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Oscillation of the Anti-tank Missile System Fagot Fired on the Elastic Ground

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***Abstract*–This paper presents a method for establishment and solution the system of differential equations to describe the oscillation of the 9K111 anti-tank missile system (Fagot) fired on the elastic ground. The analysis of the results obtained is used for the research, design, and manufacture spare parts of the anti-tank missile 9K111 in conditions of Vietnam. Results of this paper are also the scientific basis to develop an understanding of the oscillation of the 9K111 anti-tank missile system. Results are able to evaluate the state of the system after repairs, improvements.**

Keywords-Fagot; Anti-tank missile; Dynamics; System of differential equations

I. Introduction

The 9K111 anti-tank missile system (Fig. 1) is a second-generation tube-launched guided missile system of the Soviet Union that is equipped for many armies in the world. This is the kind of the relatively modern missile system designed for the long range precision fires. These missiles are very useful in Viet Nam army because of the easy and fast maneuverability as well as the suitable for the military tactics.

Figure 1. System 9K111 anti-tank missiles (Fagot).

However, because this system has been equipping for a long time in the army, the state of it is degraded and unreliable operations of some parts that need to overcome in time. Replacements of the failed part face many difficulties because almost of spare parts are bought from the foreign. In order to truly master the process, the in-depth study of the phenomenon that occurs when firing is required. The theoretical model, a theoretical basis need to build completely and reliably to evaluate the quality of the system 9K111.

On the other hand, the research results of this paper formulate the method of position selection as well as evaluating the system's ability to balance on the elastic ground.

Based on the dynamics mathematical model, using the second order Lagrange equations, the oscillation of the anti-tank missile system Fagot fired on the elastic ground is solved. The results of the oscillation of the 9K111 anti-tank missile system (Fagot) fired on the elastic ground are represented graphically.

II. Building System of Equations Describing the Oscillation of Fagot System

A. Assumptions

Model of oscillation of the system 9K111 was established on the basis of its analysis, which include moments of forces, forces, structure characteristics, and the system operations during the whole process of launching.

The system of differential equations was solved using the following assumptions:

- Details and detail assemblies are considered as rigid bodies,
- The ground is flat and homogeneous, the effect of it on the system is modeled by springs and dampers,
- The mass distribution of system is replaced with the mass centralization of system and moments of inertia of objects are located in the center of parts of the system,
- Details are considered as symmetrical, the symmetry axis goes through the center of mass of the object,
- No eccentricity, no deflection of thrusts,
- Not mention the change of the projectile mass when it moves on the launcher.

From these assumptions, the computational model of the oscillation for anti-tank missiles 9K111 system is given in Fig. 2.

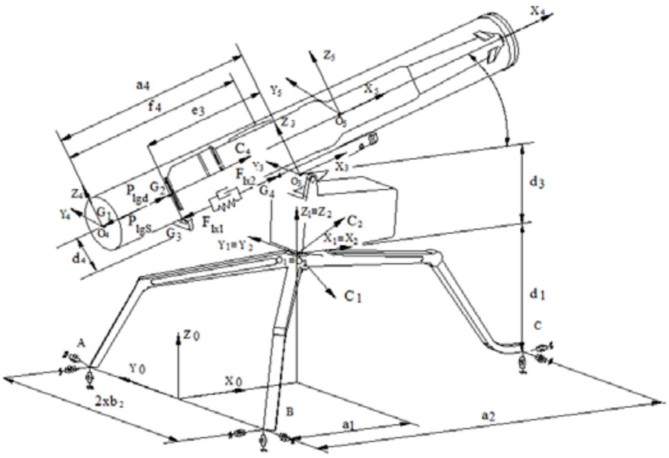

Figure 2. Oscillation calculation model of system 9K111.

B. Bodies in the Model

Based on the structure of the real model, the linkages of the components as well as assumptions mentioned above, the 9K111 system dynamics model (Fig. 2) was set up as follows:

- Body 1: Tripod, mass m_1, center of gravity at C_1, moment of inertia of tripod to focus C_1 is I_1,
- Body 2: Traverser parts, mass m_2, center of gravity at C_2, moment of inertia traverser parts to focus C_2 is I_2,
- Body 3: Elevation parts, mass m_3, center of gravity at C_3, moment of inertia of body to focus C_3 is I_3,
- Body 4: Launching tube, mass m_4, center of gravity at C_4, moment of inertia of body to focus C_4 is I_4;
- Body 5: Missile 9M111M, mass m_5, center of gravity at C_5, moment of inertia of body to focus C_5 is I_5;

Mass, central position and moment of inertia of all bodies are determined through SolidWorks software.

C. Coordinate Systems

To investigate system of rigid body dynamics, the coordinate systems are built as follows:

- Fixed coordinate system $R_0=\{O_0X_0Y_0Z_0\}$

  where: O_0 is the midpoint between the two legs behind the mass center of system as shown in Fig. 2; the $\vec{X}_0$ axis is oriented towards the barrel muzzle, it is the intersection of the plane of symmetry of the system and the background; the $\vec{Z}_0$ axis is parallel to the direction of spin axis and oriented vertically upwards; $\vec{Y}_0$-axis is perpendicular to the $\vec{X}_0$-axis and $\vec{Z}_0$-axis.

- $R_1=\{O_1X_1Y_1Z_1\}$

  where: O_1 is the intersection point of the spin axis of the launcher and the plane of the training circle placed in the tripod. The $\vec{X}_1$, $\vec{Y}_1$, $\vec{Z}_1$ are shown as in Fig. 2. X_1 is parallel and has the same direction with X_0 axis, Z_1 is parallel and has the same direction with Z_0 axis, and Y_1 is parallel and has the same direction with Y_0 axis.

- $R_2=\{O_2X_2Y_2Z_2\}$: is coordinate system of the body 2, in the current state of model, R_2 coincides with R_1.
- $R_3=\{O_3X_3Y_3Z_3\}$: is coordinate system of the body 3. The $\vec{X}_3$, $\vec{Y}_3$, $\vec{Z}_3$ are shown as in Fig. 2. X_3 axis is parallel with the launcher and oriented towards the muzzle of launcher.
- $R_4=\{O_4X_4Y_4Z_4\}$: is coordinate system of the body 4. O_4 is the intersection point of the axis of the launching tube with launching tube back. The $\vec{X}_4$, $\vec{Y}_4$, $\vec{Z}_4$ are shown as in Fig. 2.
- $R_5=\{O_5X_5Y_5Z_5\}$: is coordinate system of the body 5. O_5 coincides with the center of gravity of body 5. The $\vec{X}_5$, $\vec{Y}_5$, $\vec{Z}_5$ are shown as in Fig. 2.

From the assumptions, the system has 8 independently generalized coordinates q_j ($j = 1 \div 8$): q_1 - the translation of body 1 along the $\vec{X}_0$; q_2 - translation of body 1 along the $\vec{Y}_0$; q_3 - translation of body 1 along the $\vec{Z}_0$; q_4 - the rotation around the axis $\vec{X}_0$; q_5 - the rotation around the axis $\vec{Y}_0$; q_6 - the rotation around the axis $\vec{Z}_0$, q_7 - translation of object 4 along the $\vec{X}_4$ and q_8 - translation of object 5 along the $\vec{X}_5$.

D. Objects Position Determining by Generalized Coordinates

- $R_{O_i}^{(0)}$ and $r_P^{(0)}$ is the vector algebra in inertial coordinate system R_0 (O_i is the origin of the system R_i, P is any point): $\overrightarrow{R_{O_i}} = \overrightarrow{O_0O_i}$, $\overrightarrow{r_P} = \overrightarrow{O_0P}$; $u_P^{(i)}$ is the vector algebra in coordinate system R_i ($\overrightarrow{u_P} = \overrightarrow{O_iP}$); A_j^i is the transformation matrix mapping the coordinate system R_i to the coordinate system R_j. A_j^i is the matrix depending on the generalized coordinates.

- Coordinates of the point P in fixed coordinate system: $r_P^{(0)} = R_{O_i}^{(0)} + A_0^i.u_P^{(i)}$

- Coordinates of the point P in the coordinate system corresponding to the object: $u_P^{(j)} = A_j^i.u_P^{(i)}$

- The transformation matrix mapping the coordinate system O_i to the coordinate system O_0 is A_0^i

• Angular velocity vector of the *ith* object in the coordinate system O_0: $\overline{\omega}_i = \dot{A}_0^i.A_0^i$.

E. External Forces Acting on the Mechanical System

Forces acting on the mechanical system include: the firing force, the gravitational force of objects, and the force of elastic ground:

- The gravitational force of objects:

Gravitational force $\overrightarrow{P_{ig}}$ acting on the object is located at the center of mass of objects, perpendicular to the $O_0X_0Y_0$ horizontal plane and oriented towards the background. In the fixed coordinate system, the gravitational force of the object i:

$$P_{ig}^{(0)} = \begin{bmatrix} 0 & 0 & -m_i.g \end{bmatrix}^T \quad (1)$$

(g is the gravitational acceleration).

- Propellant gases pressure forces acting on the launching tube (P_{lgS})

During the firing process, system Fagot 9K111 appears 2 components axial forces acting on the launching tube: kinetic force (F_{en}) is the force generated by the expansion of propellant gases when low pressure chamber changes its volume to cause the recoil of the launching tube. Jet force (F_{p}) is generated by ejecting gas to the back through the nozzle, and this force acts in the direction opposite the kinetic force. The P_{lgS} force is determined from the result of internal ballistics of the rocket motor of 9M111M missiles. The direction of the P_{lgS} force is in the direction opposite the projectile motion and located at the point: G_1. In the coordinate system of body 4, the P_{lgS} force is presented:

$$P_{\mathrm{lgS}} = [-P_{\mathrm{lgS}} \ \ 0 \ \ 0]^T \quad (2)$$

The firing force is determined by the formula:

$$P_{\mathrm{lgS}} = F_{\mathrm{dn}} - F_{\mathrm{p}}$$

In there [4]:

$$F_{\mathrm{dn}} = p\int_{s_{\mathrm{th}}}^{s} ds = p\left(s - s_{\mathrm{th}}\right)$$

$$F_{\mathrm{P}} = \{\varphi_1\varphi_2[F_{\mathrm{r}}(\xi, k) - 0{,}693] - 0{,}555\}.p.S_{\mathrm{th}}$$

φ_1, φ_2: the coefficient counting losses when gas flows through nozzle; $F_{\mathrm{r}}(\xi, k)$: aerodynamic function; p: the pressure in the chamber; g: gravitational acceleration; U_{a},U_{th}: the velocity of gas flow in out-section and throat-section of nozzle; S_{a},S_{th}: the area in the out-section and throat-section of nozzle; p_{a}, p_{th}: the gas pressure in the out-section and throat-section of nozzle.

The interior ballistics of the launch engine 9M111M missile is solved, using software Maple. Firing force that depends on time is determined as shown in Fig. 3.

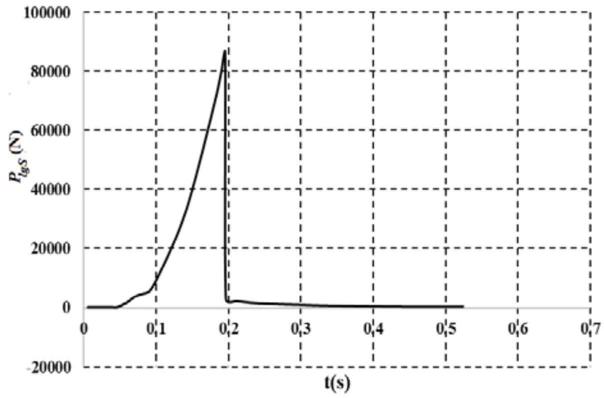

Figure 3. The gas force on the launching tube vs time.

• Gas force acting on the missile (P_{lgd}): the direction of this force coincides with the direction of movement of projectile, and is placed at the point G_2 - bottom center of projectile. In the coordinate system of body 4, the P_{lgS} force is presented

$$P_{\mathrm{lgd}} = [P_{\mathrm{lgd}} \ \ 0 \ \ 0]^T \quad (3)$$

• The force of the springs and dampers acting on object 3 (F_{lx1}) and object 4 (F_{lx2}): these two forces are parallel to the axis of the launching tube, in opposite directions, and located at point G_3 and G_4. Magnitude of these forces:

$$F_{\mathrm{lx3}} = F_{\mathrm{lx4}} = k_{\mathrm{a}}(l_0 + q_7) + b_{\mathrm{a}}.\dot{q}_7 \quad (4)$$

• in there: l_0 – the initial compression of the spring – damper system, k_{a} – stiffness of the springs, b_{a} – drag coefficient of reduction

• The force of elastic ground acts on the mechanical system at points A, B, C. In the fixed coordinate system:

$$P_{nenA}^{(0)} = \left(k_A.\Delta r_A^{(0)} + c_A.\dot{r}_A^{(0)}\right); P_{nenB}^{(0)} = \left(k_B.\Delta r_B^{(0)} + c_B.\dot{r}_B^{(0)}\right);$$
$$P_{nenA}^{(0)} = \left(k_C.\Delta r_C^{(0)} + c_C.\dot{r}_C^{(0)}\right) \quad (5)$$

In there: $\Delta r_A^{(0)}, \Delta r_B^{(0)}, \Delta r_C^{(0)}$ is the initial displacement vector and $\dot{r}_A^{(0)}, \dot{r}_B^{(0)}, \dot{r}_C^{(0)}$ velocities of points A, B, C in a fixed coordinate system; k_A, k_B, k_C and c_A, c_B, c_C the drag coefficient matrix elastic and viscous of the background.

F. Establishment of the System of Differential Equations of the Mechanical System

To establish equations using Lagrange equations, the matrices formula of equations is written according to [4]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T^{\Sigma}}{\partial \dot{q}_j}\right) - \frac{\partial T^{\Sigma}}{\partial q_j} = Q_j \quad (j = 1..8) \quad (6)$$

In there: T^{Σ} - the total kinetic energy of the system; q_j - generalized coordinates j; Q_j - generalized forces corresponding to the generalized coordinates q_j .

$$T^{\Sigma} = \sum_{1}^{5} T_i = \frac{1}{2}\left(\dot{R}_i^{(0)T}.m_i.\dot{R}_i^{(0)} + \varpi_i^T.A_0^i.I_i.A_0^{iT}.\varpi_i\right) \quad (7)$$

$$Q_j = \sum_{i=1}^{5} P_{ig}^{(0)T}.\frac{\partial R_i^{(0)}}{\partial q_j} + P_{\lg S}^{(0)T}.\frac{\partial r_{G_1}^{(0)}}{\partial q_j} + P_{\lg d}^{(0)T}.\frac{\partial r_{G_2}^{(0)}}{\partial q_j} + F_{1x1}^{(0)T}.\frac{\partial r_{G_3}^{(0)}}{\partial q_j} + F_{1x2}^{(0)T}.\frac{\partial r_{G_4}^{(0)}}{\partial q_j} + P_{nenA}^{(0)T}.\frac{\partial r_A^{(0)}}{\partial q_j} + P_{nenB}^{(0)T}.\frac{\partial r_B^{(0)}}{\partial q_j} + P_{nenC}^{(0)T}.\frac{\partial r_C^{(0)}}{\partial q_j} \quad (8)$$

In there: R_i – center vector coordinates of *ith* object in fixed coordinates O_0; m_i – mass matrix of *ith* object; ϖ_i – vector angular velocity of *ith* object; I_i – moment of inertia of the *ith* object in the coordinate system O_i.

$$m_i = \begin{pmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{pmatrix}; \qquad I_i = \begin{pmatrix} J_x^i & 0 & 0 \\ 0 & J_y^i & 0 \\ 0 & 0 & J_z^i \end{pmatrix}.$$

Results of the establishing process are 8 differential equations which are used to describe the movement of objects in the system. Tab. 1 shows parameters selected for the mathematical model.

TABLE I. INPUT DATA FOR THE MODEL

Symbol	Unit	Value
a_1	cm	24.5
a_2	cm	75
a_4	cm	41.5
b_2	cm	23
d_1	cm	22.7
d_3	cm	13.6
d_4	cm	6.9
e_3	cm	12
φ	°	30
u_1	cm	(0, 0, -9)
u_2	cm	(6, 0, 6)
u_3	cm	(0, 0, 0)
u_4	cm	(43.7, 0, 0)
u_5	cm	(24.5,0,0)
I_1	kg.m	(1600,1400,1600)
I_2	kg.m	(1800,1100,2500)
I_3	kg.m	(2100,1600,1000)
I_4	kg.m	(1900,1100;750)
I_5	kg.m	(3200,2500,1950)
m_1	kg	5
m_2	kg	2
m_3	kg	0.5
m_4	kg	10
m_5	kg	12.9
K_{lx}	N/m	$4.5e^3$
K_c	N/m	$11e^3$

III. CALCULATION RESULTS

The differential equations system of the oscillation of the 9K111 anti-tank missile system is solved using the software Maple. The results are shown in Fig. 4- Fig. 7.

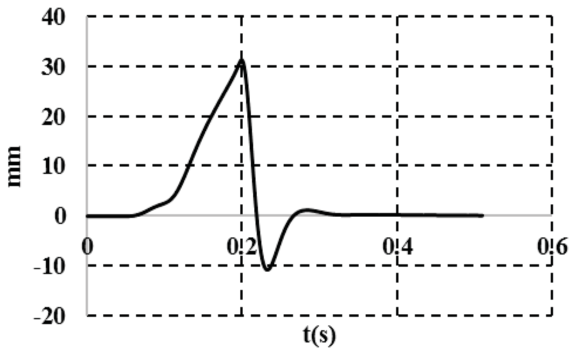

Figure 4. Oscillation of launcher stand in the O_0X_0 axis.

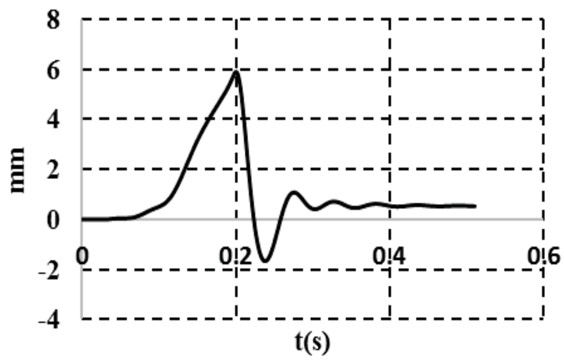

Figure 5. Oscillation of launcher stand in the O_0Z_0 axis.

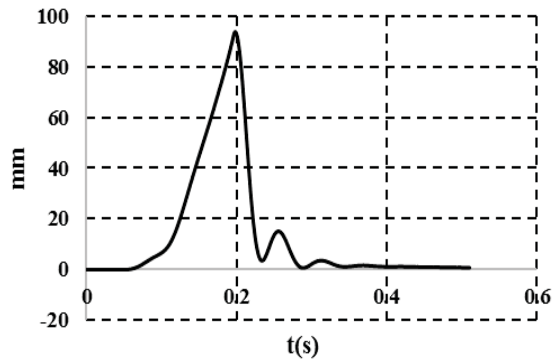

Figure 6. Oscillation of the muzzle of the launching tube in the O_0X_0 axis.

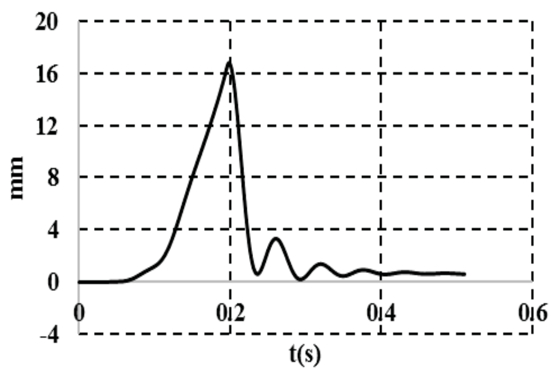

Figure 7. Oscillation of the muzzle of the launching tube in the O_0Z_0 axis.

Based on the results obtained, the oscillation of the system was investigated relatively in agreement with the experiment when the missile launch. Oscillation of the objects reaches the maximum value at the time $t = 0.195$ s. When the pressure in the chamber of the missile motor reaches a critical value to penetrate the rear nozzle and reaction forces generated by gas ejected through the rear nozzle eliminate the firing force. Therefore, the oscillations are extinguished rapidly under the effect of the elastic element. The oscillation of the launch tube is greatest, especially in direction O_0X_0 axis (Fig. 6), this is due to launch tubes directly affected by firing forces. At time $t = 0.516$ s, the projectile leaves the launch tube. At this time, the oscillation of the system is close to the equilibrium position, thus ensuring the stability of the missile leaving the launching tube.

IV. CONCLUSION

The paper has built the system of rigid multibody dynamics to establish the system of differential equations to solve the oscillation of the anti-tank missile system 9K111. This system is built and solved using the second order Lagrange equation. In this model, all objects, coordinate systems, and generalized coordinates were chosen in the general way that allows the study of the system dynamics of the Fagot system when it launched.

The calculation expressions are built in the form of matrices, vector algebra, algorithm diagram and intuitive solution method. The study results are used to determine the influence of the parameters of the launcher structure to the oscillations of the system. This is also the scientific basis for calculating reliability, design, improvement, exploiting anti-tank missile system 9K111.

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