# The Dynamic of Working Equipment of the Remotely Controlled Bomb Disposal Machine 

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#### Abstract

This paper presents the method of establishes the system of differential equations of motion of the system based on Lagrange-2 equation and Denavit-Hartenberg transformation; and presents calculations of working equipment dynamics of remotely controlled bomb disposal machine improved from the Komatsu PC 200-6 hydraulic excavator.


Keywords- remotely controlled machine; dynamic of working equipment; bombs; hydraulic excavator

## I. Introduction

During military operations and often even at peace time, it is necessary to have specialized machines to approach, dig and lift bombs, mines and explosive objects in the underground in difficult terrain conditions for purposes of their disposal. Converting a hydraulic excavator into remotely controlled bomb disposal machine is one of the solutions that is proposed to carry out this work (Fig. 1).


Figure 1. The remotely controlled bomb disposal machine

## 1-boom; 2-arm; 3-bucket; 4-bomb clamp

The conversion of the hydraulic excavator requires integration of bomb clamps, underground ultrasonic device and remote control system ensuring safety for operators during the process of digging up and lifting bombs. Currently, we have not seen any scientific publication about this sort of machine. However, there are quite a lot of scientific publications about the dynamics of hydraulic excavators. The research of dynamic of working equipment is the basis for perfecting the working equipment structure of the remotely controlled bomb disposal machine. In this paper, we will study the dynamics of working equipment of remotely controlled bomb disposal machine improved from Komatsu PC200-6 hydraulic excavator. This machine is capable of digging and picking up bombs, mines,
explosive objects at a depth of 5 m underground, weighing up to 1 ton, diameter up to 42 cm in difficult terrain conditions such as hills.

## II. ESTABLISHING THE SYSTEM OF DIFFERENTIAL EQUATIONS

## 1. Calculation assumption

Dynamical model of working equipment of remotely controlled bomb disposal machine includes 5 stitches ( 0 fixed stitch, 1- boom stitch, 2-arm stitch, 3-bucket stitch, 4bomb clamp stitch) (Fig. 2).

For simplicity and without losing the generality of the calculating model, there are following assumptions:

- The machine stands still on the horizontal ground, skipping the elasticity and horizontal and vertical tilt angles of the background;
- The process of diging and lifting bombs takes place on the vertical symmetry plane of the machine;


Figure 2. Dynamical model of working equipment

- All stictches are considered as absolute stiff;
- Friction in hinge joints linking the stitches is negligible.


## 2. Selection of Coordinate Systems

To study the dynamic of working equipment of remotely controlled bomb disposal machine, we have to choose the coordinate system of the studied system, comprising from 4 components:
$\theta_{1}$ - swing angle determines the position of the boom compared to machine body;
$\theta_{2}$ - swing angle determines the position of the arm compared to boom;
$\theta_{3}$ - swing angle determines the position of the bucket compared to arm;
$\theta_{4}$ - swing angle determines the position of the bombs clamp compared to arm.

## 3. Calculation of Kinetic Energy, Potential Energy, Dissipated Energy

In all equations, $\mathrm{m}_{\mathrm{i}}$ and $\mathrm{J}_{\mathrm{i}}$ are the mass and moment of inertia of the i -stitch respectively $(i=1 \div 4)$. Let $\mathrm{G}_{\mathrm{i}}$ be central point of i -stitch. Using the Denavit-Hartenberg transformation [ $2,4,7,9$ ], we determine the central coordinates of the stitches in the absolute coordinate system. We can assume:
$\theta_{\mathrm{ijkl}}=\theta_{\mathrm{i}}+\theta_{\mathrm{j}}+\theta_{\mathrm{k}}+\theta_{1}, \dot{\theta}_{\mathrm{ijkl}}=\dot{\theta}_{\mathrm{i}}+\dot{\theta}_{\mathrm{j}}+\dot{\theta}_{\mathrm{k}}+\dot{\theta}_{1}, \beta_{\mathrm{ijkl}}=\beta_{\mathrm{i}}+\beta_{\mathrm{j}}+\beta_{\mathrm{k}}+\beta_{1} ;$
$\mathrm{a}_{1}=\mathrm{OO}_{1}, \mathrm{a}_{2}=\mathrm{O}_{1} \mathrm{O}_{2}, \mathrm{a}_{3}=\mathrm{O}_{1} \mathrm{O}_{3}, \mathrm{a}_{4}=\mathrm{O}_{3} \mathrm{O}_{4}, \mathrm{~L}_{1}=\mathrm{OG}_{1}, \mathrm{~L}_{2}=\mathrm{O}_{1} \mathrm{G}_{2}$ $\mathrm{L}_{3}=\mathrm{O}_{3} \mathrm{G}_{3}, \mathrm{~L}_{4}=\mathrm{O}_{2} \mathrm{G}_{4}$

The total potential energy of the system is determined by the following expression $[2,4,10]$ :

$$
\begin{align*}
& \Pi=m_{1} g L_{1} \sin \left(\theta_{1}+\beta_{2}\right)+m_{2} g\left[L_{2} \sin \left(\theta_{12}+\beta_{3}\right)+a_{1} \sin \theta_{1}\right]+\ldots \\
& m_{3} g\left[L_{3} \sin \left(\theta_{123}+\beta_{4}\right)+a_{3} \sin \theta_{12}+a_{1} \sin \theta_{1}\right]+\ldots  \tag{1}\\
& +m_{4} g\left[L_{4} \sin \left(\theta_{124}-\beta_{15}\right)+a_{2} \sin \left(\theta_{12}-\beta_{1}\right)+a_{1} \sin \theta_{1}\right]
\end{align*}
$$

The total kinetic energy of the system is determined by the following expression [2,4,10]:

$$
\begin{align*}
& \mathrm{T}=\frac{1}{2} \mathrm{~m}_{1}\left(\mathrm{~L}_{1}^{2}+\mathrm{J}_{1}\right) \dot{\theta}_{1}^{2}+\frac{1}{2} \mathrm{~J}_{2} \dot{\theta}_{2}^{2}+\frac{1}{2} \mathrm{~J}_{3} \dot{\theta}_{3}^{2}+\frac{1}{2} \mathrm{~J}_{4} \dot{\theta}_{4}^{2} \\
& +\frac{1}{2} \mathrm{~m}_{2}\left[\mathrm{~L}_{2}{ }^{2} \dot{\theta}_{12}^{2}+\mathrm{a}_{1}^{2} \dot{\theta}_{1}^{2}+2 \mathrm{a}_{1} \mathrm{~L}_{2} \dot{\theta}_{12} \dot{\theta}_{1} \cos \left(\theta_{2}+\beta_{3}\right)\right]+  \tag{2}\\
& +\frac{1}{2} \mathrm{~m}_{3}\left[\begin{array}{l}
\mathrm{L}_{3}{ }^{2} \dot{\theta}_{123}^{2}+\mathrm{a}_{3}{ }^{2} \dot{\theta}_{12}^{2}+\mathrm{a}_{1}{ }^{2} \dot{\theta}_{1}^{2}+2 \mathrm{a}_{1} \mathrm{a}_{3} \dot{\theta}_{12} \dot{\theta}_{1} \cos \theta_{2}+ \\
+2 \mathrm{~L}_{3} \mathrm{a}_{3} \dot{\theta}_{123} \dot{\theta}_{12} \cos \left(\theta_{3}+\beta_{4}\right)+2 \mathrm{~L}_{3} \mathrm{a}_{1} \dot{\theta}_{123} \dot{\theta}_{1} \cos \left(\theta_{23}+\beta_{4}\right)
\end{array}\right]+ \\
& +\frac{1}{2} \mathrm{~m}_{4}\left[\begin{array}{l}
\mathrm{L}_{4}^{2} \dot{\theta}_{124}^{2}+\mathrm{a}_{2} \dot{\theta}_{12}{ }^{2}+\mathrm{a}_{1} \dot{\theta}_{1}^{2}+2 \mathrm{a}_{1} \mathrm{a}_{2} \dot{\theta}_{12} \dot{\theta}_{1} \cos \left(\theta_{2}-\beta_{1}\right)+ \\
+2 \mathrm{~L}_{4} \mathrm{a}_{2} \dot{\theta}_{124} \dot{\theta}_{12} \cos \left(\theta_{4}-\beta_{5}\right)+2 \mathrm{~L}_{4} \mathrm{a}_{1} \dot{\theta}_{124} \dot{\theta}_{1} \cos \left(\theta_{24}-\beta_{15}\right)
\end{array}\right]
\end{align*}
$$

Dissipated energy is zero: $\Phi=0$

## 4. Calculation of the Force

The external force of the studied system consists of forces from the corresponding cylinders $\overrightarrow{\mathrm{F}}_{\mathrm{XL1}}, \overrightarrow{\mathrm{~F}}_{\mathrm{XL} 2}, \overrightarrow{\mathrm{~F}}_{\mathrm{XL} 3}, \overrightarrow{\mathrm{~F}}_{\mathrm{XL} 4}$, which are converted into the corresponding torque of $M_{1}, M_{2}, M_{3}, M_{4}$. There are also pressure $\overrightarrow{\mathrm{F}}_{\mathrm{K} 1}, \overrightarrow{\mathrm{~F}}_{\mathrm{K} 2}$ of bomb effects on bomb clamp and bucket.

Let $\alpha_{1}$ be bomb hugging angle, f - the friction coefficient between the bomb shell and the bucket as well as the bomb clamp, $\mathrm{m}_{5}$ - mass of bomb. We can then state:

$$
\mathrm{F}_{\mathrm{K} 1}=\mathrm{F}_{\mathrm{K} 2}=\frac{\mathrm{m}_{5} \mathrm{~g}}{2 \mathrm{f} \sin \frac{\alpha_{1}}{2}}
$$

The total virtual work of external forces is determined by the following expression $[1,4]$ :

$$
\begin{align*}
\delta \mathrm{A}= & \mathrm{M}_{1} \cdot \delta \theta_{1}-\mathrm{M}_{2} \cdot \delta \theta_{2}-\mathrm{M}_{3} \cdot \delta \theta_{3}+\mathrm{M}_{4} \cdot \delta \theta_{4}+ \\
& +\sum_{\mathrm{i}=1}^{4} \mathrm{~F}_{\mathrm{k} 1}^{\mathrm{x}} \frac{\partial \mathrm{x}_{\mathrm{T}_{\mathrm{i}}}}{\partial \theta_{\mathrm{i}}}+\sum_{\mathrm{i}=1}^{4} \mathrm{~F}_{\mathrm{k} 1}^{\mathrm{y}} \frac{\partial \mathrm{y}_{\mathrm{T}_{\mathrm{i}}}}{\partial \theta_{\mathrm{i}}}+\sum_{\mathrm{i}=1}^{4} \mathrm{~F}_{\mathrm{k} 2}^{\mathrm{x}} \frac{\partial \mathrm{x}_{\mathrm{T}_{2}}}{\partial \theta_{\mathrm{i}}}+\sum_{\mathrm{i}=1}^{4} \mathrm{~F}_{\mathrm{k} 2}^{\mathrm{y}} \frac{\partial \mathrm{y}_{\mathrm{T}_{2}}}{\partial \theta_{\mathrm{i}}} \tag{3}
\end{align*}
$$

External force vector of the system is:

$$
\mathrm{Q}=\left[\begin{array}{l}
\mathrm{Q}_{1}  \tag{4}\\
\mathrm{Q}_{2} \\
\mathrm{Q}_{3} \\
\mathrm{Q}_{4}
\end{array}\right]=\left[\begin{array}{l}
\mathrm{F}_{\mathrm{XL1}} \frac{\mathrm{OAxOB} \sin \left(\theta_{1}+\delta_{1}+\delta_{6}\right)}{\sqrt{\mathrm{OB}^{2}+\mathrm{OA}^{2}-2 \mathrm{x}_{\mathrm{A}} \mathrm{OB} \cos \left(\theta_{1}+\delta_{1}\right)-2 \mathrm{y}_{\mathrm{A}} \mathrm{OB} \sin \left(\theta_{1}+\delta_{1}\right)}} \\
-\mathrm{F}_{\mathrm{XL2}} \cdot \frac{\mathrm{O}_{1} \mathrm{C} \cdot \mathrm{O}_{1} \mathrm{D} \cdot \sin \left(\theta_{2}+\delta_{3}+\delta_{5}\right)}{\sqrt{\mathrm{O}_{1} \mathrm{D}^{2}+\mathrm{O}_{1} \mathrm{C}^{2}+2 \mathrm{O}_{1} \mathrm{C} \times \mathrm{O}_{1} \mathrm{D} \cos \left(\theta_{12}+\delta_{3}-\delta_{5}\right)}} \\
\mathrm{F}_{\mathrm{K} 2} \times \mathrm{O}_{3} \mathrm{~T}_{2}-\mathrm{F}_{\mathrm{xL} 3} \cdot \mathrm{MN} \cdot \sin \left(\alpha_{2}+\alpha_{3}\right) \\
\mathrm{F}_{\mathrm{XL} 4} \frac{-\mathrm{O}_{1} \mathrm{CxO}_{1} \mathrm{D} \cos \left(\theta_{4}-\delta_{7}-\beta_{7}\right)}{\sqrt{\mathrm{O}_{1} \mathrm{H}^{2}+\mathrm{a}_{2}{ }^{2}+\mathrm{O}_{2} \mathrm{P}^{2}-2 \mathrm{O}_{1} \mathrm{Hx} \mathrm{O} \mathrm{O}_{2} \mathrm{P} \cos \left(\theta_{34}+\beta_{146}-\beta_{7}\right)-}}+\ldots \\
-2 \mathrm{O}_{1} \mathrm{Ha}_{2} \cos \beta_{6}+2 \mathrm{a}_{2} \mathrm{O}_{2} \mathrm{P} \cos \left(\theta_{34}+\beta_{14}-\beta_{7}\right) \\
+\mathrm{F}_{\mathrm{K} 1} \times \mathrm{O}_{2} \mathrm{~T}_{1} \sin \left(\theta_{4}-\delta_{8}\right)
\end{array}\right]
$$

## 5. Differential Equations of Motion of the System

Applying the Lagrange-2 equation to establish the differential equations of motion of the system [10], the general form of the Lagrange-2 equation results in:

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{~T}}{\partial \dot{\theta}_{\mathrm{i}}}\right)-\frac{\partial \mathrm{T}}{\partial \theta_{\mathrm{i}}}+\frac{\partial \Phi}{\partial \dot{\theta}_{\mathrm{i}}}+\frac{\partial \Pi}{\partial \theta_{\mathrm{i}}}=\mathrm{Q}_{\mathrm{i}}(\mathrm{i}=\overline{1 \div 4}) \tag{5}
\end{equation*}
$$

in which : T- the total kinetic energy; $\Pi$ - the total potential energy; $\Phi$ - dissipated energy; $\mathrm{Q}_{\mathrm{i}}$ - the external force corresponding coordinate i

From (1), (2), (4), (5) we establish the system of differential equations of motion of the system consisting of the following four equations:

$$
\left[\begin{array}{llll|}
\mathrm{m}_{11} & \mathrm{~m}_{12} & \mathrm{~m}_{13} & \mathrm{~m}_{14}  \tag{6}\\
\mathrm{~m}_{21} & \mathrm{~m}_{22} & \mathrm{~m}_{23} & \mathrm{~m}_{24} \\
\mathrm{~m}_{31} & \mathrm{~m}_{32} & \mathrm{~m}_{33} & \mathrm{~m}_{34} \\
\mathrm{~m}_{41} & \mathrm{~m}_{42} & \mathrm{~m}_{43} & \mathrm{~m}_{44}
\end{array}\right]\left[\begin{array}{l}
\ddot{\theta}_{1} \\
\ddot{\theta}_{2} \\
\ddot{\theta}_{3} \\
\ddot{\theta}_{4}
\end{array}\right]=\left[\begin{array}{l}
\mathrm{F}_{1} \\
\mathrm{~F}_{2} \\
\mathrm{~F}_{3} \\
\mathrm{~F}_{4}
\end{array}\right]
$$

$\mathrm{m}_{11}=\mathrm{m}_{1}\left(\mathrm{~L}_{1}{ }^{2}+\mathrm{J}_{1}\right)+\mathrm{m}_{2}\left[\mathrm{~L}_{2}{ }^{2}+\mathrm{a}_{1}{ }^{2}+2 \mathrm{a}_{1} \mathrm{~L}_{2} \cos \left(\theta_{2}+\beta_{3}\right)\right]+\mathrm{m}_{3}\left(\mathrm{~L}_{3}{ }^{2}+\mathrm{a}_{3}{ }^{2}\right)+\ldots$
$+\mathrm{m}_{3}\left[\mathrm{a}_{1}{ }^{2}+2 \mathrm{a}_{1} \mathrm{a}_{3} \cos \theta_{2}+2 \mathrm{~L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)+2 \mathrm{~L}_{3} \mathrm{a}_{1} \cos \left(\theta_{23}+\beta_{4}\right)\right]+\ldots$
$+m_{4}\left[\begin{array}{l}L_{4}{ }^{2}+a_{2}{ }^{2}+a_{1}{ }^{2}+2 L_{4} a_{1} \cos \left(\theta_{24}-\beta_{15}\right)+\ldots \\ +2 L_{4} a_{2} \cos \left(\theta_{4}-\beta_{5}\right)+2 a_{1} a_{2} \cos \left(\theta_{2}-\beta_{1}\right)\end{array}\right]$
$\mathrm{m}_{12}=\mathrm{m}_{2}\left[\mathrm{~L}_{2}{ }^{2}+\mathrm{a}_{1} \mathrm{~L}_{2} \cos \left(\theta_{2}+\beta_{3}\right)\right]+\mathrm{m}_{3} \mathrm{a}_{1} \mathrm{a}_{3} \cos \theta_{2}+\mathrm{m}_{4}\left(\mathrm{~L}_{4}{ }^{2}+\mathrm{a}_{2}{ }^{2}\right)+\ldots$
$+\left[\mathrm{L}_{3}{ }^{2}+\mathrm{a}_{3}{ }^{2}+\mathrm{L}_{3} \mathrm{a}_{1} \cos \left(\theta_{23}+\beta_{4}\right)+2 \mathrm{~L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)\right]+\ldots$
$+\mathrm{m}_{4}\left[+\mathrm{a}_{1} \mathrm{a}_{2} \cos \left(\theta_{2}-\beta_{1}\right)+2 \mathrm{~L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)+\mathrm{L}_{4} \mathrm{a}_{1} \cos \left(\theta_{24}+\theta_{4}-\beta_{15}\right)\right]$
$\mathrm{m}_{13}=\mathrm{m}_{3}\left[\mathrm{~L}_{3}{ }^{2}+\mathrm{L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)+\mathrm{L}_{3} \mathrm{a}_{1} \cos \left(\theta_{23}+\beta_{4}\right)\right]$
$\mathrm{m}_{14}=\mathrm{m}_{4}\left[\mathrm{~L}_{4}{ }^{2}+\mathrm{L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)+\mathrm{L}_{4} \mathrm{a}_{1} \cos \left(\theta_{24}-\beta_{15}\right)\right]$
$\mathrm{m}_{22}=\frac{1}{2}\left(\mathrm{~m}_{2} \mathrm{~L}_{2}{ }^{2}+\mathrm{J}_{2}\right)+\mathrm{m}_{3}\left[\mathrm{~L}_{3}{ }^{2}+\mathrm{a}_{3}{ }^{2}+2 \mathrm{~L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)\right]+\ldots$
$+\mathrm{m}_{4}\left[\mathrm{~L}_{4}{ }^{2}+\mathrm{a}_{2}{ }^{2}+2 \mathrm{~L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)\right]$
$\mathrm{m}_{21}=\mathrm{m}_{2}\left[\mathrm{~L}_{2}{ }^{2}+\mathrm{a}_{1} \mathrm{~L}_{2} \cos \left(\theta_{2}+\beta_{3}\right)\right]+\mathrm{m}_{3} \mathrm{~L}_{3} \mathrm{a}_{1} \cos \left(\theta_{23}+\beta_{4}\right)+\mathrm{m}_{3} \mathrm{~L}_{3}{ }^{2}+\ldots$
$+\mathrm{m}_{3}\left[\mathrm{a}_{3}{ }^{2}+\mathrm{a}_{1} \mathrm{a}_{3} \cos \theta_{2}+2 \mathrm{~L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)\right]+\mathrm{m}_{4} \mathrm{~L}_{4} \mathrm{a}_{1} \cos \left(\theta_{24}-\beta_{15}\right)+\ldots$
$+\mathrm{m}_{4}\left[\mathrm{~L}_{4}{ }^{2}+\mathrm{a}_{2}{ }^{2}+\mathrm{a}_{1} \mathrm{a}_{2} \cos \left(\theta_{2}-\beta_{1}\right)+2 \mathrm{~L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)\right]$
$\mathrm{m}_{23}=\mathrm{m}_{3}\left[\mathrm{~L}_{3}{ }^{2}+\mathrm{L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)\right] ; \mathrm{m}_{24}=\mathrm{m}_{4}\left[\mathrm{~L}_{4}{ }^{2}+\mathrm{L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)\right]$
$\mathrm{m}_{31}=\mathrm{m}_{3}\left[\mathrm{~L}_{3}{ }^{2}+\mathrm{L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)+\mathrm{L}_{3} \mathrm{a}_{1} \cos \left(\theta_{23}+\beta_{4}\right)\right]$
$\mathrm{m}_{32}=\mathrm{m}_{3}\left[\mathrm{~L}_{3}{ }^{2}+\mathrm{L}_{3} \mathrm{a}_{3} \cos \left(\theta_{3}+\beta_{4}\right)\right] ; \mathrm{m}_{33}=\mathrm{m}_{3} \mathrm{~L}_{3}{ }^{2}+\mathrm{J}_{3} ; \mathrm{m}_{34}=0$
$m_{41}=m_{4}\left[L_{4}^{2}+L_{4} a_{2} \cos \left(\theta_{4}-\beta_{5}\right)+L_{4} a_{1} \cos \left(\theta_{24}+\theta_{4}-\beta_{15}\right)\right]$
$\mathrm{m}_{42}=\mathrm{m}_{4}\left[\mathrm{~L}_{4}{ }^{2}+\mathrm{L}_{4} \mathrm{a}_{2} \cos \left(\theta_{4}-\beta_{5}\right)\right] ; \mathrm{m}_{43}=0 ; \mathrm{m}_{44}=\mathrm{m}_{4} \mathrm{~L}_{4}{ }^{2}+\mathrm{J}_{4}$
$\mathrm{F}_{1}=\mathrm{Q}_{1}+\mathrm{m}_{3}\left[\mathrm{~L}_{3} \mathrm{a}_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3}+\mathrm{L}_{3} \mathrm{a}_{1} \sin \left(\theta_{23}+\beta_{4}\right) \dot{\theta}_{23}\right] \dot{\theta}_{3}+\ldots$
$+\mathrm{m}_{4}\left[\mathrm{~L}_{4} \mathrm{a}_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4}+\mathrm{L}_{4} \mathrm{a}_{1} \sin \left(\theta_{24}-\beta_{15}\right) \dot{\theta}_{24}\right] \dot{\theta}_{4}-\ldots$
$-\mathrm{m}_{1} \mathrm{~g} \mathrm{~L}_{1} \cos \left(\theta_{1}+\beta_{2}\right)-\mathrm{m}_{2} \mathrm{~g}\left[\mathrm{~L}_{2} \cos \left(\theta_{12}+\beta_{3}\right)+\mathrm{a}_{1} \cos \theta_{1}\right]-\ldots$
$-\mathrm{m}_{3} \mathrm{~g}\left[\mathrm{~L}_{3} \cos \left(\theta_{123}+\beta_{4}\right)+a_{3} \cos \theta_{12}+\mathrm{a}_{1} \cos \theta_{1}\right]-\ldots$
$-\mathrm{m}_{4} \mathrm{~g}\left[\mathrm{~L}_{4} \cos \left(\theta_{124}-\beta_{15}\right)+\mathrm{a}_{2} \cos \left(\theta_{12}-\beta_{1}\right)+\mathrm{a}_{1} \cos \theta_{1}\right]$
$+\left[\begin{array}{l}m_{2} a_{1} L_{2} \sin \left(\theta_{2}+\beta_{3}\right) \dot{\theta}_{2}+m_{4} L_{4} a_{1} \sin \left(\theta_{24}-\beta_{15}\right) \dot{\theta}_{24}+\ldots \\ +m_{4}\left[2 a_{1} a_{2} \sin \left(\theta_{2}-\beta_{1}\right) \dot{\theta}_{2}+2 L_{4} a_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4}\right]+\ldots \\ +m_{3}\left[2 a_{1} a_{3} \sin \theta_{2} \dot{\theta}_{2}+2 L_{3} a_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3}+\right]+\ldots \\ +2 m_{3} L_{3} a_{1} \sin \left(\theta_{23}+\beta_{4}\right) \dot{\theta}_{23}\end{array}\right]\left(2 \dot{\theta}_{1}+\dot{\theta}_{2}\right)$
$\mathrm{F}_{2}=\mathrm{Q}_{2}-\mathrm{m}_{2} \mathrm{a}_{1} \mathrm{~L}_{2} \sin \left(\theta_{2}+\beta_{3}\right) \dot{\theta}_{1}^{2}-\mathrm{m}_{4} \mathrm{~g}\left[\mathrm{~L}_{4} \cos \left(\theta_{124}-\beta_{15}\right)+\mathrm{a}_{2} \cos \left(\theta_{12}-\beta_{1}\right)\right]+\ldots$
$\left[\begin{array}{l}m_{3}\left[a_{1} a_{3} \sin \theta_{2} \dot{\theta}_{2}+2 L_{3} a_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3}+L_{3} a_{1} \sin \left(\theta_{23}+\beta_{4}\right) \dot{\theta}_{23}\right]+\ldots \\ +m_{4}\left[a_{1} a_{2} \sin \left(\theta_{2}-\beta_{1}\right) \dot{\theta}_{2}+2 L_{4} a_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4}+L_{4} a_{1} \sin \left(\theta_{24}-\beta_{15}\right) \dot{\theta}_{24}\right]\end{array}\right] \dot{\theta}_{1}+\ldots$
$+\mathrm{m}_{3} \mathrm{~L}_{3} \mathrm{a}_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3}\left(2 \dot{\theta}_{2}+\dot{\theta}_{3}\right)+\mathrm{m}_{4} \mathrm{~L}_{4} \mathrm{a}_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4}\left(2 \dot{\theta}_{2}+\dot{\theta}_{4}\right)+\ldots$
$-\mathrm{m}_{3}\left[\mathrm{a}_{1} \mathrm{a}_{3} \dot{\theta}_{12} \dot{\theta}_{1} \sin \theta_{2}+\mathrm{L}_{3} \mathrm{a}_{1} \dot{\theta}_{123} \dot{\theta}_{1} \sin \left(\theta_{23}+\beta_{4}\right)\right]+\ldots$
$-m_{4}\left[\mathrm{a}_{1} \mathrm{a}_{2} \dot{\theta}_{12} \dot{\theta}_{1} \sin \left(\theta_{2}-\beta_{1}\right)+\mathrm{L}_{4} \mathrm{a}_{1} \dot{\theta}_{124} \dot{\theta}_{1} \sin \left(\theta_{24}-\beta_{15}\right)\right]+\ldots$
$-\mathrm{m}_{2} \mathrm{~g} \mathrm{~L}_{2} \cos \left(\theta_{12}+\beta_{3}\right)-\mathrm{m}_{3} \mathrm{~g}\left[\mathrm{~L}_{3} \cos \left(\theta_{123}+\beta_{4}\right)+\mathrm{a}_{3} \cos \theta_{12}\right]+\ldots$
$\mathrm{F}_{3}=\mathrm{Q}_{3}+\mathrm{m}_{3}\left[\mathrm{~L}_{3} \mathrm{a}_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3}+\mathrm{L}_{3} \mathrm{a}_{1} \sin \left(\theta_{23}+\beta_{4}\right) \dot{\theta}_{23}\right] \dot{\theta}_{1}-\ldots$
$-m_{3} g L_{3} \cos \left(\theta_{123}+\beta_{4}\right)+m_{3} L_{3} a_{3} \sin \left(\theta_{3}+\beta_{4}\right) \dot{\theta}_{3} \dot{\theta}_{2}+\ldots$
$-m_{3}\left[L_{3} a_{3} \dot{\theta}_{123} \dot{\theta}_{12} \sin \left(\theta_{3}+\beta_{4}\right)+L_{3} a_{1} \dot{\theta}_{123} \dot{\theta}_{1} \sin \left(\theta_{23}+\beta_{4}\right)\right]$
$\mathrm{F}_{4}=\mathrm{Q}_{4}+\mathrm{m}_{4}\left[\mathrm{~L}_{4} \mathrm{a}_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4}+\mathrm{L}_{4} \mathrm{a}_{1} \sin \left(\theta_{24}-\beta_{15}\right) \dot{\theta}_{24}\right] \dot{\theta}_{1}+\ldots$
$+\mathrm{m}_{4} \mathrm{~L}_{4} \mathrm{a}_{2} \sin \left(\theta_{4}-\beta_{5}\right) \dot{\theta}_{4} \dot{\theta}_{2}-\mathrm{m}_{4} \mathrm{gL} \mathrm{L}_{4} \cos \left(\theta_{124}-\beta_{15}\right)-\ldots$
$-m_{4}\left[L_{4} \mathrm{a}_{2} \dot{\theta}_{124} \dot{\theta}_{12} \sin \left(\theta_{4}-\beta_{5}\right)+\mathrm{L}_{4} \mathrm{a}_{1} \dot{\theta}_{124} \dot{\theta}_{1} \sin \left(\theta_{24}-\beta_{15}\right)\right]$
By solving the system of equations (6), we can fully describe the motion of all stitches in the system.

## III. RESULTS AND DISCUSSION

We have simulated all the parts of the remotely controlled bomb disposal machine improved from the Komatsu PC 200-6 hydraulic excavator by using Autodesk Inventor software. In this way, we can get the geometric and structural parameters of
working equipments of this machine. Therefore, these are the parameters to solve the system of differential equations of the system. The value of all input parameters are given below:
$\mathrm{m}_{1}=1253 \mathrm{~kg} ; \mathrm{m}_{2}=618 \mathrm{~kg} ; \mathrm{m}_{3}=585 \mathrm{~kg} ; \mathrm{m}_{4}=60 \mathrm{~kg} ; \mathrm{m}_{5}=1260 \mathrm{~kg}$;
$\mathrm{J}_{1}=1.29 \times 10^{4} \mathrm{kgm}^{2} ; \mathrm{J}_{2}=8.2 \times 10^{2} \mathrm{kgm}^{2} ; \mathrm{J}_{3}=5.2 \times 10^{2} \mathrm{kgm}^{2}$;
$\mathrm{J}_{4}=29.4 \mathrm{kgm}^{2} ; \mathrm{L}_{1}=2.678 \mathrm{~m} ; \mathrm{L}_{2}=0.9 \mathrm{~m} ; \mathrm{L}_{3}=0.655 \mathrm{~m}$;
$\mathrm{L}_{4}=0.512 \mathrm{~m} ; \mathrm{a}_{1}=5.526 \mathrm{~m} ; \mathrm{a}_{2}=2.874 \mathrm{~m} ; \mathrm{a}_{3}=3.124 \mathrm{~m}$;
$\mathrm{a}_{4}=1.406 \mathrm{~m} ; \mathrm{a}_{5}=1.183 \mathrm{~m} ; \mathrm{OA}=0.828 \mathrm{~m} ; \mathrm{OB}=2.182 \mathrm{~m}$;
$\mathrm{OC}=2.77 \mathrm{~m} ; \mathrm{O}_{1} \mathrm{C}=3.212 \mathrm{~m} ; \mathrm{O}_{1} \mathrm{D}=0.825 \mathrm{~m} ; \mathrm{O}_{1} \mathrm{E}=0.82 \mathrm{~m}$;
$\mathrm{O}_{1} \mathrm{~K}=2.626 \mathrm{~m} ; \mathrm{O}_{1} \mathrm{H}=1.391 \mathrm{~m} ; \mathrm{O}_{2} \mathrm{H}=1.485 \mathrm{~m} ; \mathrm{O}_{2} \mathrm{P}=0.325 \mathrm{~m} ;$
$\mathrm{O}_{2} \mathrm{~T}_{1}=0.82 \mathrm{~m} ; \mathrm{O}_{3} \mathrm{~N}=0.45 \mathrm{~m} ; \mathrm{O}_{3} \mathrm{~K}=0.534 \mathrm{~m} ; \mathrm{O}_{3} \mathrm{~T}_{2}=0.712 \mathrm{~m}$;
$\mathrm{KM}=0.487 \mathrm{~m} ; \mathrm{MN}=0.60 \mathrm{~m} ; \mathrm{x}_{\mathrm{A}}=0.556 \mathrm{~m} ; \mathrm{y}_{\mathrm{A}}=0.613 \mathrm{~m}$;
$\beta_{1}=0.04 \mathrm{rad} ; \beta_{2}=0.29 \mathrm{rad} ; \beta_{3}=0.20 \mathrm{rad} ; \beta_{4}=0.72 \mathrm{rad} ; \mathrm{f}=0.78$
$\beta_{5}=0.04 \mathrm{rad} ; \beta_{6}=0.041 \mathrm{rad} ; \beta_{7}=0.54 \mathrm{rad} ; \beta_{7}=0.10 \mathrm{rad} ; \alpha_{1}=60^{\circ}$
$\beta_{9}=1.80 \mathrm{rad} ; \delta_{1}=0.37 \mathrm{rad} ; \delta_{2}=0.42 \mathrm{rad} ; \delta_{3}=2.86 \mathrm{rad} ;$
$\delta_{4}=0.74 \mathrm{rad} ; \delta_{5}=0.36 \mathrm{rad} ; \delta_{6}=0.83 \mathrm{rad} ; \delta_{8}=0.24 \mathrm{rad} ;$
By using Matlab software $[1,3,7]$ with initial condition: $\theta_{1}=0.52 \mathrm{rad} ; \theta_{2}=4.47 \mathrm{rad} ; \theta_{3}=6.16 \mathrm{rad} ; \theta_{4}=4.37 \mathrm{rad}$, $\dot{\theta}_{1}=\dot{\theta}_{2}=\dot{\theta}_{3}=\dot{\theta}_{4}=0$, we proceed to solve the system of differential equations of motion of the system during clamping bomb, resulting in the graphs of rotation angle, velocity, acceleration of all stitches. When the bomb diameter is $42 \mathrm{~cm}(\mathrm{~d}=42 \mathrm{~cm})$, we have the following diagrams:


Figure 3. Diagram of angular displacement of stitches when $\mathrm{d}=42 \mathrm{~cm}$


Figure 4. Diagram of angular velocity
of stitches when $\mathrm{d}=42 \mathrm{~cm}$


Figure 5. Diagram of angular acceleration

$$
\text { of stitches when } \mathrm{d}=42 \mathrm{~cm}
$$

From the survey results, we can see that the law of motion of all stitches is consistent with the law of actual movement of working equipment of remotely controlled bomb disposal machine. Boom and arm do not fluctuate during clamping process, so $\theta_{1}$ and $\theta_{2}$ have constant value, $\dot{\theta}_{1}$ and $\dot{\theta}_{2}$ are zero (Fig. 4). In Fig. 3, we can see that $\theta_{4}$ increases and $\theta_{3}$ decreases over time, because in fact the bomb clamp and bucket move in the opposite direction, approaching each other to clamp the bomb. Since the 17 th second, the angular velocity of the bomb clamp and bucket tends to slow down, their angular velocity tends to zero (Fig. 4). Since the 20th second, the bomb clamp and bucket stop moving, then both touch the bomb and they create a necessary clamping force enough to keep the bomb from detonation and falling during the the lift and movement of the bomb. At the end of the clamping process, we can determine the deviation $\Delta \theta=\theta_{4}-\theta_{3}=0.49 \mathrm{rad}$, this value corresponds to the arc of the circle that has a diameter of 0.42 m and touches with both of bomb clamp and bucket. The diameter of this circle is suitable for the diameter of the bomb.

Considering the whole process of clamping bomb, the deviation of the angular displacements of all stitches is: $\Delta \theta_{1}=0 \mathrm{rad}, \Delta \theta_{2}=0 \mathrm{rad}, \Delta \theta_{3}=6.16-5.15=1.01 \mathrm{rad}$ and $\Delta \theta_{4}=5.64-4.37=1.27 \mathrm{rad}$. In general, the value of angular velocity and angular acceleration of all stitches is very small (Fig. 4 and Fig. 5), this shows that the movement of all stitches is quite smooth. This is a necessary requirement when the remotely controlled bomb disposal machine interacts with the bomb.

One of the requirements for the remotely controlled bomb disposal machine is that it can grip bombs of different diameters. We also had a similar survey of the clamping process for a bomb with a smaller diameter. The survey results with the bomb which its diameter is 27 cm are shown in the following diagrams:


Figure 6. Diagram of angular acceleration of the stitches when $\mathrm{d}=27 \mathrm{~cm}$


Figure 7. Diagram of angular acceleration
of stitches when $\mathrm{d}=42 \mathrm{~cm}$
From the survey results in this case, we can see that the bomb clamp and bucket need more time to finish clamping bomb process (Fig. 6). At the end of this process, the deviation $\Delta \theta_{3}=6.16-5.09=1.07 \mathrm{rad}$ corresponds to $61.33^{\circ}$ and $\Delta \theta_{4}=5.7-4.37=1.33 \mathrm{rad}$ corresponds to $76.24^{\circ}$. Since the 24th second, the angular velocity of the bomb clamp and bucket are zero (Fig. 7) when they touch with bomb and stop moving. In two cases, we see that the timing of the clamping process is fast or slow depending on the diameter of bomb and the initial position of the bomb clamp and bucket, but the shape of diagrams is similar.

## IV. CONCLUSON

The paper presented a method to set up and solve the differential equations describing the movement of working equipment of remotely controlled bomb disposal machine improved from the Komatsu PC200-6 hydraulic excavator during clamping bomb process.

In particular, the research results are the basis for manufacturing the remotely controlled bomb disposal machine in reality. In a similar way, we can study to improve other types of hydraulic excavators into the remotely controlled bomb disposal machine to enhance adaptability to terrain such as: Komatsu PC120-6, Hyundai R140W -9S, Hyundai R60W9S, Hitachi ZX130-5G, ...

In the future, it will be also necessary to perform a survey of bombs, which might be picked up and moved by the bomb disposal machine suggested by this paper. Survey results are the basis for determining the reasonable structural and working parameters of the working equipment of the remotely controlled bomb disposal machine. This study will also create a database for remote control problems. In fact, the requirement is that the remotely controlled bomb disposal machine must be highly adaptable to the working terrain and it can dig and pick up other types of bombs with a maximum allowable diameter of 42 cm . Because each bomb has different permissible maximum clamping force, it is necessary to adjust clamping force accordingly, so the survey results are the basis for us to determine and adjust the maximum clamping force required for each different bomb to ensure safety during clamping and pick up them without detonating on the move.

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