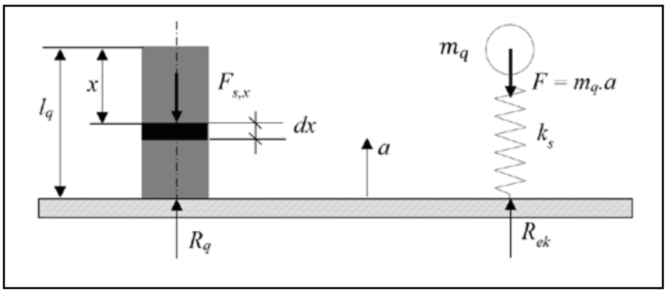

Figure 2. Model to determine the equivalent stiffness of the bullet.

The stiffness of the equivalent spring can be analytically determined in two ways (variant 1 and 2). According to the first variant, the stiffness of the equivalent spring is obtained by comparing the identical deformation of the solid and of the spring that substitutes it. According to the second variant, the stiffness of the equivalent spring is obtained by comparing the identical stress induced by inertial force when braking of the bullet upon impact with the target.

Procedure (1st variant) – comparing the deformation:

- on the cross-section of the cylindrical solid at a distance of x from its upper surface, the axial inertial force $F_{s,x}$ induces the stress $\sigma(x)$:

$$\sigma(x) = \frac{F_{s,x}}{S} = \frac{\rho x S a}{S} = \rho x a , \qquad (10)$$

where S is the cross-section area of the cylindrical solid and ρ is the density of the solid's material;

- relative axial deformation ε(x) for segment dx is:

$$\varepsilon(x) = \frac{u(x)}{dx} , \qquad (11)$$

where $u(x)$ is absolute axial deformation of the segment with the length of dx;

- according to the Hook's law, we have:

$$\varepsilon(x) = \frac{u(x)}{dx} = \frac{\sigma(x)}{E_c} = \frac{\rho x a}{E_c} \qquad (12)$$

$$\Leftrightarrow u(x) = \frac{\rho x a}{E_c} dx , \qquad (13)$$

where E_c is the elastic modulus in compression of the bullet's material [Pa]. The value of E_c can be determined by a static test of compression strength;

- integrating the equation (13) for x: $0 \div l_q$, we determine the absolute axial deformation for the whole solid:

$$u(l_q) = \int_0^{l_q} \frac{\rho x a}{E_c} dx , \qquad (14)$$

$$\Leftrightarrow u(l_q) = \frac{\rho x a}{E_c}\frac{l_q^2}{2} = m_q \frac{l_q}{2SE_c}.a , \qquad (15)$$

The deformation of the equivalent spring substituted the solid u_{ek} is:

$$u_{ek} = \frac{R_{ek}}{k_s} = \frac{m_q.a}{k_s} , \qquad (16)$$

where R_{ek} is the inertial force caused by the target on the equivalent spring.

Comparing the equation (15) and (16), we obtain the resulting relation for stiffness of the spring k_s according to the 1st variant ($u(l_q) = u_{ek}$):

$$k_s = \frac{2SE_c}{l_q} = \frac{\pi d^2 E_c}{2l_q} , \qquad (17)$$

where d is the diameter (caliber) of the solid (bullet) [m].

Substituting (9) and (17) to (8), we have the limit velocity:

$$v_{mez} = \sqrt{\frac{2}{m_q}\left(\frac{k_p}{n+1}\left(\frac{E_c \pi d^2}{2k_p}\varepsilon_{x,\max}\right)^{\frac{n+1}{n}} + \frac{E_c \pi d^2}{4}\varepsilon_{x,\max}^2\right)} . \qquad (18)$$

Procedure (2nd variant): comparing the inertial forces:

- inertial force R_q applied at the bottom of the solid is determine by:

$$R_q = S\sigma(l_q) , \qquad (19)$$

where $\sigma(l_q)$ is the axial stress at the bottom of the solid;

- according to the Hook's law for the bullet's material in the elastic deformation:

$$\sigma(l_q) = \varepsilon_{x,\max} E_c = \frac{u(l_q)}{l_q} E_c , \qquad (20)$$

where $u(l_q)$ is the total deformation of the solid;

- substituting (20) to (19), we have:

$$R_q = \frac{SE_c}{l_q} u(l_q) . \qquad (21)$$

The inertial force applied on the equivalent spring by the target:

$$R_{ek} = m_q.a = k_s.u_{ek} , \qquad (22)$$

where u_{ek} is the deformation of the equivalent spring.

On the basis of the equilibrium of deformations of the solid and spring $u(l_q) = u_{ek}$ and the comparison of the relations (21) and (22), we obtain the resulting relation for stiffness of the spring k_s according to the 2st variant:

$$k_s = \frac{SE_c}{l_q} = \frac{\pi d^2 E_c}{4l_q} . \qquad (23)$$

The relation (20) corresponds to the conditions of static testing of the solid in tension. Comparing relation (14) and (20), it is obvious that the value of the stiffness according to (20) is halved.

The real bullet has not cylindrical shape. When testing the strength of the bullet and calculating of the limit velocity, the equivalent diameter of the bullet d_{ek} is used by assuming that the volume of the cylinder with diameter of d_{ek} and height l_q (equal the length of the bullet) is equal to that of the real bullet.

The Tab. I shows the comparison of the limit velocity calculation according to three different variants for different type of frangible bullet when impacting on a circular steel plate with a diameter of $D = 0.5$ m, a thickness of $h = 10$ mm and a stiffness of $60.55 \cdot 10^5$ N.m^{-1}, which is restrained around its circumference, an elastic modulus of $E_c = 2.1 \cdot 10^{11}$ Pa according to [3], the assumption $n = 1$. The calculation was carried out for 4 different types of frangible bullet caliber of 9 mm Luger:

- experimental bullet SR with a mass of 5.16 g, which was produced by cold pressing from the composite of Fe and Sn;
- experimental bullet BiCu with a mass of 7 g, which was produced by cold pressing from the composite of Bi and Cu;

TABLE I. COMPARISON OF THE CALCULATING VALUES OF LIMIT VELOCITY v_{MEZ} ACCORDING TO DIFFERENT MODELS

		Bullet / solid	Sinterfire	SR	Bi100 7.62	BiCu	BiCu Sinter
Bullet and target properties		d_{ek} [m]	0.00831	0.00847	0.00731	0.00851	0.00851
		l_q [m]	0.01605	0.01367	0.01	0.01415	0.01415
		m_q [kg]	0.00648	0.005167	0.004	0.007	0.007
		ρ [kg.m^{-3}]	7443	6650	9515	8669	8669
		E_c [10^{9}.Pa]	9.48	3.12	3.67	5.71	5.88
		$\varepsilon_{x,max}$ [1]	0.0181	0.0176	0.0171	0.0208	0.0210
		k_p [10^{6}.N.m^{-1}]	6.055	6.055	6.055	6.055	6.055
v_{mez} [m.s^{-1}]	According to	model of Ing. Rydlo [3]	96.9	38.0	36.1	63.8	71.3
		new model (1st variant)	98.3	38.9	37.0	69.9	72.5
		new model (2nd variant)	51.2	21.2	20.0	36.9	38.4

- experimental bullet BiCu Sinter, which is BiCu bullet sintered at 220 °C for 60 minutes in air after pressing;
- commercial frangible bullet Sinterfire with a mass of 6.48 g.

In addition to the bullet caliber of 9 mm, a cylindrical solid of material 100 % Bi (so-called Bi100) with a diameter of 7.62 mm and a weight of 4 g was used for comparison.

The results of the calculation show that the values of v_{mez} determined by analytical relation of 1st variant and relation according to Ing. Rydlo are close. The values of v_{mez} determined according to the 2nd variant are significantly smaller.

III. COMBINATION OF THEORETICAL AND EXPERIMENTAL METHODS FOR DETERMINING THE LIMIT VELOCITY OF THE FRANGIBLE BULLET

In this part of the article, the new combination (theoretical and experimental) method for determining the limit velocity of frangible bullet is presented based on the experimental values of frangibility factor determined at higher initial resp. impact velocity (100 – 500 m/s).

A. Frangibility factor of the frangible bullet

The frangibility factor of the frangible bullet is a quantification of the capability to disintegrate of the bullet when hitting on a defined obstacle / target. To determined the experimental frangibility factor (FF_E), it is necessary to carry out shooting experiments. After hitting of the bullet on the target and its disintegration, it is necessary to capture fragments by the suitable trap. The total mass of the all captured fragments compared with the original mass of the bullet must be maximized (min. 90 %) so that as the results of evaluation frangibility of the frangible bullet are representative. To calculate the FF_E, the captured fragments are divided into 5 size classes according to the external dimensions of the individual fragments. Fragments are differentiated using sieves of defined mesh size. The frangibility factor FF_E is then determined according to the following equation:

$$FF_E = 100\sum_{i=1}^{5} K_{mi}\frac{m_{ci}}{m_q}, \qquad (24)$$

where: m_{ci} (i = 1 – 5) is the total mass of fragments in the particular size class i [kg], m_q is the original mass of bullet [kg], K_{mi} is the size coefficient.

For each size class the size coefficient K_{mi} is established. K_{mi} as a weighting number expresses the preference of the individual size class of the bullet's fragments. Due to the preference of disintegration of frangible bullet into small fragments, the values of K_{mi} are higher in the class, which corresponds to the smaller size of fragments and vice versa. In the Tab. II the dimensions of fragments and the size coefficient for each class are shown, which have been used to determine the frangibility factor in this article. The values of these size coefficients in the meaning of modified weighting numbers express the fact that the size class of fragments up to 0.5 mm has crucial importance for achieving the required high value of FF_E, while the meaning of the last size class of above 5 mm is negligible (applies for the bullet caliber of 6 – 10 mm).

TABLE II. SIZE CLASS OF FRAGMNETS

Size class of fragments *i*	1	2	3	4	5

Range of fragments dimension [mm]	(0, 0.5]	(0.5, 1]	(1, 2]	(2, 5]	(5, l_q]*
Size coefficient K_{mi}	1	0.75	0.5	0.25	0.01

*. l_q is the length of bullet.

Frangibility factor FF_{EM} reaches the values in the interval from 0 to 100 %. The zero value express the fact that the disintegration does not occur when the bullet impacts on a standard target with a certain velocity (the bullet remained compact). The value FF_{EM} = 100 % is indicative of the complete disintegration of the bullet with all the fragment in the 1st class. This method was presented in [7, 8].

In practice, it is impossible to capture all fragments up to 100 % of the original weight of the bullet (escape of very fine particles from the trap through the clearances, their swirling after opening the trap). The impossibility of collecting fragments from entire volume of the bullet's material is the lack of the method that reduces the objectivity of the frangibility evaluation. Based on the experience from a range of experiments, we can accept the assumption that the missing (unfound) fragments (in the case of ensuring reliable tightness of the trap) have character of very fine grain, which belongs to the smallest size class (1st class). Adding the missing mass of fragments into the 1st class the value of modified frangibility factor FF_{EM} will be determined, which differs from FF_E. This article will use the **frangibility factor FF_{EM}**, which allows objectively to quantify frangibility of the frangible bullet.

B. Determination of limit velocity using factor FF_{EM}

This article summarizes the results of shooting experiments for some experimental bullets. The discrete function $FF_{EM} = f(v_d)$ is determined from shooting experiments. FF_{EM} can be approximated by the exponential function in the form:

$$FF_{EM}\,[\%] = 100 - a.\exp(b.v_d^{\alpha}) \,, \qquad (25)$$

where v_d is impact velocity of the bullet [m.s^{-1}], a, b, α are constants.

Substituting zero of functional value $FF_{EM} = 0$, then $v_d = v_{mez}$ and we have:

$$v_{mez} = \left(\frac{1}{b}\ln\left(\frac{100}{a}\right)\right)^{1/\alpha} \,, \qquad (26)$$

where v_{mez} is limit impact velocity of the bullet. This is the velocity at which the bullet begins disintegrate into fragments.

The above-mentioned frangible bullet were used in experiment. In the cartridges the nitrocellulose smokeless powder S-011 was utilized. The changes of impact velocity of the bullet were realized by changing the powder's mass of the cartridge. The cartridges were fired to a fixed target (Hardox 450) on the rear wall of the trap at distance of 5 m from the barrel muzzle. The results are shown in the Fig. 3. The constant a, b, α for each bullet and the limit velocity given in the Tab. III are determined from the experimental results by the relation (26).

In the experimental determination of calculated limit velocity of the bullet SR the values of FF_{EM} corresponding to the significant low velocity in the range of 37.3 – 91.2 m.s^{-1} were evaluated. The approximated function FF_{EM} is in agreement with the experiment, as confirmed by the gray triangular points in the left of the graph in Fig. 3. Based on the evidence on the bullet after hitting, it was found that the longitudinal axis of the bullet was deflected normal to the plate (i.e. from the arrival direction of the bullet) up to 30 – 40°, indicating instability of the bullet due to non-standard, very low initial velocity. In an oblique impact, the bullet is easier to shatter into fragments than that in a perpendicular direction. Regarding this fact, it can be assumed that limit velocity of the bullet will be somewhat higher.

TABLE III. VALUES OF COEFFICIENT OF APPROXIMATED FUNCTION AND LIMIT VELOCITY OF THE BULLETS

Bullet	SR	Sinterfire	Bi100 7.62	BiCu Sinter	BiCu
a	110.73	157.10	131.46	3762.50	1057.80
b	-0.00106	-0.008609	-0.013	-1.47	-0.90
α	1.3079	0.9231	1.122	0.2359	0.285
v_{mez}	32.8	73.0	15.1	46.0	29.4

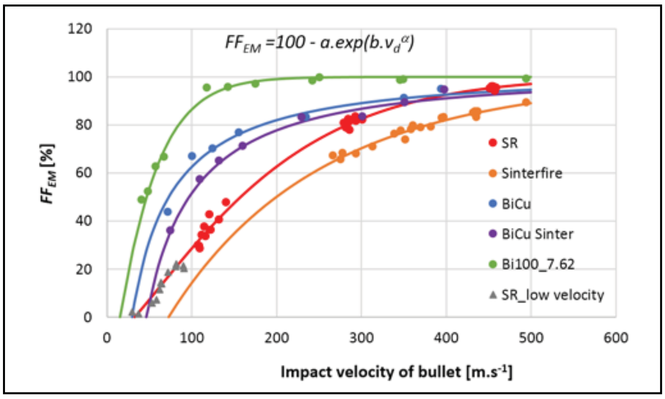

Figure 3. Dependence of frangibility factor FF_{EM} of the experimental bullet on the impact velocity.

These results confirm that the limit velocity of the frangible bullet can be theoretically estimated using approximated function FF_{EM} determined from the values obtained at higher velocities.

IV. DETERMINING THE LIMIT VELOCITY OF FRANGIBLE BULLET BY SIMULATION OF INTERACTION OF THE BULLET WITH THE TARGET

Within the analysis of material of experimental and commercial frangible bullet, the data of bullet's material were obtained by axial test and radial test of cylindrical sample (i.e. Brazilian test – BT). The cylindrical samples were produced by cutting off the ogival parts of the frangible bullets mentioned above. The properties of materials are shown in Tab. IV. The elastic modulus of the material is determined by axial compression testing.

These data are not sufficient to simulate the interaction of the frangible bullet with a hard target. To complement the required characteristic of the material, the challenging experiments would be required. Therefore, the simplified simulation model has been set up based on the acquired experience, which can be described by the following points:

- the material of the bullet is isotropic and homogeneous;
- equation of state: the material of the bullet is defined with a linear EOS;
- strength model of the bullet: the Drucker-Prager model is applied, which is used for brittle materials such as soil, rock, concrete, etc. In the Autodyn system, there are three types of Drucker-Prager model: Linear, Stassi and Piecewise. When simulating the frangible bullet, the model Drucker-Prager "Stassi", which requires a yield stress uniaxial tension and a yield stress uniaxial compression, is used. The yield stress uniaxial compression is the yield compressive stress $\sigma_{0.2}$. The yield stress uniaxial tension of the material corresponds to 90 % of the tensile yield strength determined in Brazilian test;
- Poisson ratio of the bullet's material is 0.28 (estimated by comparing the results of simulation and experiment);
- failure model of the bullet: the model Pmin (Hydro tensile limit pressure) is used, whose value is defined by tensile strength determined in the Brazilian test;
- the target is modeled with a material that is completely rigid (material model Rigid in Autodyn).

TABLE IV. MATERIAL PROPERTIES OF THE BULLET

Bullet / Material	SR	Sinterfire	BiCu	BiCu sintr	Bi100
Density [kg.m^{-3}]	6650	7440	8670	8670	9525
Ultimate compression strength [MPa]	69.1	319	119	124	67.2
Yield compressive stress of material $\sigma_{0,2}$ [MPa]	60.7	280	103	105	60.7
Tensile strength according to BT [MPa]	14.1	49.9	17.4	19.0	7.7
Elastic modulus [MPa]	1760	5264	5711	5881	3893
Poisson ratio	0.28	0.28	0,28	0.28	0.28
Bulk modulus [MPa]	1333	3988	4327	4455	2949
Shear modulus [MPa]	687.5	2056	2231	2297	1521

TABLE V. COMPARISON OF THE VALUES OF LIMIT VELOCITY OF THE FRANGIBLE DETERMINED BY DIFFERENT METHODS

Bullet		SR	Sinterfire	BiCu	BiCu sintr	Bi100 _7.62
Limit velocity v_{mez} [m/s] according to:	**1st variant**	39	98	70	73	37
	2nd variant	21	51	37	38	20
	FF_{EM} curve	33	73	29	46	15
	simulation	70	85	50	65	15
Average of v_{mez} [m/s]		**41**	**77**	**47**	**56**	**22**
Standard deviation σ [m/s]		18.1	17.3	15.5	14.1	9.0
Relative standard deviation σ / v_{mez} [1]		**0.44**	**0.23**	**0.33**	**0.25**	**0.42**

The simulation results are shown in the Tab.V. The simulation values v_{mez} are (with the exception of the Bi100) generally higher than the average of the values obtained by other methods. The results in Tab. V show that the values of limit velocity according to simulation lie within the interval values determined by other methods for all experimental bullet (excluding SR). The simulation limit velocity of the bullet SR significantly differs from the calculated values and from the value derived from the FF_{EM} curve, which testified a lower in agreement of simulation input with reality.

V. CONCLUSION

The values of limit velocities determined according to the 4 methods described above are shown in Tab. V. It is obvious that the achieved results differ for individual bullet, however the agreement of the order is reached. The greatest consistency between the values of limit velocity determined by the individual method is achieved with Sinterfire and BiCu sintr bullets, the highest relative scatter of the values is for SR bullet and the cylinder Bi100.

From the Tab. V, it can be estimated from the average values of v_{mez} that the bismuth cylinder with the diameter of 7.2 mm and the bullet SR have the highest brittleness, conversely the most compact bullet is the commercial bullet Sinterfire, which is pressed at a high pressure and is additionally sintered. The mixture of metal powders of BiCu generally have the ability to achieve the higher compactness of the bullet than the pure bismuth powder. Comparing the values of limit velocity of the BiCu bullet and BiCu sintr bullet shows that all models are sensitive to increasing of coherence of the bullet's material induced by sintering (the values of limit velocity for all cases of the bullet BiCu sintr are higher).

The limit velocity of the bullet can be used as one of the quantification of frangibility of frangible bullets. On the basis of above results, it can be stated that the determination of limit velocity of the bullet is relatively complex problem, which is principally solvable by various methods but with the absence of a clear agreement between achieved results. Achieving order alignment between the values of limit velocity according to individual method can be considered as a successful achievement. For objectification of determination of limit velocity, the average value of limit velocity calculated from the values of individual method for evaluation frangible bullet can be used.

This article defines the limit velocity of the frangible bullet when hitting on the steel plate, a very hard obstacle. It can be assumed that the limit velocity of the bullet increases with the decreasing of resistance of the obstacle and in some couples of obstacle / bullet the limit velocity will be infinitely high, i.e. the disintegration of the bullet will not occur. From the point of view of practical applications, the determination of the limit velocity of the ultrafrangible bullet upon impact on a target made of soft tissue substitution (ballistic gelatin or gel) is an interesting problem. These bullets are characterized by the extreme wounding potential and their limit velocity can be used even in the legal classification of the frangible bullet.

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